Elasticities:

measure of how an endogenous var. changes w. exogenous var.

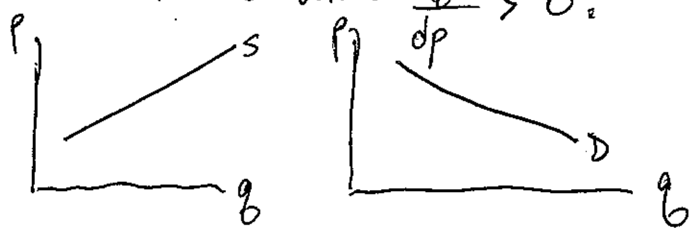
Question: When p changes by 1%, by how much does q change?

$$\frac{\%\Delta q}{\%\Delta p} = \frac{\Delta q / q}{\Delta p / p} = \frac{\Delta q}{\Delta p} \cdot \frac{p}{q}$$

$$\xrightarrow{\lim} \frac{dq}{dp} \cdot \frac{p}{q} \equiv \varepsilon_{q,p}$$

What would we generally expect sign of this to be for supply curves? Demand?

Supply: Positive ("Law of Supply")
Demand: Negative ("'Law' of Demand")
Not really a law. Giffen goods are those with $\frac{dq_s}{dp} > 0$.

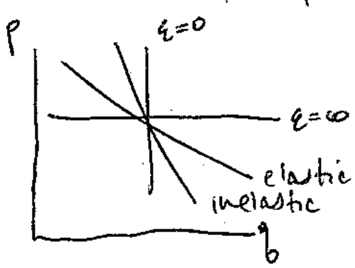


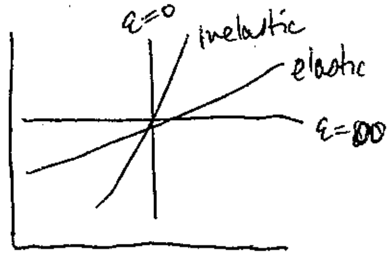
N.B. Economists often actually normalize elasticities to be positive (when sign is clear); i.e.,

$$\varepsilon_{q,p} \equiv \left| \frac{dq}{dp} \cdot \frac{p}{q} \right|$$

Different levels of elasticity

$\boxed{\varepsilon}$	$\boxed{\text{Nature of demand/supply/...}}$
0	Perfectly inelastic
(0,1)	Highly inelastic
1	Unit elastic
>1	Highly elastic
∞	Perfectly elastic





More inelastic S/D curves are "flatter" (ie, more horizontal)

Tables: Elasticities of demand (ie, "own-price demand elasticities")

2.1 • Addictive/necessary → <u>inelas</u>

• High fraction of expenditures → elastic

2.2, 2.3 • Good substitutes available → elastic

2.8 • long run → more elastic (<u>BUT</u>, as above, not always)

Another useful formulation

$$\frac{d[\log q]}{d[\log p]} = \underbrace{\frac{d \log q}{dp}} \cdot \frac{dp}{d \log p}$$

natural log

$$= \frac{d \log q}{dq} \cdot \frac{dq}{dp} \cdot \frac{1}{d \log p / dp}$$

$$= \frac{1}{q} \cdot \frac{dq}{dp} \cdot p = \frac{dq}{dp} \cdot \frac{p}{q} = \varepsilon_{q,p} \,!$$

$$\boxed{\text{Recall: } ① \ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \ ; \ ② \ \frac{d \log x}{dx} = \frac{1}{x} \ ; \ ②b \ \int \frac{dx}{x} = \log x + k}$$

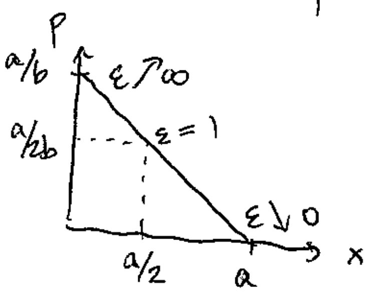
Consider examples

① Linear demand

$x = a - bp \longrightarrow$

$\varepsilon = \left| \frac{dx}{dp} \cdot \frac{p}{x} \right|$

$= \left| (-b)\left(\frac{p}{a-bp}\right) \right| = \frac{bp}{a-bp}$



Note at $p = \frac{a}{2b}$, $\varepsilon\left(p = \frac{a}{2b}\right) =$

$\frac{\cancel{b}\left(\frac{a}{2\cancel{b}}\right)}{a - \cancel{b}\left(\frac{a}{2\cancel{b}}\right)} = \frac{a/2}{a/2} = 1$!

② Constant elasticity demand

$x = ap^{-b} \longrightarrow$

$\varepsilon = \left| \frac{dx}{dp} \cdot \frac{p}{x} \right|$

$= \left| -\cancel{a}bp^{\cancel{-b-1}} \cdot \cancel{p} \cdot \frac{1}{\cancel{ap^{-b}}} \right|$

$= |-b| = b$

Where could I have come up with this crazy thing?

$\frac{dx}{dp} \cdot \frac{p}{x} = -b \Rightarrow \frac{dx}{x} = -b\,\frac{dp}{p} \Big) \int$

$\int \frac{dx}{x} = -b \int \frac{dp}{p}$

$$\int \frac{dx}{x} = -b \int \frac{dp}{p}$$

remember natural log

$$\log x = -b\left[\log p + k\right]$$

$$= \log p^{-b} - bk$$

$$x = \underbrace{e^{-bk}}_{\equiv a} p^{-b} = a p^{-b}$$

Could also get this using alternate formulation

$$\varepsilon_{x,p} = \left| \frac{d \log x}{d \log p} \right| \quad \text{since}$$

$$x = a p^{-b} \Leftrightarrow \log x = \log a - b \log p$$

$$\frac{d \log x}{d \log p} = -b$$

as before!

Elasticity and total expenditure

(can think similarly for revenue)

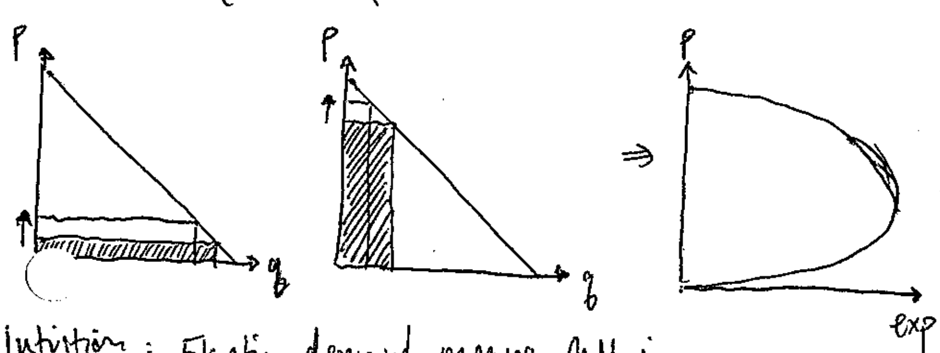
$$\text{expenditure} = \underset{p}{\text{price}} \cdot \underset{q(p)}{\text{quantity}}$$

$$\frac{d\,exp}{dp} = q(p) + \frac{dq}{dp} \cdot p$$

$$= q(p)\Big[1 + \underbrace{\frac{dq}{dp} \cdot \frac{p}{q(p)}}_{-\varepsilon_{q,p}}\Big]$$

$$= q(p)\left[1 - \varepsilon\right]$$

$$\begin{cases} > 0 & \text{if } \varepsilon < 1 \text{ (inelastic)} \\ = 0 & \text{if } \varepsilon = 1 \text{ (unit elastic)} \\ < 0 & \text{if } \varepsilon > 1 \text{ (elastic)} \end{cases}$$



Intuition: Elastic demand means fall in quantity outweighs increased price

This is not the only elasticity (e.g., of demand), since demand depends on things other than "own" price.

$q_a(p_a)$: Demand curve

$q_a(\underbrace{p_a, p_b, I, \ldots}_{\text{can define elasticities wrt any!}})$: Demand function

- own-price elasticity : $\dfrac{\partial x_a}{\partial p_a} \cdot \dfrac{p_a}{x_a}$ (2.7)
- cross-price elasticity : $\dfrac{\partial x_a}{\partial p_b} \cdot \dfrac{p_b}{x_a}$ (2.5, 2.6)

  >0 : substitutes

  <0 : complements
- income elasticity $\dfrac{\partial x_a}{\partial I} \cdot \dfrac{I}{x_a}$ (2.4)

  >0 : normal

  <0 : inferior