Electronic supplementary material to the paper by Lehmann and Feldman 2009 "Coevolution of adaptive technology, maladaptive culture, and population size in a producer-scrounger game". *Proc. R. Soc. Lond.*

There are three appendixes (A1 to A3) followed by four figures (S1 to S4) in this supplement.

A.1. Numerical results for unequal producing and scrounging times and fitness costs

In the sub-section "Adaptations without maladaptations" of the "Results" section in the main text, we presented analytical results with equal investment into producing and scrounging ($z_{\rm i} = z_{\rm s} = z$) and no cost to individual learning ($c_{\rm i} = 0$). Here, we relax these assumptions.

If we assume that the proportion of time spent producing adaptive technology is different from that spent scrounging it ($z_{\rm i} \neq z_{\rm s}$), the analysis becomes more complicated and we present only numerical results for this case. When $z_{\rm s} > z_{\rm i}$, we observe that $\hat{p}$ decreases as $z_{\rm s}$ increases while $z_{\rm i}$ is held constant (Fig. S1). But $\hat{N}$ and $\hat{A}$ behave in the opposite way with both increasing as $z_{\rm s}$ increases while $z_{\rm i}$ is held constant (Fig. S1). Consequently, increased scrounging has the potential to increase the average level of adaptations and population size (Fig. S2). This occurs because the more time that scroungers spend scrounging, the more they aggregate adaptations from different producers (see the second term on the right hand side of eq. 2.11 of the main text), thereby raising the average stock of adaptations in the population (which varies directly with $z_{\rm s}$). This, in turn, increases the amount of adaptive technology that both producers and scroungers inherit from the parental generation (eq. 2.10 and eq. 2.11 of the main text), and the rate of innovation when $I_{{\rm A},t} > 0$ (eq. 2.12). Hence, when $z_{\rm s} > z_{\rm i}$, scrounging increases carrying-capacity. By contrast, when $z_{\rm s} < z_{\rm i}$, we observe that $\hat{p}$ increases as $z_{\rm s}$ decreases while $z_{\rm i}$ is held constant, but the equilibrium values $\hat{N}$ and $\hat{A}$ are reduced instead of increased (Fig. S1).

The main effect of introducing a fitness cost to being a producer relative to being a scrounger ($c_{\mathrm{i}} > 0$) is that it decreases $\hat{p}$ (Fig. S3), which decreases the equilibrium stock of adaptive technology, and consequently $\hat{N}$. In contrast to the case where there is no cost to producers, the equilibrium values $\hat{N}$ and $\hat{A}$ will now be affected by introducing scroungers into the population even for $z_{\mathrm{s}} = z_{\mathrm{i}}$. The presence of scroungers actually decreases both $\hat{N}$ and $\hat{A}$, everything else being constant (Fig. S3). Hence, under these conditions when producers bear a fitness cost relative to scroungers, scrounging decreases the carrying-capacity.

A.2. Multiplicative effects on fitness

When fitness costs and benefits combine multiplicatively, but everything else is the same, eq. 2.2 and eq. 2.3 of the main text are replaced by

$$w_{\mathrm{i},t} = \frac{\alpha(1 + A_{\mathrm{i},t})(1 - z_{\mathrm{i}})(1 - c_{\mathrm{M}} M_{\mathrm{i},t})(1 - c_{\mathrm{i}})}{1 + \eta N_t}, \tag{A-1}$$

and

$$w_{\mathrm{s},t} = \frac{\alpha(1 + A_{\mathrm{s},t})(1 - z_{\mathrm{s}})(1 - c_{\mathrm{M}} M_{\mathrm{s},t})}{1 + \eta N_t}. \tag{A-2}$$

We checked with numerical analysis that with these fitness functions the critical qualitative results reported in the main text are not altered. That is, when $z_{\mathrm{i}} = z_{\mathrm{s}} = z$, $c_{\mathrm{i}} = 0$, $\varphi_{\mathrm{A}} = \varphi_{\mathrm{M}} = 0$, and $\epsilon_{\mathrm{A}} = \epsilon_{\mathrm{M}} = \epsilon$, scroungers do not affect the equilibrium population size, which is illustrated in the additive case by eqs. 3.6–3.9 of the main text, and that when these equalities are not satisfied, periodic cycling of the strategies and demographic variables occurs under a large set of parameters values with $\epsilon_{\mathrm{M}} > \epsilon_{\mathrm{A}}$ or $\varphi_{\mathrm{M}} > \varphi_{\mathrm{A}}$.

A.3. Producer-scrounger equilibrium for unequal transmission rates

When $\beta_{\mathrm{M}} \neq \beta_{\mathrm{T}}$, but everything else is the same, eqs. 3.6–3.9 are replaced by

$$\hat{N} = \frac{1}{\eta}\left(\frac{\alpha(1-z)\{\epsilon + z\mu(1-x)\}}{\epsilon + \alpha c_{\mathrm{M}} z\mu x} - 1\right), \tag{A-3}$$

$$\hat{A} = \frac{\hat{p}z\mu(1-x)(1+z\beta_{\mathrm{T}}(1-\hat{p})\hat{N})}{\epsilon}, \tag{A-4}$$

$$\hat{M} = \frac{\hat{p}z\mu x(1+z\beta_{\mathrm{M}}(1-\hat{p})\hat{N})}{\epsilon}, \tag{A-5}$$

and

$$\hat{p} = \frac{1}{z\hat{N}}\left(\frac{x(1+\eta\hat{N})c_{\mathrm{M}} - (1-x)(1-z)}{x(1+\eta\hat{N})c_{\mathrm{M}}\beta_{\mathrm{M}} - (1-x)(1-z)\beta_{\mathrm{T}}}\right). \tag{A-6}$$

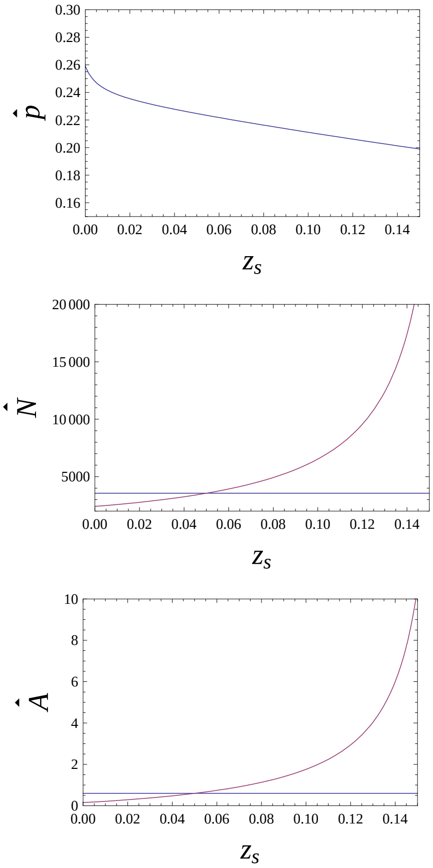

Figure S1: Equilibrium values $\hat{p}$, $\hat{N}$, $\hat{A}$ graphed as functions of the fraction z_s of time that scroungers spend scrounging, while holding constant the fraction z_i of time producers spend producing. Parameter values are the same as those in Fig. 1 of the main text except that z_s varies while $z_\mathrm{i} = 0.05$ is held constant. To gauge the effect on $\hat{N}$ and $\hat{A}$ of letting z_s vary, the horizontal lines in the second and third panel give the values of $\hat{N}$ and $\hat{A}$ for $z_\mathrm{i} = z_\mathrm{s} = 0.05$ (see Fig. 1). When $z_\mathrm{s} > z_\mathrm{i}$ the equilibrium values of both $\hat{N}$ and $\hat{A}$ increase, while for $z_\mathrm{s} < z_\mathrm{i}$ the equilibrium values of both $\hat{N}$ and $\hat{A}$ decrease.

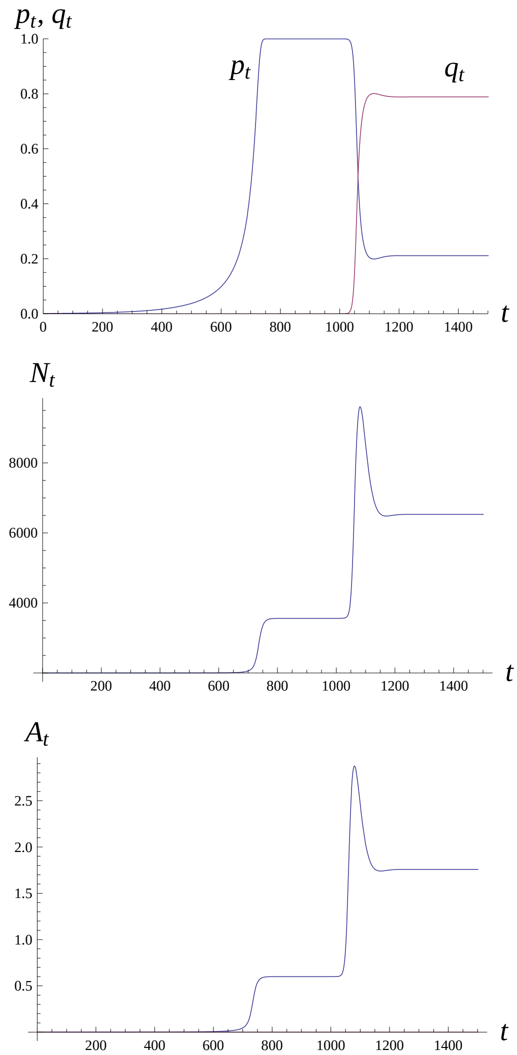

Figure S2: Dynamics of p_t, q_t, N_t and A_t when scroungers spend more time scrounging than producers do producing ($z_s > z_i$). Here $z_s = 0.1$ and $z_i = 0.05$ while the other parameter values are the same as those in Fig. 1 of the main text. As was the case in Fig. 1, producers first invade the population of innates, and go to quasi fixation, which is followed by population size and adaptive technology reaching a steady state. Scroungers subsequently invade the population of producers. However, after the invasion of scroungers, the equilibrium values $\hat{N}$ and $\hat{A}$ are now larger than they were before the invasion of scroungers. Scroungers bring an advantage to the population.

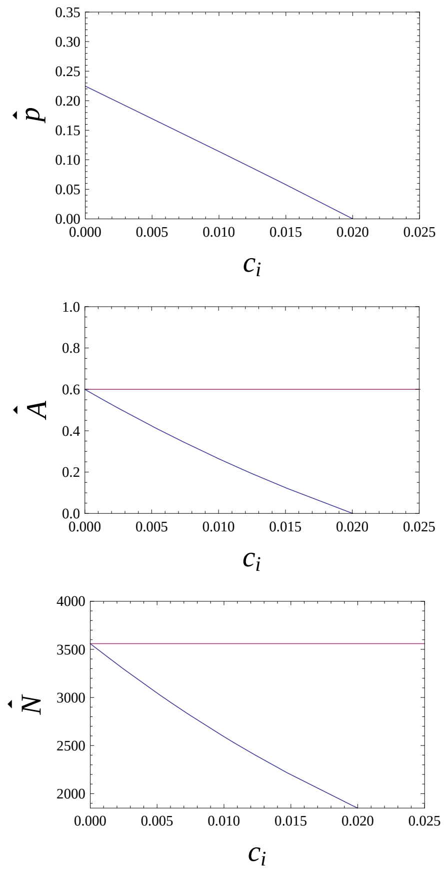

Figure S3: Equilibrium values $\hat{p}$, $\hat{N}$, $\hat{A}$ as functions of the fitness cost c_{i} to producers. Other parameter values are the same as those in Fig. 1 of the main text. The top curves in the second and third panels represent the situation where there are no scroungers in the population ($\hat{p} = 1$), and thus represent benchmarks against which the effect on carrying-capacity of including scroungers can be assessed. The lower curves in the second and third panels represents the situation where there are scroungers in the population, and show that in the presence of fitness costs to producers, adding scroungers decreases the values of both $\hat{N}$ and $\hat{A}$. This differs qualitatively feature from the case where there is no intrinsic cost to producers (eqs. 3.3–3.4 of the main text). We mention that for the value of the cost where the frequency of producers is zero, $\hat{N}$ is lower than in a population of innates, which stems from the fact that scroungers do no spend all their time in labor, and the population would thus be invaded by innates.

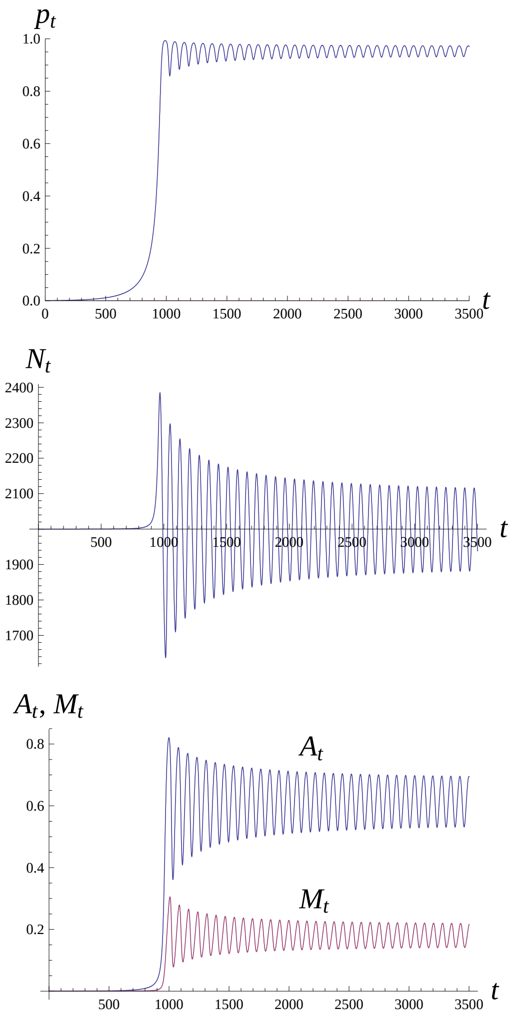

Figure S4: Dynamics of producers in the absence of scroungers ($q_0 = 0$); and N_t, A_t and M_t in the presence of maladaptations, $x = 0.1$, and cost to maladaptations, $c_{\mathrm{M}} = 1$. Parameter values are $\alpha = 3$, $\eta = 0.001$, $\epsilon_{\mathrm{A}} = 0.1$, $\epsilon_{\mathrm{M}} = 0.01$, $\mu = 2$, $\varphi_{\mathrm{A}} = 0$, $\varphi_{\mathrm{M}} = 0$, $z_{\mathrm{i}} = 0.05$, and $c_{\mathrm{i}} = 0$, otherwise the initial values are those given in Fig. 1 of the main text. Producers invade a population of innates. In so doing, they first increase the level of adaptive technology, and subsequently that of maladaptations. The dynamics of all variables then oscillate to finally settle in stable periodic cycles, which are reached regardless of the initial mixture of innates and producers in the population (no chaos was observed).