

Inflation inertia and credible disinflation[☆]

Guillermo Calvo^{a,b}, Oya Celasun^c, Michael Kumhof^{c,*}

^a Columbia University

^b NBER

^c IMF

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Abstract

We develop a model of optimizing forward-looking staggered price setting where even fully credible disinflations display a delayed and gradual inflation response and significant output losses. There is a welfare trade-off between these output losses and the gains from smaller inflationary distortions. For reasonable parameter values disinflation improves welfare, and more so if it is phased in gradually. The pricing assumption of our model yields dynamics that are similar to models of sticky information, but its state space is much simpler, thereby allowing for the application of standard linearization methods.

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1. Introduction

A large literature on monetary policy analysis uses optimizing dynamic general equilibrium models in which forward-looking agents face nominal rigidities. This literature typically builds on the original time-dependent price adjustment formulations of Taylor (1980), Rotemberg (1982), and Calvo (1983).¹ Whether the inflation specifications derived from such models can fully

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* Corresponding author. International Monetary Fund, Research Department, Modeling Division, 700 19th Street NW, Washington, DC 20431, United States. Tel.: +1 202 623 6769; fax: +1 202 623 6334.

E-mail address: mkumhof@imf.org (M. Kumhof).

¹ Comprehensive surveys can be found in Galí (2002) and Lane (2001).

account for the short-run empirical properties of inflation and output has recently been much debated.² Starting with [Fuhrer and Moore \(1995\)](#) and [Fuhrer \(1997\)](#), rational expectations staggered pricing models have been shown to display a much lower degree of structural inflation persistence than what is found in the data.³ The absence of structural persistence in these models in turn implies that monetary policy shocks have an immediate impact on inflation, whereas in the data the effects are delayed and gradual ([Mankiw, 2001](#); [Mankiw and Reis, 2002a](#)). An important implication of this rapid inflation response is that disinflationary policies are predicted to have minimal real costs, which is also inconsistent with a large body of empirical evidence ([Gordon, 1982, 1997](#); [Ball, 1994](#)).⁴

This paper proposes a tractable generalization of the [Calvo \(1983\)](#) staggered pricing model that generates inflation persistence and recessionary disinflations in a fully specified dynamic general equilibrium model where agents are forward looking and have rational expectations. Our model retains the commonly used [Calvo \(1983\)](#) assumption of an exogenous timing of price-changing opportunities. Where we depart from the existing literature is in the specification of price setting behavior. In the realistic case of a positive steady state inflation rate, we suggest that it is plausible to assume that firms employ pricing policies which keep them as close as possible to their flexible price optimum without incurring reoptimization costs. To keep the model tractable, we specifically assume that once a firm gets the chance to change its pricing policies, it jointly and optimally chooses an initial price level and a rate at which it will update its price in the future, a ‘firm-specific inflation rate’.⁵ This approach differs from two important approaches in the literature. In one ([Woodford, 2002](#)) firms choose only a price level without updating. At positive steady state inflation this has been shown to generate a monetary nonneutrality ([Rotemberg and Woodford, 1997](#)) where higher steady state inflation implies lower steady state output through increased price dispersion. This nonneutrality is removed in the second approach, starting with [Yun \(1996\)](#), where firms still choose only a price level but update their prices at the steady state inflation rate at all times.⁶ But under both of these approaches only the aggregate price level is sticky while inflation is flexible. Credible disinflation policies therefore do not cause recessions.

By contrast, when firms employ pricing rules of the kind proposed in this paper, an unexpected and permanent decline in the steady state inflation rate targeted by monetary policy entails a slow inflation response and output losses, even if the change in policy is perfectly credible. There are two main reasons for this. The first is the lingering effect of historic pricing decisions. The economy initially contains a large number of firms that have chosen their price updating rates under the previous policy, and the weighted average of such updating rates is an important component of

² See [Taylor \(1998\)](#) and [Clarida et al. \(1999\)](#) for a review of the empirical successes of this model class, and [Rudd and Whelan \(2006\)](#) for a critical view.

³ In particular, inflation in these models displays an unrealistically low degree of persistence unless marginal costs are persistent, whereas the empirical consensus is that even serially uncorrelated shocks to marginal costs have persistent effects on inflation.

⁴ Another criticism is that these models counterfactually predict that inflation should lead output. [Galí and Gertler \(1999\)](#) and [Galí et al. \(2001\)](#) argue, however that the microfounded versions of the models predict that inflation should lead real marginal costs—for which they provide supporting evidence—and that the dynamic correlation between inflation and output depends on the relationship between output and real marginal costs.

⁵ As our pricing rule does not take the simple form of setting new price levels, it is important to stress that our motivation for time-dependent nominal rigidities is not based on menu costs ([Akerlof and Yellen, 1985](#)) but instead on reoptimization costs, such as costs of information gathering, decision making, negotiation and communication. The empirical evidence presented by [Zbaracki et al. \(2004\)](#) emphasizes the importance of such reoptimization costs relative to menu costs. See also [Christiano et al. \(2005\)](#) on the reoptimization cost motivation.

⁶ Because we allow for updating, our model also removes the nonneutrality.

aggregate inflation. The second reason is the behavior of new price setters. The difference between firms' initially chosen price and the aggregate price level is the second component of aggregate inflation. But because firms have the option of spreading out their price adjustments over time, the initial price does not need to respond to a policy change as much as it does in a staggered pricing model, thereby contributing to the sluggishness of the inflation response. Finally, the slow inflation response gives rise to a recession.⁷ The transmission channels causing the recession differ depending on the monetary regime. In an open economy using the exchange rate to disinflate, the relative price of nontradable goods rises, causing a decline in the demand for nontradables and recession in that sector. In a closed economy with an interest rate rule, the real interest rate rises to reduce aggregate demand and bring down inflation.

The staggered pricing policies assumption also yields interesting results on the optimal speed of disinflation in the presence of a welfare trade-off between disinflationary output losses and the gains from smaller inflationary distortions to the consumption-leisure choice. In the open economy case we show that the optimal length of the disinflation period increases with the degree of price stickiness as measured by the average contract length. This is because a more gradual disinflation helps to reduce disinflationary output losses, and the latter is larger when prices are stickier. When prices are relatively flexible, a more rapid disinflation is preferable since gradualism lengthens the period during which agents face inflationary distortions.

The motivation for our work was first developed in Calvo et al. (2002), and is similar to that of Mankiw and Reis (2002a,b), who present a model where price setters are assumed to be able to reset their price every period but receive information only at random intervals.⁸ Their approach is equivalent to assuming that firms choose a (possibly nonlinear) price path based on the information they have when they get a random signal, and implies that aggregate inflation responds to monetary policy changes with a lag. Our model can be seen as a special version of the Mankiw and Reis model where firms set (log-) price paths with a constant slope, equal to the optimally selected updating rate. As we will show, this restriction is not critical in that the two models yield similar results with respect to aggregate inflation dynamics. But the benefit of imposing our restriction on pricing behavior is that the state space of the economy is dramatically simplified relative to models where firms set unconstrained price paths, thereby allowing for the application of standard linearization methods to solve for the equilibrium dynamics in a fully articulated microfounded model. This makes it straightforward to explore the model's quantitative predictions and their sensitivity to the values of structural parameters, and to conduct welfare analysis.⁹

⁷ Empirical tests of this model of pricing, based on Mexican data and reported in Calvo et al. (2002), are supportive of our inflation specification, suggesting that our formulation is a sensible way of capturing inflation inertia in countries with similar inflation behavior.

⁸ Burstein (in press) provides a microfounded state-dependent pricing model in which firms can set nonlinear price paths. Because of this nonlinearity, the model cannot be solved with conventional perturbation methods. Instead the paper focuses on the perfect foresight case and uses a nonlinear solution method. Devereux and Yetman (2003) study a related time-dependent pricing model. Buiter and Miller (1985) provide a continuous time pricing specification that shares some features with our model, but their pricing equations are not derived from microeconomic foundations.

⁹ In the Mankiw and Reis model firms can set nonlinear price paths, which allows them to respond to announced future disinflations at the time of the actual future reduction in money supply. This avoids the boom in real activity in advance of a disinflation which Ball (1995) shows to be an implication of the Calvo (1983) model. Since our model assumes firm-specific price paths to be linear, the possibility of small booms ahead of announced disinflations remains. Announced future disinflations are not widely observed in reality, however, and our model does generate recessions in response to more realistic gradual, anticipated disinflations, as will be shown in the open economy simulations.

The literature related to inflation inertia also encompasses models of imperfect credibility, learning, backward looking behavior, and real rigidities. A well-known explanation for inflation inertia during disinflations is lack of credibility, as in the papers by Ball (1995) and Calvo and Vegh (1993). However, in many countries where disinflations were costly the monetary authority enjoyed a high degree of credibility, as argued e.g. by Ball (1994). This is therefore only a suitable explanation for a limited number of cases. Models of learning about monetary policy have recently become popular, and clearly such models do give rise to inflation inertia. Two examples are Woodford (2002) and Erceg and Levin (2003). We see our model on structural inertia as complementary to models of learning. Until quite recently the literature mostly relied on pricing specifications that were not explicitly built on forward looking optimizing behavior. Fuhrer and Moore (1995) present a relative real-wage setting model, while Ghezzi (2001) and Clarida et al. (1999) modify the Calvo (1983) model to allow for a share of price setters to be backward looking, in the sense of using a rule of thumb that depends on lagged inflation. Christiano et al. (2005) generate inflation and output inertia in a rational expectations model by introducing a number of nominal and real rigidities, including backward looking price and wage indexation schemes. Rudd and Whelan (2005) argue that at least as far as price setting is concerned, backward looking specifications do not rest on strong microfoundations, and as such they are as open to the Lucas critique as the traditional econometric Phillips curve models they seek to replace. By generating inflation inertia in a world of rational, forward looking, and optimizing agents, our model goes at least some way in addressing this concern.

The pricing mechanism we propose is very general and can be embedded in a variety of macroeconomic models. The key features of inflation inertia and recessionary disinflations are robust across such environments. Our main interest in this paper is in a two-sector small open economy setting with a disinflationary monetary policy based on an exchange rate target path. The disinflations we have in mind are the moderate disinflations experienced by a number of countries in Latin America in recent years.¹⁰ As such the paper builds on earlier work on exchange rate based stabilizations in emerging markets surveyed in Calvo and Vegh (1999).¹¹ But we also briefly sketch how the same pricing block performs in a one-sector closed economy model where monetary policy targets inflation via an interest rate rule. This allows for a comparison of our results with the large closed economy literature on this topic, including an evaluation of the ability of our model to generate the key empirical regularities observed in industrialized economies.

The rest of the paper is organized as follows. Section 2 presents the pricing block of the model. Section 3 embeds this in a two-sector open economy model with an exchange rate targeting central bank. Section 4 discusses the solution paths of the open economy model for permanent and credible disinflations. This includes an analysis of the welfare trade-off between the output losses and efficiency gains of disinflation. Section 5 analyzes disinflation in the closed economy. Section 6 concludes.¹²

¹⁰ There are a number of moderate disinflations in (small and open) industrialized countries in the 1970s and 1980s to which this model could equally apply.

¹¹ Historically, many exchange rate based stabilizations of very high inflation have been characterized by an initial consumption boom followed by a later recession. Our model does not incorporate elements that could generate such a boom, such as lack of credibility or wealth effects. Our result that inflation inertia by itself leads to an initial recession is consistent with Calvo and Vegh (1994), who posit a wage setting equation that is not derived from optimization. Celasun (2006) shows that nontradables inflation was partially sticky during several exchange rate based disinflation experiments in the 1990s.

¹² A Technical Appendix with detailed derivations of all key equations is available from the authors upon request.

2. A model of staggered pricing policies

We assume that firms are distributed uniformly along the unit interval and have linear production functions

$$y_t(j) = \ell_t(j), \quad j \in [0, 1], \quad (1)$$

where $y_t(j)$ is output and $\ell_t(j)$ is labor input. Firms are price takers in the labor market and are monopolistically competitive in the goods market. They distribute all nominal profits $Z_t(j)$ to households in a lump-sum fashion:

$$Z_t(j) = P_t(j)y_t(j) - W_t\ell_t(j), \quad j \in [0, 1], \quad (2)$$

where $P_t(j)$ is the price of individual good j and W_t is the nominal wage.

We derive demand for variety j by noting that aggregate consumption c_t is given by a CES aggregate over varieties $c_t(j)$

$$c_t = \left(\int_0^1 c_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \quad (3)$$

with elasticity of substitution $\sigma > 1$. Household minimization of consumption expenditure implies the demand function for good j

$$c_t(j) = c_t \left(\frac{P_t(j)}{P_t} \right)^{-\sigma}, \quad (4)$$

where the aggregate price index P_t is given by:

$$P_t = \left(\int_0^1 P_t(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \quad (5)$$

As the model does not feature investment or government spending, goods market clearing for variety j requires $y_t(j) = c_t(j)$.

Following Calvo (1983), we assume that firms only get infrequent opportunities to change their pricing policies, and that these opportunities arrive as an exogenous random process. At each time t_0 and for each firm they follow an exponential distribution with probability density $\delta e^{-\delta(t-t_0)}$, and are therefore independent of their last occurrence. They are also independent across firms.

Firms maximize the expected present discounted value of real future profits each time they are allowed to change prices. The time t discount factor applied to time $s > t$ profits incorporates the intertemporal marginal rate of substitution of firms' owners, households. For the case of log preferences, which we use in both our open and closed economy applications, this equals $e^{-\int_t^s (\beta + \frac{\dot{c}_z}{c_z}) dz}$, where β is the rate of time preference. Furthermore, profits at time s are weighed by the probability $e^{-\delta(s-t)}$ that time t 's price will still be in force. The time t discount factor is therefore $e^{-\int_t^s (\beta + \frac{\dot{c}_z}{c_z} + \delta) dz}$. Firms' real marginal cost equals the real wage $w_t = W_t/P_t$, where W_t and P_t are taken as given. Crucially for this paper, whenever a firm j does receive a price-changing opportunity it determines an optimal price schedule, consisting of today's price level V_t^j and a firm-specific inflation rate v_t^j . If the price schedule of product variety j was last set at time t , we therefore have for all $s > t$ that

$$P_s(j) = V_t^j e^{v_t^j(s-t)}. \quad (6)$$

We will refer to this specification as staggered pricing *policies*. Firms maximize

$$\text{Max}_{v_t^j, v_t^j} \int_t^\infty e^{-\int_t^s (\delta + \beta + (\dot{c}_z/c_z)) dz} \left[\frac{V_t^j e^{v_t^j(s-t)}}{P_s} y_s(j) - w_s \ell_s(j) \right] ds. \quad (7)$$

subject to the production function (1), and subject to goods demand (4). Given goods market clearing and Eq. (6), goods demand can, for $s > t$, be written as

$$y_s(j) = c_s \left(\frac{V_t^j e^{v_t^j(s-t)}}{P_s} \right)^{-\sigma}. \quad (8)$$

Note that the maximization problem is identical for all firms that receive a price-changing opportunity, so that the firm index j can be dropped in stating the first-order conditions. We define the new variable $p_t \equiv V_t/P_t$, the initial relative price of new price setters. Note also that, for $s > t$, $P_s = P_t e^{\int_t^s \pi_z dz}$, where $\pi_t = \dot{P}_t/P_t$. Then we have the following first-order conditions for V_t and v_t :

$$\int_t^\infty e^{-\int_t^s (\delta + \beta + (\dot{c}_z/c_z)) dz} c_s \left(p_t e^{-\int_t^s (\pi_z - v_t) dz} \right)^{-\sigma} \left[p_t e^{-\int_t^s (\pi_z - v_t) dz} - \mu w_s \right] ds = 0, \quad (9)$$

$$\int_t^\infty e^{-\int_t^s (\delta + \beta + (\dot{c}_z/c_z)) dz} c_s \left(p_t e^{-\int_t^s (\pi_z - v_t) dz} \right)^{-\sigma} (s - t) \left[p_t e^{-\int_t^s (\pi_z - v_t) dz} - \mu w_s \right] ds = 0, \quad (10)$$

where $\mu \equiv \frac{\sigma}{\sigma-1}$. Next we linearize Eqs. (9) and (10) around the steady state,¹³ take derivatives with respect to time, and combine the resulting expressions. For any variable x , steady-state values will be denoted by \bar{x} and deviations from steady state by $\hat{x}_t = x_t - \bar{x}$. We have $\bar{\pi} = \bar{v}$, $\bar{p} = 1$, and $\bar{w} = \frac{1}{\mu}$. We obtain the following intermediate result

$$\dot{v}_t = -(\delta + \beta)^2 (\hat{p}_t - \mu \hat{w}_t), \quad (11)$$

where v_t is a jump variable. When there is a discrete change in the monetary policy regime it will be optimal for firms receiving a price-changing signal to allow discrete changes in both their current price and their firm-specific inflation rate. To complete the description of price dynamics we now turn to the aggregate price index. When firms set prices in the manner specified above, the index (5) can be rewritten as

$$P_t = \left(\delta \int_{-\infty}^t e^{-\delta(t-s)} (V_s e^{v_s(t-s)})^{1-\sigma} ds \right)^{\frac{1}{1-\sigma}}. \quad (12)$$

To obtain an expression for the aggregate inflation rate we first take the derivative of Eq. (12) with respect to time, and then we linearize the resulting expression around the steady state. We obtain

$$\hat{\pi}_t = \delta \hat{p}_t + \hat{\psi}_t, \quad (13)$$

¹³ All equations are linearized rather than log-linearized.

where

$$\hat{\psi}_t = \delta \int_{-\infty}^t e^{-\delta(t-s)} \hat{v}_s ds. \quad (14)$$

The variable $\hat{\psi}$ is the weighted average of currently ‘active’ firm-specific inflation rates, and it is predetermined. Eq. (13) is key to understanding inflation dynamics under staggered pricing policies. The first term on the right-hand side \hat{p} is the initial relative price of new price setters, and it is free to jump at time 0. But the second term $\hat{\psi}$ introduces a predetermined variable into aggregate inflation dynamics. It reflects inertia in the aggregate inflation rate through the historic pricing policies of firms which have not yet received a price-changing opportunity. This ability to spread out price adjustments over time in turn makes it unnecessary (and in fact very costly) to choose a large jump in \hat{p} at time 0.

Using the foregoing we can derive the system of differential equations characterizing the pricing block:

$$\dot{\psi}_t = \delta \hat{v}_t - \delta \hat{\psi}_t, \quad (15)$$

$$\dot{\pi}_t = 2\delta \hat{v}_t - (3\delta + 2\beta)\hat{\psi}_t + (\delta + 2\beta)\hat{\pi}_t - 2\delta(\delta + \beta)\mu \hat{w}_t, \quad (16)$$

$$\dot{v}_t = \frac{(\delta + \beta)^2}{\delta} \hat{\psi}_t - \frac{(\delta + \beta)^2}{\delta} \hat{\pi}_t + (\delta + \beta)^2 \mu \hat{w}_t. \quad (17)$$

We wish to contrast our model of staggered pricing policies with a more conventional Calvo (1983) formulation, which assumes that price setters choose only their initial price level. Rather than allowing firms to choose their updating rate also, this is typically augmented by the Yun (1996) assumption that firms update their prices at the steady state inflation rate. Under these assumptions one obtains the equivalent of Eq. (13) as

$$\hat{\pi}_t = \delta \hat{p}_t, \quad (18)$$

meaning that inflation dynamics around the steady state is driven exclusively by the initial price levels chosen by current price setters. Furthermore, one obtains the following New Keynesian Phillips Curve:

$$\dot{\pi}_t = \beta \hat{\pi}_t - \delta(\delta + \beta)\mu \hat{w}_t. \quad (19)$$

To fully characterize the dynamic behavior of this economy, the system of differential Eqs. (15), (16) and (17) in ψ , π and v , or alternatively Eq. (19) for the Calvo–Yun case, must be closed with an equation that determines the evolution of real marginal cost w . To do so we must first describe the demand side and government behavior, and define equilibrium. We do so separately for the open economy case and the closed economy case.

3. Aggregate demand and equilibrium in the open economy

We consider a small open economy which consists of a continuum of identical price-taking infinitely-lived households, a continuum, indexed by $j \in [0,1]$, of monopolistically competitive, infinitely-lived nontradable-goods producing firms that set prices as described in Section 2, and a

government that targets the path of the nominal exchange rate.¹⁴ The economy trades goods with the rest of the world and purchasing power parity is assumed to hold in the tradables sector. Normalizing the foreign price level to one, this implies that the nominal price of tradables equals the nominal exchange rate E_t . The nominal price level of nontradable goods is denoted by P_t and is given by Eq. (5), with associated inflation rate π_t . The relative price of tradables and nontradables, which will be referred to as the real exchange rate, is $e_t = E_t/P_t$. The economy can also freely borrow from or lend to the rest of the world, and uncovered interest parity is assumed to hold:

$$i_t = r + \varepsilon_t. \quad (20)$$

Here r is the exogenous, constant and positive real interest rate on international bonds b_t denominated in units of tradable goods, $\varepsilon_t = \dot{E}_t/E_t$ is the rate of exchange rate depreciation, and i_t is the nominal interest rate on domestic currency denominated bonds.

3.1. Households

Households maximize lifetime utility, which depends on their consumption of homogenous tradable goods c_t^* , heterogeneous nontradable goods c_t as defined by Eq. (3), and utility from leisure $1 - \ell_t$, where 1 is the fixed endowment of time and ℓ_t is total labor supply to firms. To rule out inessential dynamics we assume that households' personal discount rate β satisfies $\beta = r$. The representative household's objective function is:

$$\text{Max} \int_0^{\infty} [\gamma \ln(c_t^*) + (1 - \gamma) \ln(c_t) + \kappa \ln(1 - \ell_t)] e^{-rt} dt. \quad (21)$$

Households are subject to a cash in advance constraint for their purchases of tradables and nontradables:

$$m_t \geq \alpha \left(c_t^* + \frac{c_t}{e_t} \right), \quad (22)$$

where $m_t (M_t)$ are real (nominal) money balances, with $m_t = M_t/E_t$, and α is constant inverse velocity. The opportunity cost of holding one unit of money is equal to the nominal interest rate, which given our assumption of predetermined positive exchange rate depreciation (see below) and uncovered interest parity must be greater than zero. The cash-in-advance constraint will therefore be binding at all times. Households receive a fixed endowment of tradable goods y^* and government lump-sum transfers in terms of tradables τ_t . From firms they receive nominal wages $W_t \ell_t$ and nominal lump-sum profit distributions $\int_0^1 Z_t(j) dj$. Their flow budget constraint is

$$\dot{b}_t = r b_t - \dot{m}_t - \varepsilon_t m_t + y^* - c_t^* + \tau_t + \frac{W_t \ell_t}{E_t} + \frac{\int_0^1 Z_t(j) dj}{E_t} - \frac{\int_0^1 P_t(j) c_t(j) dj}{E_t}. \quad (23)$$

¹⁴ We take the specification of monetary policy as a given. It is well known that exchange rate targeting is not necessarily an optimal policy for a small open economy. For a recent comprehensive treatment of this issue, see [Devereux et al. \(in press\)](#).

After imposing the no-Ponzi condition $\lim_{t \rightarrow \infty} (b_t + m_t)e^{-rt} \geq 0$, we can write consumers' lifetime budget constraint as:

$$b_0 + m_0 + \int_0^{\infty} \left(y^* + \tau_t + \frac{W_t \ell_t}{E_t} + \frac{\int_0^1 Z_t(j) dj}{E_t} \right) e^{-rt} dt \geq \int_0^{\infty} \left(c_t^* + \frac{\int_0^1 P_t(j) c_t(j) dj}{E_t} + i_t m_t \right) e^{-rt} dt. \quad (24)$$

The representative household maximizes Eq. (21) subject to Eqs. (22) and (24), with Eq. (22) binding. The first-order conditions are Eq. (4), Eq. (24) holding with equality, and

$$\frac{\gamma}{c_t^*} = \lambda(1 + \alpha i_t), \quad (25)$$

$$\frac{c_t}{c_t^*} = e_t \frac{1 - \gamma}{\gamma}, \quad (26)$$

$$w_t = \frac{\kappa c_t (1 + \alpha i_t)}{(1 - \ell_t)(1 - \gamma)}, \quad (27)$$

where $w_t = W_t/P_t$ is the real wage in terms of nontradables, and λ is the constant multiplier of the lifetime budget constraint (24), equal to the shadow value of lifetime wealth. Eq. (25) states that the marginal utility of tradables consumption equals the marginal utility of wealth times the effective price of consumption, given by the purchase price plus the cost of holding the money balances necessary to conduct transactions. Eq. (26) states that the marginal rate of substitution between tradables and nontradables equals their relative price, the real exchange rate. Eq. (27) states that the real wage equals the marginal rate of substitution between consumption and leisure, corrected for a monetary distortion that is increasing in the nominal interest rate. This reflects the negative effect of deviations from the Friedman rule on steady state output.

3.2. Government

The government owns a stock of net foreign assets h_t , issues nominal money balances M_t , and makes lump-sum transfers τ_t . Its flow budget constraint is

$$\dot{h}_t = r h_t + \dot{m}_t + \varepsilon_t m_t - \tau_t. \quad (28)$$

By imposing the transversality condition $\lim_{t \rightarrow \infty} (h_t - m_t)e^{-rt} = 0$ one obtains the government's lifetime constraint:

$$h_0 - m_0 + \int_0^{\infty} (i_t m_t - \tau_t) e^{-rt} dt = 0. \quad (29)$$

A *government policy* is defined as a list of time paths $\{E_t, \tau_t\}_{t=0}^{\infty}$ such that, given the time paths $\{M_t, i_t\}_{t=0}^{\infty}$, the constraint (29) holds. Fiscal policy $\{\tau_t\}_{t=0}^{\infty}$ is assumed to be Ricardian. As for exchange rate policy $\{E_t\}_{t=0}^{\infty}$, we assume that the government reduces inflation by a surprise announcement at time 0 of a reduction in the rate of exchange rate depreciation from its initial level ε^h . We consider two variants of this policy. The first is an instantaneous, permanent and

credible disinflation to ε^l . The second is a gradual disinflation with $\varepsilon_t = \varepsilon^h - \eta t$ for $t < (\varepsilon^h - \varepsilon^l) / \eta$ and $\varepsilon_t = \varepsilon^l$ for $t \geq (\varepsilon^h - \varepsilon^l) / \eta$.

3.3. Equilibrium

The list of time paths $\{b_t, h_t, m_t, c_t^*, y_t^*, \ell_t, c_t, y_t, \ell_t(j), c_t(j), y_t(j), j \in [0, 1]\}_{t=0}^\infty$ is an allocation. A price system is a list of time paths $\{i_t, P_t, W_t, P_t(j), V_t^j, v_t^j, j \in [0, 1]\}_{t=0}^\infty$. Finally let $f_t = b_t + h_t$, the economy's overall level of net foreign assets. Then equilibrium is defined as follows:

A perfect foresight equilibrium given f_0 is an allocation, a price system, and a government policy such that

- (a) given the government policy and the price system, the allocation solves the household's problem of maximizing Eq. (21) subject to Eqs. (22) and (24), with Eq. (22) binding,
- (b) given the government policy, the restrictions on price setting, and the sequences $\{P_t, W_t, c_t\}_{t=0}^\infty$, the sequences $\{V_t^j, v_t^j, y_t(j), \ell_t(j), j \in [0, 1]\}_{t=0}^\infty$ solve firms' problem of maximizing Eq. (7) subject to Eqs. (1) and (8),
- (c) the nontradable goods market clears for all goods and at all times,

$$y_t(j) = c_t(j) \quad \forall t, \forall j \in [0, 1], \tag{30}$$

- (d) the labor market clears at all times,

$$\ell_t = \int_0^1 \ell_t(j) dj \quad \forall t. \tag{31}$$

Eqs. (29), (24) holding with equality, and the definition of equilibrium imply that the following aggregate budget constraint must hold:

$$f_0 + \frac{y^*}{r} = \int_0^\infty c_t^* e^{-rt} dt. \tag{32}$$

Using this constraint with the first-order condition (25), the equilibrium paths of tradables consumption and therefore of net foreign assets can be computed independently from the rest of the economy because they are functions only of endowments (f_0 and y^*) and of exogenous world and policy variables (r and ε_t).

3.4. Complete dynamic system

Using the above equilibrium conditions we can derive a differential equation for real marginal cost w . To do so we linearize and differentiate Eqs. (25), (26) and (27). Letting $\hat{\varepsilon}_t = \varepsilon_t - \varepsilon^l$, we obtain

$$\dot{w}_t = -\frac{\hat{\pi}_t}{\mu(1-\bar{\ell})} + \frac{\hat{\varepsilon}_t}{\mu(1-\bar{\ell})} - \frac{\bar{\ell}}{\mu(1-\bar{\ell})} \frac{\alpha}{1+\alpha\bar{v}} \dot{\varepsilon}_t. \tag{33}$$

This equation says that a real appreciation, in other words a rising relative price of nontradables, is associated with a fall in real wages in the nontradables sector. The reason is that the higher relative price lowers the demand for nontradables output and therefore for labor, which

depresses the real wage. The second and third terms on the right-hand side, which we denote by $g(t)$, depend on the form of the exogenous disinflation profile chosen by the government.

We now show that w_t is a predetermined variable. Consider the possibility of jumps at time 0. Eq. (27) has two components driving w_t , nontradables demand c_t (equal to labor supply ℓ_t to a first-order approximation) and exchange rate depreciation ε_t . Nontradables demand is determined by Eq. (26). In that equation the real exchange rate is predetermined because nominal exchange rates are exogenous and predetermined and nontradables prices are sticky, while the tradables consumption path is a function only of exogenous lifetime resources and of government policies. This makes c_t a predetermined variable.¹⁵ Jumps in ε_t are also exogenous. Therefore w_t is a predetermined variable, along with ψ_t . The variables v_t and π_t are free to jump at time 0. The full dynamic system for this economy is represented by Eqs. (15–17), and (33). In matrix notation it takes the following form:

$$\begin{bmatrix} \dot{\psi}_t \\ \dot{v}_t \\ \dot{\pi}_t \\ \dot{w}_t \end{bmatrix} = \begin{bmatrix} -\delta & \delta & 0 & 0 \\ \frac{(\delta + \beta)^2}{(\delta + \beta)^2} & 0 & -\frac{(\delta + \beta)^2}{\delta} & (\delta + \beta)^2 \mu \\ -(3\delta + 2\beta) & 2\delta & (\delta + 2\beta) & -2\delta(\delta + \beta)\mu \\ 0 & 0 & -\frac{1}{\mu(1 - \bar{\ell})} & 0 \end{bmatrix} \begin{bmatrix} (\psi_t - \varepsilon^l) \\ (v_t - \varepsilon^l) \\ (\pi_t - \varepsilon^l) \\ (w_t - \bar{w}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ g(t) \end{bmatrix}. \quad (34)$$

In the Appendix we prove that this system has two eigenvalues with positive real parts and two with negative real parts. Given our results about the number of predetermined variables this proves that the system is saddle path stable. It can further be shown numerically that for a very large range of relevant parameter values¹⁶ all roots are real.

If Calvo–Yun pricing behavior is assumed instead, the dynamic system takes the form:

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{w}_t \end{bmatrix} = \begin{bmatrix} \beta & -\delta(\delta + \beta)\mu \\ -\frac{1}{\mu(1 - \bar{\ell})} & 0 \end{bmatrix} \begin{bmatrix} (\pi_t - \varepsilon^l) \\ (w_t - \bar{w}) \end{bmatrix} + \begin{bmatrix} 0 \\ g(t) \end{bmatrix}. \quad (35)$$

This system can also be shown to be saddle path stable.

4. Model solution and discussion

4.1. Calibration

We proceed to assign parameter values and compute solution paths for the model.¹⁷ The time unit for calibration of stock-flow ratios is one quarter. We consider a relatively moderate disinflation from 20% to 10%, which is of the order of magnitude of several recent Latin American disinflations. The parameters α and r are calibrated based on sample averages for Brazil between the beginning of 1995 and the end of 1998, corresponding to the main period of that country's exchange rate based disinflation plan. Inverse velocity α is set equal to the average of the M2/GDP

¹⁵ See Ghezzi (2001) and Calvo and Vegh (1994) for similar arguments.

¹⁶ We searched over $\delta \in [0.05, 2]$, leaving all other values at those used below in the calibration.

¹⁷ The parameter values are not necessarily chosen to match a particular disinflation episode, but rather to provide a realistic qualitative and quantitative illustration of the predictions of the model.

ratio, giving $\alpha = 1.12$. The real marginal cost of Brazil's borrowing in international capital markets is given by the nominal Brady bond yield adjusted for US inflation, which equals 12.7% p.a. The log-linear specification of the utility index implies an intertemporal elasticity of substitution of one. Empirical estimates of this elasticity are typically below one, as in [Reinhart and Vegh \(1995\)](#). However, see [Ogaki and Reinhart \(1998\)](#) and [Eckstein and Leiderman \(1992\)](#) for examples of estimates closer to one. A 50% share of tradables in consumption γ is empirically reasonable for most countries, see [De Gregorio et al. \(1994\)](#). The value for the proportion of time spent working in the pre-disinflation steady state is $1/3$, based on the evidence cited in [Cooley and Prescott \(1995\)](#). The elasticity of substitution between goods varieties is calibrated in line with much of the literature at $\sigma = 6$, implying a steady state markup of 20%. Finally we calibrate $\delta = 1/4$, which implies an average duration of pricing policies of four quarters. Note that in our model firms *change* prices every quarter but only *reoptimize* their pricing policies more infrequently. As such the model is not inconsistent with the recent empirical evidence for price *changes* of [Bils and Klenow \(2004\)](#), [Klenow and Kryvtsov \(2005\)](#), and [Goloso and Lucas \(2003\)](#), which points to an average frequency of price changes (in the U.S.) of once every 1.5 quarters for consumer prices. The assumption of quarterly price changes and less frequent reoptimization is shared with models using [Yun \(1996\)](#)-style updating or [Christiano et al. \(2005\)](#)-style backward indexation. In that literature, the assumption that prices are *reoptimized* on average every four quarters is conventional, and supported by estimation results such as in [Altig et al. \(2004\)](#). The impulse responses shown below are in terms of percent deviations from the initial steady state.

4.2. Dynamic response to disinflation

[Fig. 1](#) shows equilibrium paths for an unanticipated, instantaneous and permanent disinflation from 20% p.a. to 10% p.a. The solid line represents the case of staggered pricing policies while the dotted line shows the impulse responses for a Calvo–Yun model. Due to the permanent nature of the disinflation there is no intertemporal substitution in tradables consumption, which remains flat throughout. But in the nontradables sector the removal of a substantial inflationary distortion eventually leads to an almost 2% output expansion. The conventional wisdom is that this long-term gain has a short-term cost in that a recession is necessary to bring down inflation. The conventional wisdom is however not evident in the results of the Calvo–Yun model, where a recession is not necessary to bring down inflation. In fact, nontradables inflation immediately drops even below its long-run value so as to depreciate the real exchange rate and to allow nontradables output to start expanding towards its new and higher steady state level. By contrast, in our model aggregate inflation does not immediately jump close to its new lower steady state level; instead, the adjustment in inflation is delayed and gradual. As discussed before, this is due to the lingering influence of historic pricing decisions on current aggregate inflation, combined with firms' ability to avoid large up-front jumps in their price level when they can spread their price adjustments out over time. The profit maximizing choice at the outset of the disinflation is a strong downward adjustment of firm-specific inflation rates, which is due to the permanent and credible nature of the disinflation, combined with a reduction in the initial price by a relatively modest 3%–5%. The combination of these effects brings down aggregate inflation, but only at a rate fast enough to reach 10% after a period of three to four quarters. During this transition period the nontradables price level therefore rises at a faster rate than the tradables price, the real exchange rate appreciates, and demand for nontradables contracts. The output losses reach 2% at their lowest point, at which time nontradables inflation starts to undershoot exchange rate depreciation, thereby starting to depreciate the real exchange rate to its new equilibrium level and ending the

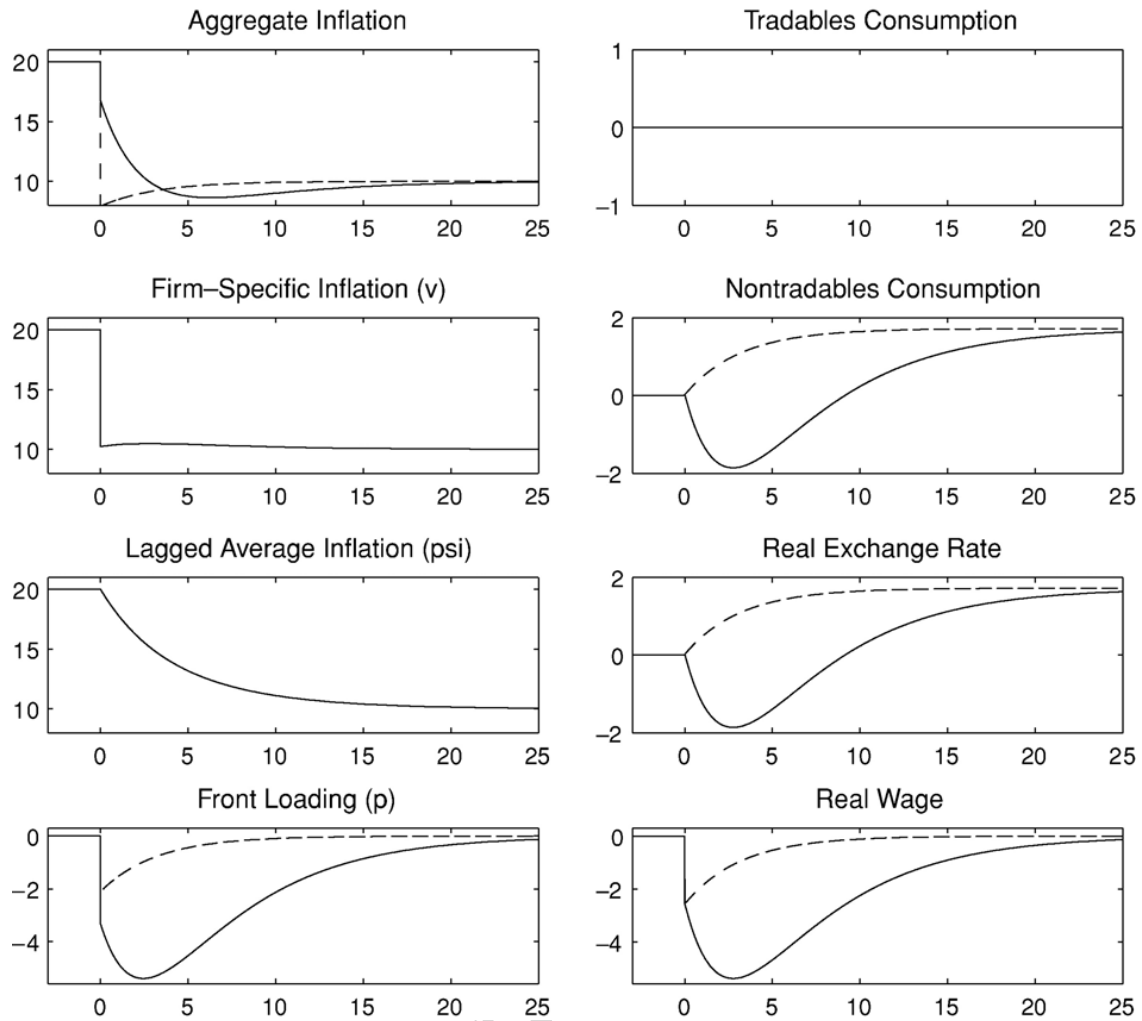


Fig. 1. Instantaneous disinflation in the open economy.

recession.¹⁸ The nontradables recession is fairly long-lived at around a two-year duration. The profile of aggregate inflation exhibits both a delayed and gradual initial response to the monetary policy shock and persistent deviations from steady state inflation following the shock.

The foregoing raises an interesting question. An instantaneous disinflation represents a ‘shock treatment’ that puts a premium on reaching the new, higher output steady state in the shortest time possible. But this forces the real exchange rate to appreciate very strongly and causes a deep recession. A more gradual disinflation, by allowing nontradables inflation to keep pace with tradables inflation, could avoid this recession, albeit at the cost of reaching the new steady state later, and at the cost of inducing intertemporal substitution in consumption. The latter arises because a gradual disinflation, by Eq. (25), leads to an upward-sloping profile in tradables consumption. Given the predetermined relative price of tradables and nontradables, this will also affect the profile of nontradables consumption especially at the outset of the disinflation program.

¹⁸ The decline in nontradables consumption is more rapid than the decline in inflation, hence the deepest point of the recession precedes the trough in inflation. This suggests that output movements lead those of inflation in response to the disinflationary policy. We note that satisfactorily assessing the dynamic correlation between inflation and output is beyond the scope of this paper, as doing so requires a fully specified stochastic DSGE model with staggered pricing policies. For the closed economy case this is done by Juillard et al. (in press), who subject such a model to Bayesian econometric analysis and show that the model does well in matching the data.

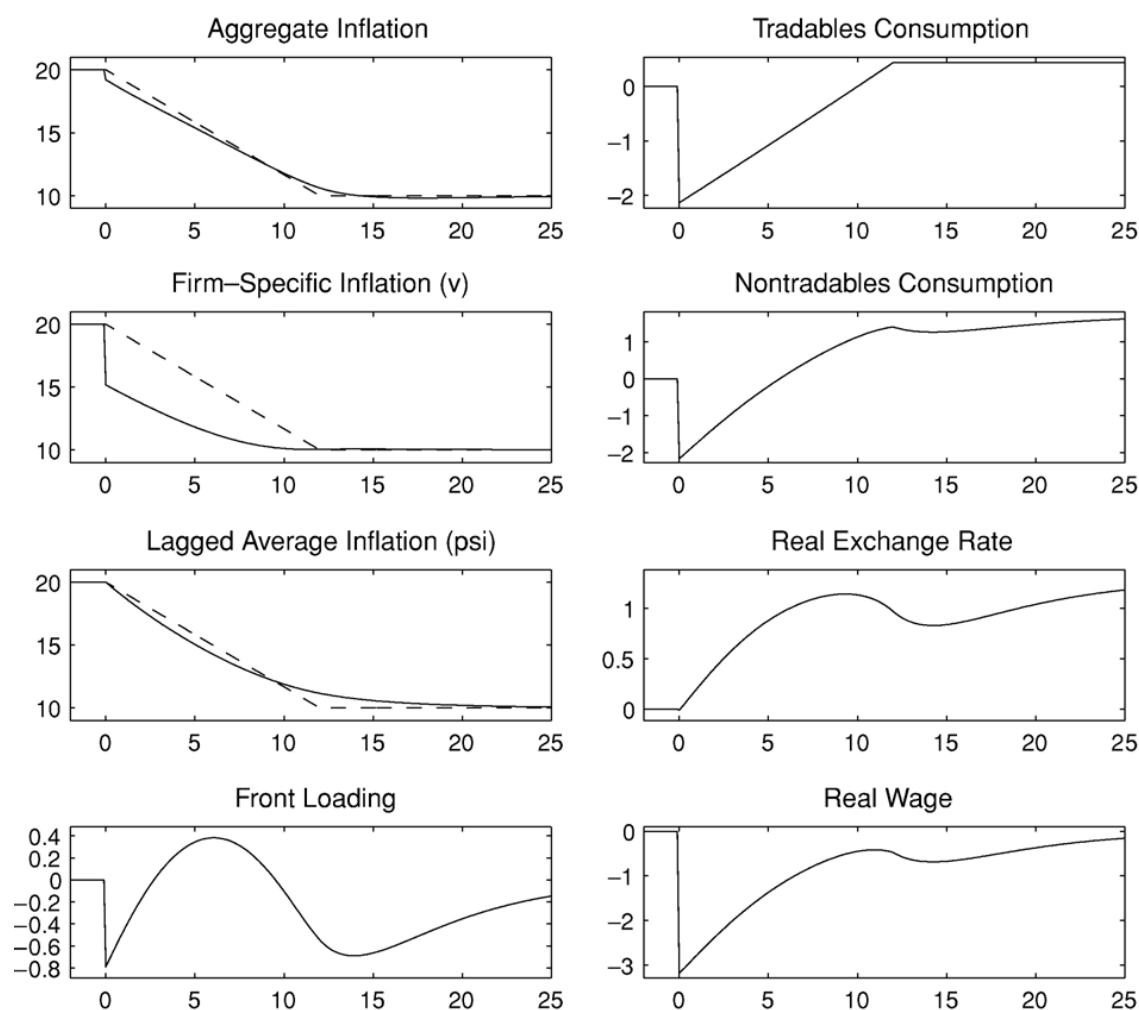


Fig. 2. Gradual disinflation in the open economy.

Fig. 2 illustrates a disinflation policy in which the central bank gradually, over a period of 12 quarters, lowers the rate of exchange rate depreciation from 20% to 10%.¹⁹ The rate of exchange rate depreciation is shown as the broken line.²⁰ We observe that in this gradual disinflation experiment the real exchange rate does not appreciate to generate a contraction of nontradables demand *relative* to tradables demand as it does in the sudden disinflation. But the slow pace of disinflation does lead to an *absolute* contraction caused by intertemporal substitution in tradables and nontradables. The other cost of gradualism, reaching the new and higher steady state output later, is not very significant in this simulation. To evaluate the overall desirability of gradualism we need a metric that takes into account all of these costs and benefits. We therefore now turn to welfare analysis.

4.3. Welfare

We evaluate the costs and benefits of different degrees of gradualism in disinflation using the compensating variation in lifetime consumption suggested by Lucas (1987). The net welfare gain

¹⁹ This type of experiment was originally suggested by John Taylor, who examines the question of gradualism in disinflation in Taylor (1983).

²⁰ We do not show results for the Calvo–Yun model in this case.

accomplished by a permanent reduction in the rate of exchange rate depreciation is defined as the fraction by which consumers' original steady state consumption basket $\bar{c}^* \gamma \bar{c}^{1-\gamma}$, for a given labor supply \bar{L} , would have to be increased to make them indifferent between their lifetime utility in the old, high inflation steady state and the lifetime utility achieved along the equilibrium path to the new, low inflation steady state. For the disinflation shown in Fig. 1 this calculation produces a welfare gain of 0.086%. This is a small gain from a 10% reduction in inflation. The main reason is that this is a net gain after taking account of the sizeable welfare costs of the nontradables recession.

Next we explore the sensitivity of our welfare results to three parameters, the size of the inflationary distortion as determined by the cash-in-advance parameter α , the degree of price stickiness as measured by the average contract length $1/\delta$, and the degree of gradualism as measured by the length of the disinflation period in quarters, which is in turn a function of the linear decline parameter η of the exchange rate depreciation profile. The results are presented in Fig. 3 below in terms of three contour plots of net utility gains representing three different assumptions about the size of inflationary distortions. Each contour plot varies gradualism along the horizontal axis and price stickiness along the vertical axis. The top plot represents our baseline calibration of $\alpha=1.12$. It shows that, for reasonable average contract lengths of up to four quarters, a disinflation from 20% p.a. to 10% p.a. produces small net welfare gains of around

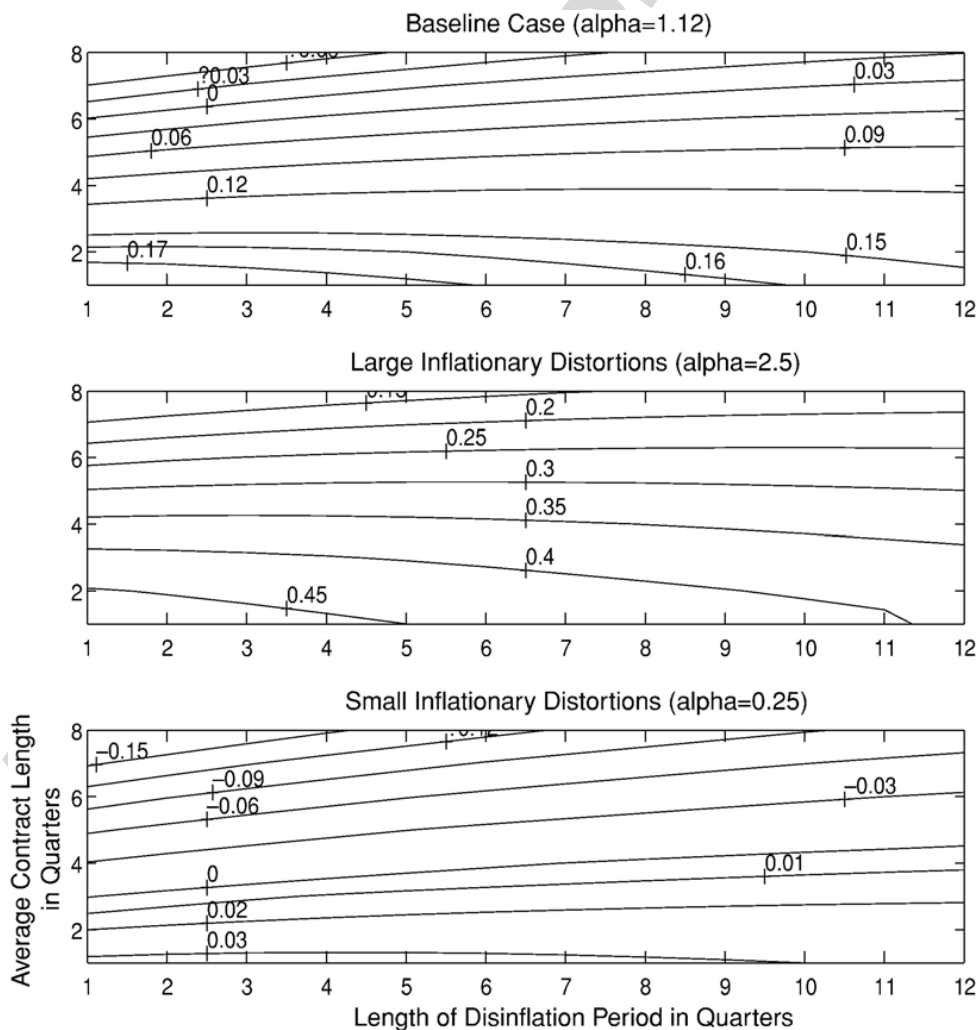


Fig. 3. Welfare gain contours.

0.1%–0.2%. As price stickiness decreases, the relative price of tradables and nontradables can adjust more quickly, so that the losses from the transitory nontradables recession decrease and the new steady state is reached more quickly. For completely flexible prices there would not be any nontradables recession at all and output would smoothly rise towards its higher steady state level. Regarding variations in the degree of gradualism, we observe that when prices are very sticky (average contract length of four quarters or more), increasing the length of the disinflation period is always beneficial because it reduces the very sizeable losses from the nontradables recession. But when prices are very flexible (average contract length of two quarters or less), increasing the length of the disinflation period is always detrimental because it unnecessarily puts off getting to the beneficial low inflation steady state. For intermediate degrees of price stickiness there is an optimal intermediate degree of gradualism that increases very steeply with the degree of price stickiness. For the range of generally accepted values of the parameter δ , the foregoing suggests that *some* gradualism in disinflation improves welfare.

The second subplot shows that for sufficiently large inflationary distortions — implied by large values of the cash-in-advance parameter α — the welfare gains can become quite substantial. On the other hand, the third subplot shows that for very small inflationary distortions and conventionally assumed contract lengths of three to five quarters it may be necessary to disinflate extremely gradually to avoid overall welfare losses.

5. Staggered pricing policies in the closed economy

We consider a closed economy which consists of a continuum of identical price-taking infinitely-lived households, a continuum, indexed by $j \in [0,1]$, of monopolistically competitive, infinitely-lived firms that set prices as described in Section 2, and a government that targets inflation through an interest rate feedback rule as in Taylor (1993). The notation is identical to the open economy case, except that notation previously reserved for the nontradables sector now applies to the economy's single productive sector. The representative household's objective function is

$$\text{Max} \int_0^{\infty} [\ln(c_t) - \nu \ln(h_t) + \kappa \ln(1 - \ell_t) + \theta \ln(M_t/P_t)] e^{-\beta t} dt. \quad (36)$$

There are two differences here to the utility function in Eq. (21). Both are made to ensure easy comparability to the dominant specification in the closed economy literature. The first is that money enters the utility function separably, which makes the first-order condition for money redundant. More importantly, we assume habit persistence in consumption, where the stock of habits evolves as:²¹

$$\dot{h}_t = \rho(c_t - h_t). \quad (37)$$

This feature has been found important in the literature (Christiano et al., 2005) in order to generate realistic dynamics of real variables. Households hold bonds $b_t = B_t/P_t$ with nominal return i_t . Their flow budget constraint is

$$\dot{b}_t = (i_t - \pi_t)b_t - \dot{m}_t - \pi_t m_t + w_t \ell_t + \int_0^1 (Z_t(j)/P_t) dj - c_t - \tau_t. \quad (38)$$

²¹ The constant relative risk aversion utility specification with habits follows Carroll et al. (2000).

We apply the methods of Optimal Control (Arrow and Kurz, 1970) to determine households' optimum. By Eqs. (36)–(38), and letting $a_t = b_t + m_t$, the Hamiltonian is

$$\begin{aligned} \mathcal{H} = & \ln(c_t) - v \ln(h_t) + \kappa \ln(1 - \ell_t) + \theta \ln(M_t/P_t) \\ & + \lambda_t \left[(i_t - \pi_t)a_t - i_t m_t + w_t \ell_t + \int_0^1 (Z_t(j)/P_t) dj - c_t - \tau_t \right] \\ & + \gamma_t [\rho c_t - \rho h_t], \end{aligned} \quad (39)$$

where λ_t and γ_t are the co-state variables for government liabilities a_t and habit h_t . The following are the necessary conditions for the optimum:

$$c_t = (\lambda_t - \rho \gamma_t)^{-1}, \quad (40)$$

$$\frac{\kappa}{1 - \ell_t} = \lambda_t w_t, \quad (41)$$

$$\frac{\dot{\lambda}_t}{\lambda_t} = \beta - (i_t - \pi_t), \quad (42)$$

$$\dot{\gamma}_t = (\beta + \rho)\gamma_t + \frac{v}{h_t}. \quad (43)$$

Monetary policy is governed by the following interest rate rule:

$$i_t = \beta + \bar{\pi} + \phi_\pi(\pi_t - \bar{\pi}) + \phi_y(y_t - \bar{y}). \quad (44)$$

The inflation target $\bar{\pi}$ is the nominal anchor of the economy. We consider an instantaneous, permanent, and credible disinflation experiment to illustrate the workings of the model. Specifically, we assume a policy that lowers the inflation target $\bar{\pi}$ from 5% to 4% p.a. The calibration of the model parameters δ , σ and steady state labor is the same as in the open economy model. We set $\beta = 0.03/4$ for a 3% p.a. rate of time preference and steady state real interest rate.²² The habit parameters are $v = 0.8$ and $\rho = 0.9$. The monetary policy rule is calibrated with $\phi_\pi = 2.0$ and $\phi_y = 0.5$. We linearize the model around the steady-state to obtain a system of six linear differential equations that can be found in the Technical Appendix. The impulse responses shown in Fig. 4 display solid lines for the case of staggered pricing policies and broken lines for the Calvo–Yun case.

As in the open economy case, impulse responses for the Calvo–Yun model are at odds with conventional wisdom and with empirical evidence for the U.S. such as Mankiw (2001). Aggregate inflation immediately drops from 5% to 4%, exhibiting neither a delayed and gradual initial response to the monetary policy shock nor persistent deviations from steady state inflation following the shock. There is no need for an increase in the real interest rate and a recession to bring this disinflation about, and all real variables remain unchanged at their steady state values.²³ For staggered pricing policies the dynamic adjustment of the economy is much more consistent with empirical regularities. For the same reasons as in the open economy case, the initial response of aggregate inflation is delayed and gradual and the deviations from the new steady state are highly persistent. The monetary policy rule, with a new lower inflation target $\bar{\pi}$, calls for a contractionary

²² For this calibration we have in mind the US rather than an emerging market.

²³ The absence of a positive output effect of disinflation is due to the way in which money enters the model.

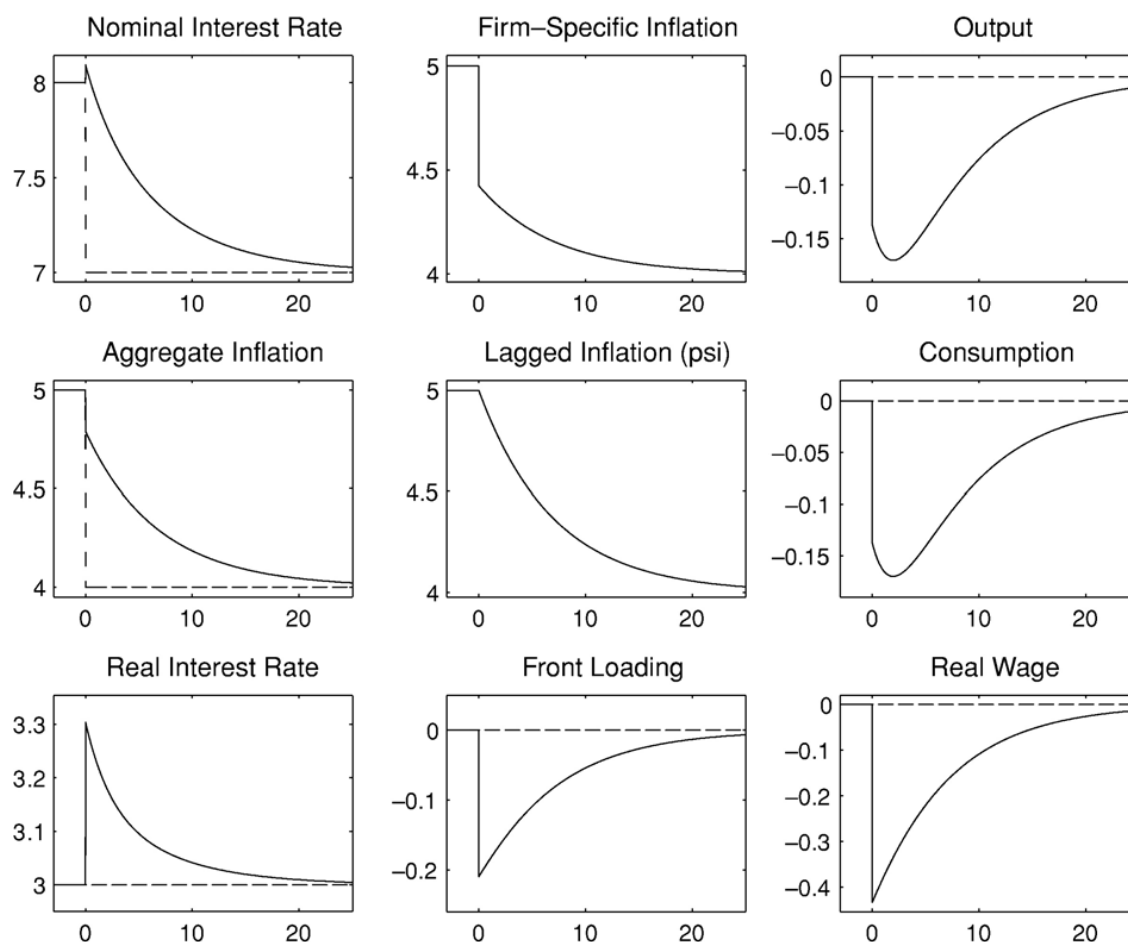


Fig. 4. Instantaneous disinflation in the closed economy.

response that raises the real interest rate and induces a recession. The sacrifice ratio, or undiscounted output cost in percentage points of initial GDP of a one percentage point reduction in inflation, is 1.81 for this disinflation. This is the same as the sacrifice ratio computed by Ball (1994) for the Volcker disinflation in the U.S. Moreover, as emphasized by Mankiw (2001), the peak impact of (un-)employment and output is reached before the peak impact on inflation.²⁴

The experiment in Fig. 4 also allows us to assess whether the staggered pricing policies model generates another important empirical regularity, the positive correlation between real activity and the change in inflation. As discussed in Estrella and Fuhrer (2002), and Mankiw and Reis (2002a), optimizing staggered pricing models counterfactually predict a negative correlation between these variables. For the Calvo–Yun inflation specification this negative correlation is clearly evident in the impulse responses in Fig. 4, and analytically in Eq. (19), where the derivative of the change in inflation with respect to real activity is negative. While a similar analytical derivation is not possible in our model, the impulse responses for output and inflation clearly represent a significant improvement over the Calvo–Yun specification. In the permanent disinflation experiment shown in Fig. 4, output is below its steady state value for a prolonged time and inflation is declining throughout, suggesting a positive contemporaneous correlation.

Overall, the results of our closed economy disinflation experiment, while involving somewhat different transmission channels, are very similar to the open economy application. They suggest

²⁴ This can also be shown to hold for additive shocks to the monetary policy rule (44).

that the assumption of staggered pricing policies is consistent with the key empirical features of moderate disinflation episodes.

6. Conclusion

This paper has proposed a model of inflation inertia in an environment of maximizing, rational, forward-looking agents. It has set out one way to reconcile rational expectations staggered pricing models with important empirical regularities that have been hard to explain, namely structural inflation persistence, gradual inflation response to monetary shocks, and recessionary disinflations. In a two-sector open economy that disinflates through an exchange rate target path, the recession takes place either because of intertemporal substitution when the disinflation happens gradually, or because of an increase in the relative price of nontradables when the disinflation is instantaneous. These transmission channels are different from the real interest rate channel in a closed economy model.

In the proposed framework it becomes possible to conduct an explicit welfare analysis of disinflations that quantifies the trade-off between initial recessions due to nominal rigidities and eventual efficiency gains due to the removal of inflationary distortions. Under our baseline parameterization and for a wide range of plausible inflationary distortions the efficiency gains are found to be larger, and the optimal degree of gradualism in disinflating is shown to increase with the degree of price stickiness.

An attractive feature of this approach is that it accomplishes these objectives while otherwise remaining firmly within the ‘New Keynesian’ modelling tradition and retaining the assumption of rational expectations. This opens up rich possibilities for expanding an already large research agenda.

Appendix A. Roots of the dynamic system

The characteristic equation of system (34), for simplicity but without loss of generality evaluated at the original steady state $\bar{c} = \bar{c} = 1/3$, can be derived as

$$\lambda^4 - 2\beta\lambda^3 + \lambda^2(\beta^2 - 3\delta(\delta + \beta)) + \lambda(3\delta^2\beta + 3\delta\beta^2) + \frac{3}{2}(\delta^2(\delta + \beta)^2) = 0. \quad (\text{A.1})$$

In the following we make use of Theorem 1.2.12 in [Horn and Johnson \(1985, p. 42\)](#). Let the 4×4 coefficient matrix in Eq. (34) be denoted by A , and the four roots by $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. Then it must be true that

$$\lambda_1\lambda_2\lambda_3\lambda_4 = \det(A) = \frac{3}{2}(\delta^2(\delta + \beta)^2) > 0. \quad (\text{A.2})$$

There must therefore be zero, two or four roots with negative real part. Furthermore,

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = \text{tr}(A) = 2\beta > 0. \quad (\text{A.3})$$

This rules out the case of four roots with negative real part. For the final part of the proof, let $\chi_{x,y,z}$ be the 3×3 principal minor of A associated with columns and rows x, y and z . Then the theorem states that the following must hold:

$$\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4 = \chi_{1,2,3} + \chi_{1,2,4} + \chi_{1,3,4} + \chi_{2,3,4}. \quad (\text{A.4})$$

For the sake of our argument, let the roots λ_3 and λ_4 have positive real parts. We compute the right-hand side and rewrite the left-hand side of Eq. (A.4) to get

$$(\lambda_1 + \lambda_2)\lambda_3\lambda_4 + \lambda_1\lambda_2(\lambda_3 + \lambda_4) = -3(\delta^2\beta + \delta\beta^2) < 0. \quad (\text{A.5})$$

The second term on the left-hand side is positive, and therefore we must have

$$\lambda_1 + \lambda_2 < 0. \quad (\text{A.6})$$

This requires that these two roots be either real and negative or complex with negative real parts. As mentioned in the text, it can be established numerically that they are in fact real for all interesting parameter values. The same is true for the positive roots.

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