INTERNATIONAL CURRENCY PORTFOLIOS

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Abstract

This paper develops a theory of international currency portfolios that holds in general equilibrium, and that is therefore not subject to the criticisms directed at the portfolio balance literature of the 1980s. It shows that, under plausible assumptions about fiscal policy, the relationship between the rates of return of different currency bonds is not correctly described by an arbitrage relationship but instead also depends on outstanding bond stocks. Other findings are: (1) There is a monotonically increasing relationship between domestic interest rates and the portfolio share of domestic currency denominated assets. This relationship is steep at low levels of government debt, and almost flat at high levels of government debt. (2) Optimal private sector foreign currency positions are negative, and their size is decreasing in exchange rate volatility. Under volatile exchange rates large negative aggregate net foreign asset positions can only be rationalized by assuming large public sector borrowing from foreign governments. (3) For a baseline economy with zero net foreign assets, open market sales of domestic government debt lead to valuation gains (losses) when the country as a whole has a short (long) position in foreign currency. (4) A fiscal theory of exchange rate determination is compatible with general equilibrium in a two-country world.

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1 Introduction

This paper develops an equilibrium theory of international currency portfolios. It thereby reconnects with the portfolio balance literature of the 1980s, which has since been criticized for its partial equilibrium nature. The paper shows that, under more plausible assumptions about fiscal policy than have hitherto been considered in the literature, the currency composition of international bond portfolios remains determinate even in a full general equilibrium model. In other words, bonds denominated in different currencies are imperfect substitutes, so that the relationship between their rates of return is not correctly described by an arbitrage relationship but instead also depends on outstanding bond stocks. The model features two countries, two goods, two capital stocks, two monies, and two currency denominations of internationally traded bonds.

The model implies that monetary policy can affect not only the level of inflation, via a target path for the nominal anchor, but also interest rates and the volatility of inflation, via balance sheet operations. Balance sheet operations and interest rate changes in turn affect real allocations through their effects on relative prices and on aggregate household wealth.

The paper illustrates the properties of portfolio equilibria by way of a detailed sensitivity analysis that explores different assumptions about fiscal policy, different volatilities of monetary and fiscal shocks, different initial government balance sheet positions, and discrete government balance sheet operations. We find that there is a monotonically increasing relationship between domestic interest rates and the portfolio share of domestic currency denominated assets, that optimal private sector exposure to foreign currency is negative and its size decreasing in exchange rate volatility, and that the valuation effects of open market bond sales are positive if the country as a whole has a negative exposure to foreign currency.

The older partial equilibrium versions of portfolio balance theory developed in the 1980s include Henderson and Rogoff (1982), Kouri (1983), and Branson and Henderson (1985), and was recently also used by Blanchard, Giavazzi and Sa (2005).

Obstfeld and Rogoff (1996) dismiss portfolio balance theory as partial equilibrium reasoning because it omits the government budget constraint. This point is made most comprehensively in an important paper by Backus and Kehoe (1989) on sterilized foreign exchange intervention.[1] Using only an arbitrage condition, they show that under complete asset markets, or under incomplete asset markets and a set of spanning conditions, changes in the currency composition of government debt require no offsetting changes in monetary and fiscal policies to satisfy both the government's and households' budget constraints. Consequently this 'strong form' of intervention is irrelevant for equilibrium allocations and prices. 'Weak form' government intervention in asset markets generally does require offsetting changes in monetary and/or fiscal policies to meet the government budget constraint, but their impact can as easily be attributed to these monetary and/or fiscal changes as to the intervention per se. Intervention is therefore not an independent policy instrument.

This is a powerful theoretical argument. But to obtain it one needs to make the very strong assumption that arbitrary monetary and fiscal policies are available following asset market interventions, while in practice these policies are much more likely to either be exogenous or to follow rules that are completely independent of interventions. Our paper represents an exploration of such rules. This exercise can be interpreted as imposing additional constraints on the form of 'weak form' interventions. We can then ask how asset market interventions affect equilibrium allocations and prices conditional on the form of these rules. Specifically, we ask whether intervention can be effective as a *second* independent instrument of *monetary* policy, taking as given a monetary rule for the nominal anchor and the set of fiscal policy rules.

Grinols and Turnovsky (1994)[2] ask a similar question. In a small open economy model with domestic and foreign government debt and domestic money they show that while

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1 The argument also applies to unsterilized intervention, which in a conventional setting should only affect the levels of prices and the nominal exchange rate but not interest rates and allocations. See Sargent and Smith (1988) on the irrelevance of open market operations in foreign currencies.

2 See also Obstfeld (1982).

stochastic money growth appears to give rise to currency risk in partial equilibrium, this disappears once the fiscal use of stochastic seigniorage has been accounted for. The latter consists of lump-sum transfers of all stochastic seigniorage revenue back to households. This policy fully hedges households against currency risk in general equilibrium. Therefore, despite volatile exchange rates, there is no imperfect asset substitutability, a version of uncovered interest parity holds, and government asset market interventions therefore have no effects on equilibria.

The specification of fiscal policy as full lump-sum transfers, together with the small open economy assumption, is critical for this result. It is however not a very plausible description of real world fiscal policy. We therefore replace it with a set of three assumptions that is more reasonable, if possible empirically supported, and whose significance can be tested by way of sensitivity analysis.

First, we introduce fiscal spending shocks that are exogenous in that they are not automatically financed by offsetting tax changes, but that instead induce price level movements that revalue the government's nominal liabilities. With such shocks domestic currency bonds become imperfect substitutes for other asset classes because their underlying real return characteristics are different in a way that cannot be hedged. The evidence presented in Click (1998) strongly suggests that such shocks are an important feature of the data. He finds, in a large cross-section of countries, that most permanent government spending is financed by conventional tax revenue while transitory government spending is financed mainly by seigniorage.

Second, we assume that shocks that originate abroad, and that affect the real value of the government's net liabilities, are allowed to change the government's net foreign asset position rather than leading to price level or tax changes. As we will show, this assumption supports the previous one by allowing for a fiscal channel for price level and exchange rate determination in a general equilibrium open economy setting. This is significant because, as shown by Dupor (2000) and Daniel (2001), the conventional fiscal theory of the price level breaks down in a two-country world.

Third, we assume that lump-sum transfers are not only limited to the seigniorage associated with monetary policy shocks, they are also asymmetric in that the domestic government only compensates domestic agents but not foreigners for their losses on domestic currency assets. It seems sufficient to appeal to plausibility to defend this assumption. It has powerful implications by making exchange rate risk much more important in considering cross-border nominal asset holdings.

Our model generates mainly theoretical propositions regarding the existence of international currency portfolios, and regarding their dependence on key structural characteristics. We appeal to empirical evidence and sensitivity analysis to support some of our key modeling choices. But the more fundamental empirical question concerns the availability of econometric evidence for the existence of a portfolio channel in the determination of interest rates and exchange rates. This literature is surveyed in Sarno and Taylor (2001), who conclude that studies done in the 1990s, a key one being Dominguez and Frankel (1993), have been generally supportive of a portfolio channel. But they typically find a quantitatively small effect of asset stocks on risk premia. This however does not make portfolio balance models uninteresting, for a number of reasons. First, one has to be careful in interpreting studies whose estimating equations are not derived from optimizing foundations and which therefore exhibit significant differences from the portfolio equations derived in this paper. Second, a small response of risk premia to stock changes is precisely what our model predicts for the large industrialized countries for which these studies have been conducted. The reason is that the sensitivity of interest differentials to asset stocks declines with the outstanding stock of domestic currency nominal liabilities, which are large in industrialized countries. But this also suggests that results should be very different for developing countries, where very little empirical work has so far been done. Third, our most interesting result is arguably the optimality of portfolio non-diversification among different currencies, and the closely related results on valuation gains from open market operations.

These results, which hold even when risk premia are very insensitive to stocks, cannot be tested by standard portfolio balance regressions. Instead they require bond stock data by currency and by holder, which unfortunately are very hard to find. Some data do however exist for the U.S., and they broadly support our results. For example, Burger and Warnock (2003) show that only around a quarter of U.S. investors' bond portfolios consists of foreign issues, of which the vast majority is in dollars. And United States Treasury (2008) shows that of the U.S. treasury securities held by foreigners, only about one third is held by the foreign private sector.

The recent theoretical literature has made great strides in incorporating portfolio theory into state of the art monetary dynamic general equilibrium models. Papers include Devereux and Sutherland (2006), Tille and van Wincoop (2008), Engel and Matsumoto (2006), and Coeurdacier, Kollman and Martin (2007). However, in one critical sense these papers take a different focus to the earlier portfolio balance literature. This is that they only allow agents to choose portfolios of exclusively *private* equity and debt. While there is a role for government in setting nominal interest rates, there is no role for *government* asset market operations in nominal debt. There is a reason for this choice. The key problem in this new literature is to find *solution* methods for portfolio equilibria, but in general there is no question as to their *existence*. This is because all the assets in question are either equity with idiosyncratic return characteristics, or debt whose return characteristics depend on these idiosyncratic returns. Imperfect asset substitutability is therefore a given. The problem in the older portfolio balance literature however was precisely that, for government asset market operations in nominal debt, imperfect asset substitutability can *not* be taken as a given.

Our paper is of course also related to the literature on interest rate risk premia in open economy models without portfolio features. As shown in Lewis (1995), empirical risk premia have been both large in absolute value and highly variable in industrialized countries, and they are known to have been even larger in developing countries. An attempt at explaining that fact has to take into account both default and currency risk. The focus of this paper is on currency risk.[3] Engel (1992) and Stulz (1984) show that in flexible price monetary models monetary volatility per se will not give rise to any currency risk premium. Engel (1999), using the frameworks of Obstfeld and Rogoff (1998, 2000) and Devereux and Engel (1998), shows that sticky prices are required to generate a risk premium. But the source of the risk premium in such models is the covariance of consumption and the exchange rate. This makes it difficult to rationalize large absolute-value risk premia because consumption is not very variable.[4] A general equilibrium portfolio model such as ours introduces portfolio considerations as a second and potentially very powerful source of interest rate differentials.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 calibrates and computes a baseline version of the model. Section 4 discusses how the characteristics of the baseline economy vary with key parameters and policy choices. Section 5 illustrates the effects of discrete government open market operations. Section 6 concludes. Mathematical details are presented in a comprehensive Technical Appendix.

2 The Model

The world economy consists of two countries, Home and Foreign. Foreign variables are denoted by adding a superscript asterisk $*$ to the corresponding symbol for Home variables. Whenever the conditions characterizing Home and Foreign are symmetric, we limit our discussion to Home. Each economy is composed of a continuum of identical infinitely lived households and a government. The asset and liability structure of public and private sectors in each country is shown in Figure 1 by way of balance sheets.[5] We use a continuous time stochastic monetary portfolio choice model to derive households' optimal consumption and

[3] There is a well-established and growing literature on default risk. The early contributions include Eaton and Gersovitz (1981) and Aizenman (1989). More recent contributions include Kletzer and Wright (2000), Kehoe and Perri (2002), Uribe and Yue (2006) and Arellano (2008).

[4] For alternative and more recent approaches see Lustig and Verdelhan (2007) and Burnside, Eichenbaum, Kleshchelski and Rebelo (2007).

[5] These balance sheets anticipate one result of the paper, the fact that households short foreign currency bonds.

portfolio decisions.[6] Government is characterized by an initial balance sheet position and by a set of fiscal and monetary policy rules.

2.1 Uncertainty

2.1.1 Exogenous Processes

There are two sources of risk in each economy, shocks to the nominal money supply and shocks to government spending. We define a two-dimensional Brownian motion $W_t = [W_t^M \quad W_t^{M*}]'$ consisting of the two shocks to the growth rates of nominal money supplies dM_t and dM_t^*, and a two-dimensional Brownian motion $V_t = [V_t^G \quad V_t^{G*}]'$ consisting of the two shocks to exogenous government spending dG_t and dG_t^*. Finally we let $X_t = [W_t' \;\; V_t']'$. Wherever possible we will describe stochastic processes in terms of this four-dimensional process, but for some of our key results the distinction between monetary and fiscal shocks is critical, and in those cases the distinction is maintained in the notation.

Money Supply Shocks The nominal money supply follows a geometric Brownian motion with a drift process μ_t determined by the nominal anchor or inflation target of monetary policy. The vector of monetary diffusion terms is given by $\sigma_{M,t}^m = [\sigma_M^M \quad \sigma_{M,t}^{M*}]$. The first term σ_M^M (σ_{M*}^{M*} for the Foreign money supply process) is an exogenous and constant diffusion that multiplies shocks to the domestic money supply, while diffusions with respect to foreign money supply shocks are endogenous. The vector of fiscal diffusion terms is given by $\sigma_{M,t}^g = [\sigma_{M,t}^G \;\; \sigma_{M,t}^{G*}]$, where both terms are endogenous. As a general rule throughout the paper, we index endogenous drift and diffusion terms by time if they represent possibly time-varying monetary policy choices, or if they are functions of such choices. Being an Itô process, M_t is continuous, which ensures exchange rate determinacy. We have

$$\frac{dM_t}{M_t} = \mu_t dt + \sigma_{M,t} dX_t = \mu_t dt + \sigma_{M,t}^m dW_t + \sigma_{M,t}^g dV_t \quad . \tag{1}$$

[6] Useful surveys of the technical aspects of stochastic optimal control are contained in Chow (1979), Fleming and Rishel (1975), Malliaris and Brock (1982), Karatzas and Shreve (1991), and Duffie (1996). The seminal papers using this technique to analyze macroeconomic portfolio selection are Merton (1969, 1971) and Cox, Ingersoll and Ross (1985).

Fiscal Shocks Exogenous government spending is given by the Itô process[7]

$$\frac{dG_t}{K_t} = \sigma_G^G dV_t^G \quad , \tag{2}$$

where K_t denotes aggregate household physical capital, and σ_G^G ($\sigma_{G^*}^{G^*}$ for the Foreign spending process) is the diffusion term multiplying domestic spending shocks. Fiscal spending shocks affect the resources available for private consumption. In order for this to represent a risk to households in general equilibrium, it must be true that government consumption is an imperfect substitute for private consumption. We choose the simplest and most tractable assumption under which this is true, namely that government spending does not enter household utility.[8]

2.1.2 Endogenous Processes

Exchange Rates The nominal exchange rate E_t, expressed as units of Home currency per unit of Foreign currency, floats. The process E_t is endogenously determined as a function of the four exogenous stochastic processes. It follows a geometric Brownian motion with drift ε_t and diffusions $\sigma_{E,t}^m$ and $\sigma_{E,t}^g$:

$$\frac{dE_t}{E_t} = \varepsilon_t dt + \sigma_{E,t} dX_t = \varepsilon_t dt + \sigma_{E,t}^m dW_t + \sigma_{E,t}^g dV_t \quad . \tag{3}$$

To preserve symmetry in our presentation we use the notation $E_t^* = 1/E_t$ for the exchange rate of Foreign, which follows the stochastic process

$$\frac{dE_t^*}{E_t^*} = \varepsilon_t^* dt + \sigma_{E^*,t} dX_t = \varepsilon_t^* dt + \sigma_{E^*,t}^m dW_t + \sigma_{E^*,t}^g dV_t \quad . \tag{4}$$

The presence of Jensen's inequality terms implies that

$$\varepsilon_t^* = -\varepsilon_t + (\sigma_{E,t})^2 \quad , \tag{5}$$

and of course we also have $\sigma_{E^*,t}^j = -\sigma_{E,t}^j$, $j = M, M^*, G, G^*$.

Price Levels All goods are tradable, and purchasing power parity is assumed to hold. Households consume an aggregate that includes both Home and Foreign goods, with the

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7 A nonzero drift would affect feasible choices for the drift of the tax rate. But because this does not affect the presence or transmission mechanism of a portfolio channel, we ignore it without loss of generality.

8 This assumption would not seem to require an apology, as it is still the dominant choice in dynamic business cycle models.

Home CPI price level denoted by P_t. Home output is sold both to Home and Foreign, and the price of Home output in Home is denoted by Q_t. Like the nominal exchange rate, these price levels are endogenously determined and follow the processes

$$\frac{dP_t}{P_t} = \pi_{P,t}dt + \sigma_{P,t}dX_t = \pi_{P,t}dt + \sigma^m_{P,t}dW_t + \sigma^g_{P,t}dV_t \quad , \tag{6}$$

$$\frac{dQ_t}{Q_t} = \pi_{Q,t}dt + \sigma_{Q,t}dX_t = \pi_{Q,t}dt + \sigma^m_{Q,t}dW_t + \sigma^g_{Q,t}dV_t \quad . \tag{7}$$

Taxes Households are subject to a lump-sum tax dT_t levied as a proportion of wealth a^c_t. This tax follows an Itô process with adapted drift process τ_t and diffusion processes $\sigma^M_{T,t}$ and $\sigma^{M^*}_{T,t}$:

$$\frac{dT_t}{a^c_t} = \tau_t dt + \sigma^M_{T,t}dW^M_t + \phi\sigma^{M^*}_{T,t}dW^{M^*}_t \ . \tag{8}$$

The drift and diffusion terms will be determined in equilibrium from a balanced budget requirement for the government. Note that taxes respond only to money supply shocks but not to fiscal shocks. For foreign money supply shocks, a distinction between two cases turns out to be critically important. The case of $\phi = \phi^* = 1$ implies an economy with high exchange rate volatility, while $\phi = \phi^* = 0$ implies very low exchange rate volatility. We defer a more detailed discussion to Section 2.3.

2.2 Households

Preferences The representative household has time-separable logarithmic preferences[9] that depend on his expected lifetime path of a tradable goods consumption basket $\{c_t\}^\infty_{t=0}$:

$$\mathbb{E}_0 \int_0^\infty e^{-\beta t}\ln(c_t)dt \quad , \quad 0 < \beta < 1 \ , \tag{9}$$

where $\mathbb{E}_0$ is the expectation at time 0, and β is the rate of time preference. The consumption basket is in turn a Cobb-Douglas aggregate of Home produced tradables c^h_t and Foreign produced tradables c^f_t,

$$c_t = A\left(c^h_t\right)^\gamma \left(c^f_t\right)^{1-\gamma} \quad , \tag{10}$$

where $A = (\gamma)^{-\gamma}(1-\gamma)^{-(1-\gamma)}$, and the parameter γ represents home bias in consumption.

[9] Logarithmic preferences are commonly used in the open economy asset pricing and portfolio choice literature for their analytical tractability, see e.g. Stulz (1984, 1987) and Zapatero (1995).

Cash Constraint Monetary portfolio choice models often introduce money into the utility function separably because this preserves the separability between portfolio and savings decisions found in Merton (1969, 1971). However, as pointed out by Feenstra (1986), without a positive cross partial between money and consumption the existence of money cannot be rationalized through transactions cost savings. We therefore use a cash constraint instead, and we show that it is still possible to obtain highly tractable analytical solutions. Specifically, consumers are required to hold real money balances equal to a multiple α of their consumption expenditures c_t. Denoting real money balances by $m_t = M_t/P_t$, we have

$$c_t = \alpha m_t \quad . \tag{11}$$

The now very common treatment of the cash-in-advance constraint in the discrete time model of Lucas (1990) has two aspects, a cash requirement aspect and an in-advance aspect. Our own treatment goes back to the earlier Lucas (1982), which uses only the cash requirement aspect. This is due to the difficulty of implementing the in-advance timing conventions in a continuous-time framework. In the continuous time stochastic finance literature, Bakshi and Chen (1997) have used the same device.

Trading Household consumption c_t is financed from a constant real return r on physical Home capital K_t, and from the stochastic returns on three types of financial assets, (1) domestic currency denominated money M_t with a zero nominal return, (2) domestic currency denominated bonds H_t^c with nominal return $i_t^h dt$, and (3) foreign currency denominated bonds F_t^c with nominal return $i_t^f dt$. There is complete home bias in equity. We denote real asset stocks by $h_t^c = H_t^c/P_t$, $f_t^c = (F_t^c E_t)/P_t$, $k_t = (K_t Q_t)/P_t$, and total private wealth by

$$a_t^c = \frac{M_t + H_t^c + E_t F_t^c + Q_t K_t}{P_t} = m_t + h_t^c + f_t^c + k_t \; . \tag{12}$$

Portfolio shares will be denoted by $n_t^m = \frac{m_t}{a_t^c}$, $n_t^h = \frac{h_t^c}{a_t^c}$ and $n_t^f = \frac{f_t^c}{a_t^c}$, with $n_t^k = \frac{k_t}{a_t^c} = 1 - n_t^m - n_t^h - n_t^f$. We use the notation $n_t^{fin} = n_t^m + n_t^m + n_t^f$, the share of financial assets in the overall portfolio.

Budget Constraint The household budget constraint is given by

$$\begin{aligned} da_t^c &= a_t^c \left[n_t^m dr_t^m + n_t^h dr_t^h + n_t^f dr_t^f + (1 - n_t^m - n_t^h - n_t^f) dr_t^k \right] \\ &\quad - c_t dt - a_t^c [\tau_t dt + \sigma_{T,t}^M dW_t^M + \phi \sigma_{T,t}^{M^*} dW_t^{M^*}] \ , \end{aligned} \tag{13}$$

where dr_t^i is the real rate of return, in terms of the final consumption basket, on asset i. The Technical Appendix derives these returns using Itô's lemma. We denote the drift components of real asset returns by $\tilde{r}_t^m$, $\tilde{r}_t^h$, $\tilde{r}_t^f$ and $\tilde{r}_t^k$. Risk premia are given by deviations from real interest parity $\tilde{r}_t^h - \tilde{r}_t^f$. We adopt the following notation for private asset accumulation:

$$\frac{da_t^c}{a_t^c} = \mu_{a,t} dt + \sigma_{a,t} dX_t \ . \tag{14}$$

Capital Accumulation Households own and accumulate the capital stock K_t. The output $rK_t dt$ is consumed by Home and Foreign households, $c_t^h dt$ and $c_t^{h^*} dt$, and by the Home government, dG_t. We therefore have the following law of motion for the capital stock:

$$dK_t = \left(rK_t - c_t^h - c_t^{h^*} \right) dt - \sigma_G^G K_t dV_t^G \ . \tag{15}$$

Choice of Consumption Bundle The optimality conditions for domestic and foreign consumption goods are independent of the portfolio choice problem. They are standard and given by $q_t c_t^h = \gamma c_t$, $e_t q_t^* c_t^f = (1-\gamma) c_t$, where $q_t = Q_t / P_t$, $q_t^* = Q_t^* / P_t^*$, and the consumption based real exchange rate is $e_t = (E_t P_t^*) / P_t$. The CPI price index pertaining to (10) can then be derived as

$$P_t = (Q_t)^\gamma \left(E_t Q_t^* \right)^{1-\gamma} \ . \tag{16}$$

Portfolio Problem The household's portfolio problem is to maximize present discounted lifetime utility (9) subject to (11) and (13), by the appropriate portfolio choice $\left\{ n_t^m, n_t^h, n_t^f \right\}_{t=0}^{\infty}$. We solve this problem recursively using a continuous time Bellman equation. Details are discussed in the Technical Appendix. The three first order necessary conditions contain the drift and diffusions of the tax process (8). For a complete solution we therefore need a specification of fiscal policy rules, which we will develop in the following subsections.

2.3 Government

Monetary Policy Monetary policy is characterized by two policy variables. First, primary control over the *level* of inflation is achieved through a target for the **nominal anchor** consistent with an inflation target. In our model this is simply a target $\{\mu_t\}_{t=0}^{\infty}$ for money growth in equation (1). Second, we will show that control of the *volatility* of inflation can be achieved by setting a target for the **stock of nominal government debt** $\{B_t\}_{t=0}^{\infty}$, and thereby implicitly for the nominal interest rate $\left\{i_t^h\right\}_{t=0}^{\infty}$.[10]

Government Assets and Liabilities In nominal terms, the government issues domestic currency money M_t and domestic currency bonds $H_t^c + H_t^{c^*}$ (where we will show that $H_t^{c^*} < 0$) to domestic households. It also issues domestic currency bonds $H_t^{g^*}$ to and buys foreign currency bonds F_t^g from the foreign government. Domestic households determine their optimal overall holdings of domestic currency bonds H_t^c, and are indifferent between the shares of government bonds $H_t^c + H_t^{c^*}$ and of domestic currency loans to foreign households $-H_t^{c^*}$. We denote the total stock of nominal domestic currency government debt by B_t,

$$B_t = H_t^c + H_t^{c^*} + H_t^{g^*} \quad , \tag{17}$$

and its real stock of financial net wealth (this excludes discounted future tax revenue) in terms of the domestic goods basket by a_t^g,

$$a_t^g = \frac{E_t F_t^g - M_t - B_t}{P_t} \ . \tag{18}$$

We also let $f_t^g = (F_t^g E_t)/P_t$ and $h_t^{g^*} = \left(H_t^{g^*} E_t^*\right)/P_t^*$. The government's flow budget constraint is

$$\begin{aligned} da_t^g \;\; = \;\; & f_t^g dr_t^f + a_t^c \tau_t dt + a_t^c \left(\sigma_{T,t}^M dW_t^M + \phi \sigma_{T,t}^{M^*} dW_t^{M^*}\right) - k_t \sigma_G^G dV_t^G \\ & - n_t^m a_t^c dr_t^m - n_t^h a_t^c dr_t^h - e_t n_t^{h^*} a_t^{c^*} dr_t^h - e_t h_t^{g^*} dr_t^h \ . \end{aligned} \tag{19}$$

10 With imperfect asset substitutability the policy problem can always be described as either fixing interest rates and then supplying as many bonds as the market demands at that interest rate, or as fixing the quantity of bonds and allowing the interest rate to clear the market. We have chosen the latter.

Fiscal Policy The exogenous, spending component of fiscal policy is specified in (2) and the endogenous, lump-sum tax component in (8). We assume that fiscal policy meets four requirements. *First*, the expected budget balance is always zero, so that government assets a_t^g follow an Ito-process without drift. This implies the following behavior for the drift of the tax process τ_t:

$$\begin{aligned} a_t^c \tau_t &= \left(n_t^m a_t^c + n_t^h a_t^c + e_t n_t^{h^*} a_t^{c^*} + e_t h_t^{g^*} \right) \tilde{r}_t^m \\ &+ \left(a_t^c n_t^h + e_t n_t^{h^*} a_t^{c^*} + e_t h_t^{g^*} \right) i_t^h - f_t^g \tilde{r}_t^f \quad . \end{aligned} \tag{20}$$

Second, lump-sum taxes respond instantaneously to domestic (and, if $\phi = \phi^* = 1$, foreign) money supply shocks by exactly compensating households for the (net) losses from surprise inflation on their domestic currency nominal asset portfolio.[11] This implies that the diffusions of the tax process are given by

$$\begin{aligned} \sigma_{T,t}^M &= -(n_t^m + n_t^h)\sigma_{P,t}^M \quad , \\ \sigma_{T,t}^{M^*} &= -\phi(n_t^m + n_t^h)\sigma_{P,t}^{M^*} \quad . \end{aligned} \tag{21}$$

Third, domestic fiscal spending shocks are exogenous, meaning that endogenous lump-sum taxes are not available to finance them. Instead, the budget balancing role in response to these shocks falls to the price level. We obtain

$$\sigma_{P,t}^G = \sigma_G^G \left(\frac{n_t^k}{n_t^m + n_t^h} \right) \quad . \tag{22}$$

Fiscally induced price level volatility is increasing in the volatility of the fiscal shocks themselves, but it is decreasing in the amount of nominal government liabilities held in household portfolios. This is because a larger stock of nominal liabilities that can be revalued by price level movements represents a larger base of the stochastic inflation tax.

Fourth, all the remaining effects of shocks are absorbed by the government's net asset position a_t^g. This includes shocks to foreign fiscal spending and (if $\phi = \phi^* = 0$) foreign

[11] We have also computed a third case where foreign fiscal spending shocks are treated symmetrically with domestic and foreign money supply shocks. This does not yield fundamentally different insights, and we therefore do not report the results in the paper.

money supply, and shocks to the government's asset and liability positions vis-à-vis the foreign government. We obtain the following:

$$\begin{aligned} da_t^g &= f_t^g\left(\sigma_{E,t}-\sigma_{P,t}\right)dX_t+\left(e_t n_t^{h^*} a_t^{c^*}+e_t h_t^{g^*}\right)\sigma_{P,t}dX_t \\ &\quad +(1-\phi)(n_t^m a_t^c+n_t^h a_t^c)\sigma_{P,t}^{M^*}dW_t^{M^*}+(n_t^m a_t^c+n_t^h a_t^c)\sigma_{P,t}^{G^*}dV_t^{G^*} \ . \end{aligned} \tag{23}$$

The fourth assumption plays a critical role because it allows for a fiscal channel in the determination of the price level and of the exchange rate in a general equilibrium two-country setting. As shown by Dupor (2000) and Daniel (2001), the conventional fiscal theory of the price level breaks down in two-country models, put simply because a single instrument, the nominal exchange rate, cannot simultaneously ensure budget balance through nominal asset revaluation in two countries. In their argument it is assumed that the levels of spending and of taxes are exogenous. In our model shocks to spending are also exogenous, and so is the *instantaneous tax response*, which equals zero. Domestic spending shocks are therefore financed instantaneously through price level jumps that imply exchange rate jumps. But the key point is that exchange rate jumps caused by foreign spending shocks do not unbalance the domestic budget. Instead they are allowed to instantaneously change the government's net asset position. While this leads to permanent and theoretically unbounded changes in government net assets a_t^g according to (23), this does not violate government intertemporal budget balance, which instead requires only a transversality condition on a_t^g. That however continues to be satisfied because the *lump-sum drift taxes* τ_t of equation (20) are endogenous. Specifically, they ensure that the net revenue on any changes in government assets is from then onwards redistributed by way of lump-sum taxes, so that $\mathbb{E}_t\left[a_{t+\tau}^g\right]=a_t^g$, $\tau>0$. A fiscal theory of price level and exchange rate determination is therefore perfectly feasible even in a general equilibrium two-country model.

The rules (20) - (23) imply endogenous and stochastic adjustments of the two countries' foreign exchange reserves f_t^g and $h_t^{g^*}$ in response to equilibrium changes in bond and money demands. However, it can be shown that these foreign exchange acquisition rules are not part of the core equations of the model, they can instead be determined recursively once the core model has been solved for a given set of state and policy variables.

Definition 1. *A feasible government policy for Home is a list of stochastic processes* $\{\mu_t, B_t, \tau_t, \sigma^M_{T,t}, \sigma^{M^*}_{T,t}, a^g_t\}^\infty_{t=0}$ *such that, given a list of stochastic processes* $\{a^c_t,\ a^{c^*}_t,\ f^g_t,\ h^{g^*}_t,\ e_t,\ \tilde{r}^m_t,\ \tilde{r}^f_t,\ i^h_t,\ \sigma_{E,t},\ \sigma_{P,t},\ X_t\}^\infty_{t=0}$*, initial conditions* $a^c_0, a^{c^*}_0, a^g_0$*, and constant portfolio shares* $n^m_t, n^h_t, n^f_t, n^{m^*}_t, n^{h^*}_t, n^{f^*}_t$*, the conditions* (20)*,* (21)*,* (22) *and* (23) *are satisfied at all times.*

In all our policy experiments in Sections 4 and 5 we will assume that $\{B_t, B^*_t\}^\infty_{t=0}$ are deterministic sequences, and that $\mu_t, \mu^*_t, \sigma^M_M, \sigma^{M^*}_{M^*}, \sigma^G_G$ and $\sigma^{G^*}_{G^*}$ are constants. This can be shown to imply that all mean returns (including $\tilde{r}^m_t, \tilde{r}^h_t, \tilde{r}^f_t$), as well as all endogenous price and policy drifts and diffusions (including $\sigma^M_{T,t}, \sigma^{M^*}_{T,t}, \sigma_{E,t}, \sigma_{P,t}$) are functions only of these eight variables but not of the economy's state variables.

2.4 Equilibrium and Current Account

The economy's state variables at time t are given by $a^c_t, a^{c^*}_t, K_t, K^*_t, M_t, M^*_t$ and the level of net foreign assets $\tilde{f}_t$, which is given by

$$\tilde{f}_t = f^g_t + n^f_t a^c_t - e_t \left(h^{g^*}_t + n^{h^*}_t a^{c^*}_t \right) \quad . \tag{24}$$

Then equilibrium is defined as follows:

Definition 2. *An equilibrium is a set of exogenous stochastic processes* $\{X_t\}^\infty_{t=0}$*, initial conditions* $a^c_0, a^{c^*}_0, K_0, K^*_0, M_0, M^*_0, \tilde{f}_0$*, an allocation consisting of stochastic processes* $\{c_t, c^h_t, c^f_t, c^*_t, c^{f^*}_t, c^{h^*}_t, K_t, K^*_t, M_t, M^*_t, H^c_t, H^{c^*}_t, H^{g^*}_t, F^{c^*}_t, F^c_t, F^g_t\}^\infty_{t=0}$*, a price system consisting of stochastic processes* $\{i^h_t,\ i^f_t,\ E_t,\ E^*_t,\ P_t,\ P^*_t,\ Q_t,\ Q^*_t\}^\infty_{t=0}$*, and feasible government policies for Home and Foreign such that, given the exogenous stochastic processes, the initial conditions, the feasible government policies and the price system, the allocation solves households' problem of maximizing* (9) *subject to* (11) *and* (13)*.*

Current Account and GDP The current account is given by

$$\begin{aligned} & e_t q_t^* c_t^f + e_t \left(n_t^{h^*} a_t^{c^*} + h_t^{g^*} \right) \tilde{r}_t^h + n_t^{fin} a_t^c \mu_{a,t} + \sigma_G^G k_t \left(\sigma_{P,t}^G - \sigma_{Q,t}^G \right) \\ = \quad & q_t c_t^{h^*} + \left(n_t^f a_t^c + f_t^g \right) \tilde{r}_t^f dt \quad , \end{aligned} \tag{25}$$

and real GDP is

$$gdp_t = r * k_t \quad . \tag{26}$$

Portfolio Optimality Conditions The derivation of the final household optimality conditions is presented in detail in the Technical Appendix. They are

$$c_t = \frac{\beta a_t^c}{\left(1 + i_t^h / \alpha\right)} \quad , \tag{27}$$

$$n_t^{dom} = n_t^m + n_t^h = \frac{i_t^h - r - \pi_{Q,t} + \left(\sigma_{Q,t}\right)^2 - n_t^f \left(\left(\sigma_{Q,t}\right)^2 - \sigma_{E,t}\sigma_{Q,t} \right)}{\left(\sigma_{Q,t}\right)^2 - \sigma_{P,t}^M \sigma_{Q,t}^M - \phi \sigma_{P,t}^{M^*} \sigma_{Q,t}^{M^*}} \quad , \tag{28}$$

$$\begin{aligned} n_t^f \quad = \quad & \frac{i_t^f + \varepsilon_t - r - \pi_{Q,t} + \left(\sigma_{Q,t}\right)^2 - \sigma_{E,t}\sigma_{Q,t}}{\left(\sigma_{E,t}\right)^2 + \left(\sigma_{Q,t}\right)^2 - 2\sigma_{E,t}\sigma_{Q,t}} \\ & - \frac{n_t^{dom} \left(\left(\sigma_{Q,t}\right)^2 - \sigma_{E,t}\sigma_{Q,t} + \left[\sigma_{E,t}^M \sigma_{P,t}^M - \sigma_{P,t}^M \sigma_{Q,t}^M \right] + \phi \left[\sigma_{E,t}^{M^*} \sigma_{P,t}^{M^*} - \sigma_{P,t}^{M^*} \sigma_{Q,t}^{M^*} \right] \right)}{\left(\sigma_{E,t}\right)^2 + \left(\sigma_{Q,t}\right)^2 - 2\sigma_{E,t}\sigma_{Q,t}} \quad . \end{aligned} \tag{29}$$

Equation (27) is a standard condition in this model class. It states that consumption is proportional to wealth and, because of the cash constraint, negatively related to the nominal interest rate. The key novelty of this model is found in the general equilibrium portfolio balance equations (28) and (29), which demonstrate that the portfolio shares of domestic and foreign currency denominated assets are determinate even after taxes have been endogenized. The second and third terms of the denominator of (28) show that fiscal lump-sum redistribution of net seigniorage losses (21), by providing a partial hedge against price risk, increases the attractiveness of domestic currency assets.[12] The effects on foreign currency assets, in the final four terms of the numerator of (29), is not unambiguous.

[12] Producer and consumer prices are always positively correlated.

2.5 Interpretation of the Portfolio Share Equations

Equation (28) can be written approximately, ignoring covariance terms, as

$$n_t^{dom} \simeq 1 - \frac{r + \pi_{Q,t} - i_t^h}{\left(\sigma_{Q,t}\right)^2} - n_t^{for} \quad . \tag{30}$$

This shows that the share of domestic currency assets depends negatively on the risk adjusted excess return of holding domestic physical capital versus domestic currency assets, and on the share of foreign currency bonds.[13] The direct effect of raising the nominal interest rate, ceteris paribus, is a lower excess return of capital and a higher share of domestic currency assets. This is only partly offset by the indirect effect of this higher share constituting a larger inflation tax base and thereby lowering the volatility of producer prices $\left(\sigma_{Q,t}\right)^2$, and therefore the risk of holding domestic capital relative to domestic currency nominal assets. Less volatile producer prices lower the incentive to hold domestic currency assets because in terms of domestic output the value of domestic capital is predetermined at K_t, but the value of domestic currency assets H_t^c/Q_t fluctuates with, and more importantly is convex in, the producer price.

Equation (29) can be written approximately, again ignoring covariance terms, as

$$n_t^f \simeq \frac{\left(\sigma_{Q,t}\right)^2}{\left(\sigma_{Q,t}\right)^2 + \left(\sigma_{E,t}\right)^2}\left(1 - n_t^{dom}\right) - \frac{\left(r + \pi_{Q,t}\right) - \left(i_t^f + \varepsilon_t\right)}{\left(\sigma_{Q,t}\right)^2 + \left(\sigma_{E,t}\right)^2} \quad . \tag{31}$$

This shows that the share of foreign currency bonds depends negatively on the risk adjusted excess return of holding domestic physical capital versus foreign currency bonds, and on the share of domestic currency assets. A higher foreign nominal interest rate lowers the excess return of capital and drives up the share of foreign currency assets. The key difference between (30) and (31) is the presence of exchange rate volatility $\left(\sigma_{E,t}\right)^2$. This makes the foreign currency portfolio share much smaller than the domestic currency share, and in fact, as we will show, typically negative. Furthermore, increasing exchange rate volatility drives

[13] We can ignore portfolio shares of domestic currency assets at or above one, because this would correspond to a government debt to GDP ratio of over 2000%. A realistic calibration typically is in the neighborhood of $n_t^{dom} \simeq 0.02$.

the foreign share towards zero. The explanation is very straightforward. Foreign currency bonds bear exchange rate risk that is not hedged through foreign government redistribution of the associated seigniorage revenue. Domestic assets do not bear exchange rate risk, and furthermore their domestic price risk is, at least insofar as it stems from domestic money supply shocks, indirectly hedged through domestic government lump-sum redistribution of the associated seigniorage revenue.

2.6 Equilibrium Diffusions

The Technical Appendix derives a set of equilibrium conditions that determine the four diffusions for money, consumer prices, producer prices and the exchange rate. As the model features neither nominal nor real rigidities, these diffusions represent the instantaneous impulse responses of the model. The resulting standard deviations of prices and exchange rates are a critical factor in the determination of optimal portfolio shares. There are three equilibrium conditions for each shock, with the fourth condition in each case supplied by an exogenous monetary or fiscal forcing term.

The *first* condition follows directly from the stochastic differentiation of the CPI price level equation (16) and relates the CPI diffusion to the underlying diffusions of domestic and foreign goods prices and of the nominal exchange rate:

$$\sigma^j_{P,t} = \gamma\sigma^j_{Q,t} + (1-\gamma)\left(\sigma^j_{E,t} + \sigma^j_{Q^*,t}\right) \quad , \quad j = M, M^*, G, G^* \ . \tag{32}$$

The *second* condition follows from the stochastic differentiation and combination of the aggregate consumption optimality condition (27), the cash constraint (11), and the capital accumulation equation (15). We end up with the following simple results:

$$\sigma^j_{Q,t} = \sigma^j_{M,t} \quad , \quad j = M, M^*, G^* \ , \tag{33}$$

$$\sigma^G_{Q,t} = \sigma^G_{M,t} + \sigma^G_G \ . \tag{34}$$

This states that domestic producer prices move one for one with real money balances in response to all shocks except the domestic fiscal shock, where they rise in addition in response to government spending. This is because such shocks reduce the supply of domestic output that is available for private consumption.

The *third* condition follows from the stochastic differentiation of the private sector asset stock combined with the flow equation for private sector assets. We obtain:

$$\sigma^M_M = \frac{n^f_t}{n^{fin}_t}\sigma^M_{E,t} + \frac{\left(n^m_t + n^h_t\right)}{n^{fin}_t}\sigma^M_{P,t} \quad , \tag{35}$$

$$\sigma^{M^*}_{M,t} = \frac{n^f_t}{n^{fin}_t}\sigma^{M^*}_{E,t} + \phi\frac{\left(n^m_t + n^h_t\right)}{n^{fin}_t}\sigma^{M^*}_{P,t} \quad , \tag{36}$$

$$\sigma^G_{M,t} = \frac{n^k_t}{n^{fin}_t}\sigma^G_G + \frac{n^f_t}{n^{fin}_t}\sigma^G_{E,t} \quad , \tag{37}$$

$$\sigma^{G^*}_{M,t} = \frac{n^f_t}{n^{fin}_t}\sigma^{G^*}_{E,t} \quad . \tag{38}$$

In equation (35) money is exogenous, while in equations (36)-(38) money is an endogenous variable that has to accommodate changes in exchange rates and prices in order to allow the government to support its desired bond stock B_t, or more intuitively to support the nominal interest rate implied by that bond stock. Equation (35) shows that exogenous domestic money supply shocks are related to price inflation in all three financial asset classes in proportion to their share in the overall asset portfolio. Equation (36) shows that an identical relationship holds if domestic fiscal policy compensates domestic households for the effects of foreign monetary volatility ($\phi = \phi^* = 1$). The reason is that, following an expansionary foreign money shock, domestic money supply is contracted to support fiscal policy as it taxes away the valuation gains resulting from exchange rate appreciation and lower prices. This however serves to amplify the appreciation, hence the much higher exchange rate volatility under this policy. Without fiscal compensation ($\phi = \phi^* = 0$ and equations (37) and (38)), the domestic money supply response to foreign shocks is minimal, partly due to the endogenously small portfolio share of foreign assets that transmit exchange rate shocks. Domestic fiscal shocks, by (37), do of course require a powerful monetary accommodation to revalue nominal asset stocks.

A key feature of equations (32)-(38) is that in this model prices and exchange rates are determined by the interplay of goods and asset market shares. The latter is completely absent in conventional models.

2.7 Computation of Equilibrium

The interesting part of our model's dynamics is fully captured by its instantaneous impulse responses, that is by its diffusions. Beyond that, dynamic paths are not informative because the underlying asset stocks evolve as nonstationary Itô processes. But because optimal portfolio shares, marginal propensities to consume out of wealth, and capital to wealth ratios are independent of asset stocks, an analysis of equilibria for a given set of state variables conveys all the necessary information. We therefore compute the model's equilibrium at a given point in time, for a given set of state variables a_t^c,$a_t^{c^*}$,K_t,K_t^*,M_t,M_t^*,$\tilde{f}_t$, and then examine its characteristics by conducting sensitivity analysis with respect to a number of key parameters. Each equilibrium is characterized by a large simultaneous equation system that can be solved by way of a Newton algorithm.

2.8 Government Bond Market Interventions

Section 5 will study large discontinuous government open market operations that exchange bonds B_t for money M_t. Because such events are rare, they are modeled as one-off, unanticipated events. Because they are discontinuous, five of the economy's seven state variables will also be discontinuous, namely the financial asset stocks a_t^c, $a_t^{c^*}$, M_t, M_t^* and $\tilde{f}_t$, leaving only K_t and K_t^* as state variables. We therefore need five additional relationships to determine the equilibrium jumps in asset stocks.

We note first that all net asset sales have to be transacted at the new rather than the old prices. We denote pre-intervention asset stocks by a bar above the respective variable, and post-intervention stocks by a time subscript t. The effects of any intervention depend critically on the extent to which the government sterilizes changes in the domestic money stock, or equivalently on the extent to which it matches changes in its domestic currency bond liabilities by changes in its foreign currency bond assets. We parameterize this by way of the policy parameters ξ and ξ^* in the following intervention equations:

$$E_t\left(F_t^g - \bar{F}^g\right) = \xi\left[\left(H_t^c - \bar{H}^c\right) + \left(H_t^{c^*} - \bar{H}^{c^*}\right) + \left(H_t^{g^*} - \bar{H}^{g^*}\right)\right] \quad , \tag{39}$$

$$E_t^*\left(H_t^{g^*} - \bar{H}^{g^*}\right) = \xi^*\left[\left(F_t^{c^*} - \bar{F}^{c^*}\right) + \left(F_t^c - \bar{F}^c\right) + \left(F_t^g - \bar{F}^g\right)\right] \quad . \tag{40}$$

In these equations $\xi = \xi^* = 1$ corresponds to full sterilization of all domestic open market operations through foreign exchange purchases, while $\xi = \xi^* = 0$ describes open market operations that translate fully to changes in the domestic money supply. Of course the reality of any given intervention could be described by many different combinations of ξ and ξ^*.

Household behavior is described simply by the requirement that purchases and sales of financial assets have to be transacted at the new exchange rate:

$$E_t\left(F_t^c - \bar{F}^c\right) = -\left[\left(H_t^c - \bar{H}^c\right) + \left(M_t - \bar{M}\right)\right] \quad , \tag{41}$$

$$E_t^*\left(H_t^{c^*} - \bar{H}^{c^*}\right) = -\left[\left(F_t^{c^*} - \bar{F}^{c^*}\right) + \left(M_t^* - \bar{M}^*\right)\right] \quad . \tag{42}$$

Similar conditions have to be true for government purchases and sales of financial assets. The condition for Home is

$$E_t\left(F_t^g - \bar{F}^g\right) = \left(H_t^c - \bar{H}^c\right) + \left(H_t^{c^*} - \bar{H}^{c^*}\right) + \left(H_t^{g^*} - \bar{H}^{g^*}\right) + \left(M_t - \bar{M}\right) \quad . \tag{43}$$

By Walras' Law we only need this one condition for Home, because the equivalent condition for Foreign is implied by (43) in combination with (41) and (42).

To simulate government bond market interventions we first simulate the baseline model without these five equations to obtain the pre-intervention asset stocks. We then add (39)-(43) and re-simulate after changing the exogenous policy variable, specifically the desired government debt to GDP ratio $bgdp_t = B_t/(P_t * gdp_t)$, to a post-intervention value.

3 The Baseline Economy

3.1 Calibration

We adopt a symmetric baseline calibration. For consumer preferences we assume significant home bias, with 80% of domestic demand falling on domestic goods, or $\gamma = \gamma^* = 0.8$. In production we assume that capital stocks in each economy equal 25, $K_t = K_t^* = 25$, and that capital has a constant real return of four percent, $r = r^* = 0.04$. This implies a real domestic output of one in each economy. We choose personal discount rates β and β^* to normalize consumption to one, $c_t = c_t^* = 1$,[14] and we choose money stocks M_t and M_t^* to normalize consumer prices to one, $P_t = P_t^* = 1$. Together with the rest of the calibration this also implies producer prices and real exchange rates approximately equal to one. Average inflation rates are pinned down by the choice of nominal money growth rates at 3 percent per annum, $\mu_t = \mu_t^* = 0.03$.

A critical part of calibrating the model concerns financial assets to GDP ratios. For symmetry we assume that the net foreign asset position between the two countries is zero. The government's foreign exchange reserves are assumed to cover 5 percent of its money stock, a small value that is consistent with central bank practice in most advanced industrialized countries. The government debt to GDP ratios are fixed at 40 percent, by allowing nominal interest rates to take values consistent with portfolio equilibrium at those debt ratios. To set the money stock to GDP ratio we choose M1 as representing the model's monetary aggregate M_t. This is a compromise between choosing the monetary base, which would be the appropriate choice to represent money in the government's budget constraint, and choosing M2, which is more appropriate to represent the quantity of money relevant to households' spending decisions, and therefore to the calibration of velocity α. M1 is in fact much closer in size to the monetary base than to M2, which means most importantly

[14] The resulting endogenous values for personal discount rates $\beta \simeq \beta^* \simeq 0.0395$ are of course very close to those for real returns to capital.

that interest rates will have a limited direct effect on consumption decisions via the cash constraint effect in optimality condition (27) because the implied velocity is large.

We use monthly U.S. data from January 1984 through December 2007 to calibrate monetary magnitudes in both countries. This implies a money stock to GDP ratio of 12 percent, and therefore requires $\alpha \simeq \alpha^* \simeq 8.333$. The same data can also be used to estimate the volatilities of exogenous shocks. The standard deviation of U.S. M1 growth over the sample period equals 0.05. For our baseline we assume that half of this reflects endogenous money supply responses to fiscal shocks, with the other half reflecting exogenous money supply shocks. For the fiscal policy $\phi = \phi^* = 1$ this requires $\sigma_M^M = \sigma_{M^*}^{M^*} = 0.025$ and $\sigma_G^G = \sigma_{G^*}^{G^*} = 0.000736$, while for $\phi = \phi^* = 0$ it requires $\sigma_M^M = \sigma_{M^*}^{M^*} = 0.03535$ and $\sigma_G^G = \sigma_{G^*}^{G^*} = 0.000707$.

3.2 Baseline Portfolio Equilibrium

The baseline equilibrium outcome for financial asset portfolios is striking. For the case of $\phi = \phi^* = 1$ domestic households hold domestic currency denominated government bonds equal to 39.4 percent of GDP, with the remainder held by the foreign government. Because the standard deviation of the exchange rate $\sigma_E = 0.086$ is much higher than that of domestic prices $\sigma_Q = 0.05$, households dislike foreign currency exposure and make domestic currency loans to foreign households equal to only 0.75 percent of GDP, with foreign households, by symmetry, behaving identically. In other words, households go long in domestic currency but take a small short position in foreign currency. This outcome is very different from the results on optimal cross-border equity holdings, which typically find that it is optimal for households to diversify by taking a large long position in foreign equity.

For the case of $\phi = \phi^* = 0$ domestic households also hold domestic currency denominated government bonds equal to 39.4 percent of GDP. But due to much lower exchange rate risk $\sigma_E = 0.01$ at an identical price risk $\sigma_Q = 0.05$, they take much larger positions in foreign private lending, making domestic currency loans to foreign households equal to 12.1 percent of GDP and borrowing an equal amount in foreign currency from foreign households.

The market clearing nominal interest rates consistent with these portfolio equilibria equal approximately 6.75% in each country. Households' foreign lending and borrowing exposures are exactly equal only because we have assumed zero net foreign assets both in the aggregate and for governments. We will comment on alternative scenarios below.

4 Comparing Alternative Economies

In this section we consider economies for which all aspects of the calibration are identical to the baseline except in one dimension. We stress that this is different from analyzing the response of the economy to shocks by way of impulse responses. To conserve space we omit a discussion of the low exchange rate volatility case $\phi = \phi^* = 0$ whenever the results do not contain many new insights.

In the graphs used to illustrate our results, each subplot shows the value of the varied parameter or variable along the horizontal axis, and the value of a number of key endogenous variables along the vertical axis. Interest rates and inflation rates are shown in percentage points, prices and allocations are shown in percent deviations from the baseline economy, and asset stocks are shown as stock to GDP ratios.

4.1 Standard Deviation of Monetary Shocks

Figure 2 varies σ_M^M around its baseline value of $2.5 * 10^{-2}$, specifically between $0.1 * 10^{-2}$ and $10 * 10^{-2}$. Values along the horizontal axis are shown as $\sigma_M^M * 10^2$.

Figure 2a illustrates the case of high exchange rate volatility $\phi = \phi^* = 1$. In Home price volatility increases sharply as monetary volatility increases. A Home monetary expansion, by depreciating the exchange rate and thereby reducing Foreign's consumer price index, leads to valuation gains on Foreign households' domestic currency assets, and the resulting Foreign tax increase leads to a monetary contraction that is roughly equal in size to Home's monetary expansion. As a result exchange rate volatility increases even more sharply than price volatility. Higher domestic price volatility increases the demand for local currency denominated assets relative to domestic capital. But because the capital stock is predetermined, there is no significant stock substitution possibility. Instead the greater attractiveness of domestic currency assets is reflected in the fact that the government is able to pay a lower nominal and real return as volatility increases. Over the range of monetary volatilities considered here, Home nominal interest rates fall from 6.8 percent to 5.8 percent, and the predictable or drift components of real asset returns $\tilde{r}_t^h$ and $\tilde{r}_t^f$, shown as "Real Return Drifts", fall from 3.8 percent to 2.9 percent.

Unlike for capital, there is a possibility of stock substitution between different currency bonds. The increase in exchange rate volatility therefore drives the optimal amount of private domestic currency lending and foreign currency borrowing down to only 0.1 percent of GDP. It also requires an increasing mean return on foreign currency assets, and therefore a significant increase in the deviation from real interest parity $\tilde{r}_t^h - \tilde{r}_t^f$.

As for nominal returns in Foreign, we observe that a policy of compensating asset holders for Home monetary volatility keeps nominal interest rates closely aligned as monetary volatility increases. In our specific calibration Foreign interest rates in fact fall slightly more. Ceteris paribus this stimulates the relative demand for Foreign output, so that Foreign prices are relatively higher as volatility increases. Higher prices however imply lower real wealth, so that Home consumption is in fact higher than Foreign consumption at higher volatilities, despite its slightly higher interest rate. More generally, when comparing the direct effect of interest rates on consumption to the wealth effect induced by the resulting price changes, the latter is always dominant. Because lower interest rates are associated with higher volatility, this means that more volatility is associated with lower output and consumption.

Figure 2b illustrates the case of low exchange rate volatility $\phi = \phi^* = 0$. Now Home monetary volatility no longer spills over to Foreign, and even fiscal shocks no longer induce much exchange rate volatility. As a result exchange rate volatility is both very low, and increases very slowly with monetary volatility. This has two implications. First, optimal foreign currency exposures are larger, and increasing volatility makes it more rather than less desirable to borrow in foreign currency and lend in domestic currency. Second, deviations from real interest parity are very small and barely increasing in monetary volatility. Nominal interest rates however diverge sharply, as higher domestic volatility that does not spill over to Foreign allows the Home government to pay a lower nominal rate. This however puts upward pressure on domestic prices and downward pressure on wealth. Home consumption is therefore lower at higher levels of Home volatility, while Foreign consumption is higher. The larger interest rate divergence compared to Figure 2a means that these real effects are much larger.

4.2 Standard Deviation of Fiscal Shocks

Figure 3 varies σ_G^G around its baseline value of $\sigma_G^G = 0.736 * 10^{-3}$, specifically between $0.01 * 10^{-3}$ and $2.5 * 10^{-3}$. Values along the horizontal axis are shown as $\sigma_G^G * 10^3$.

In this figure we only illustrate the case of high exchange rate volatility $\phi = \phi^* = 1$. Price volatility in Home increases sharply as fiscal volatility increases. But the effect on exchange rate volatility is more muted because Foreign fiscal policy does not tax away Home induced valuation gains under this shock. Higher domestic price volatility, both absolutely and relative to exchange rate volatility, has two effects. First it increases the demand for local currency denominated assets and allows the Home government to pay lower nominal and real interest rates on its debt. Because of low volatility spillovers the Foreign interest rate is virtually unaffected. Second it strongly increases the incentive to increase private borrowing in foreign currency and private lending in domestic currency.

Figure 3 addresses the potential criticism that our baseline assumption about the share of monetary volatility that is due to fiscal factors (50 percent) was arbitrary, by varying that share between close to zero and close to 100 percent. The main conclusion is that portfolios remain determinate and well-behaved even as fiscal volatility becomes very small.

4.3 Government Debt to GDP Ratios

In our baseline calibration the government debt to GDP ratio is held constant at 40 percent. Figure 4 varies that ratio between 0 percent and 80 percent, again concentrating on the case of high exchange rate volatility $\phi = \phi^* = 1$.[15] The key implication of larger nominal government obligations is that they constitute a larger base for the stochastic inflation tax, which implies that any given shock causes less volatility of domestic prices. This can be clearly seen in equation (22) for domestic fiscal shocks, and it also obtains for foreign monetary shocks. Because less volatility decreases the amount of bonds that households are willing to hold, the government is required to pay a higher interest rate on its debt. Figure 4 shows that this effect on volatilities and interest rates gets exponentially smaller as government debt increases, which reflects decreasing returns of increases in the inflation tax base. At very low levels of outstanding government debt the real interest rate has to rise by more than 20 basis points for a 1 percentage point increase in the government debt to GDP ratio, while at more elevated debt levels that elasticity drops to around 1 basis point. Interestingly, over the relevant range these values are of the same order of magnitude as the empirical results of Engen and Hubbard (2004) and Laubach (2003), who estimate that elasticity to be between 3 and 6 basis points.

5 Open Market Operations in Government Debt

Figure 5 assumes that the economy starts from a baseline government debt to GDP ratio of 40 percent, and that thereafter, in order to change that ratio, the government has to conduct asset market interventions in accordance with equations (39)-(43). We assume that these interventions, both the policy change in Home and the endogenous response in Foreign, take the form of open market operations that exchange bonds for money, $\xi = \xi^* = 0$, without affecting the government's *stock* of foreign exchange reserves.[16] Qualitatively very similar

[15] The main results are very similar for $\phi = \phi^* = 0$.
[16] Their *value* can of course change.

effects are observed for both the high and low exchange rate volatility cases.

Open market sales increase the base of the inflation tax in Home and therefore reduce volatility of Home prices, while the volatility of Foreign prices increases. The result of these opposing trends is that exchange rate volatility is U-shaped around the 40 percent of GDP mark. This implies that private sector gross lending positions decrease with large enough open market sales or purchases. To allow Home to place additional government debt with households in the face of lower volatility it must raise the nominal interest rate. Given higher volatility in Foreign, the government there can lower interest rates.

A large unsterilized open market sale causes a large contraction of the nominal money stock. Given the cash constraint, the proportionality of consumption to the state variable wealth, and the small direct effect of interest rate changes on consumption, the contraction in the money stock must lead to a nearly proportional fall in the price level and the exchange rate, with real money balances remaining nearly constant. The lower price level increases the real value of Home wealth and supports higher consumption.

We now turn to net foreign asset valuation effects. In our baseline the public sector is assumed to take a long position in foreign currency while being short domestic currency. The private sector on the other hand optimally chooses the opposite exposure. When prices and the exchange rate fall, the public sector therefore loses as the real value of its liabilities rises, while the private sector gains as the real value of its assets rises. The overall valuation gain of Home depends on the economy's aggregate foreign currency exposure. In our high exchange rate volatility example the private sector's exposures are barely larger than those of the public sector. We therefore observe a very small positive valuation effect. Under low exchange rate volatility the private sector's positions are very much larger, and consequently valuation effects are also large at 0.2 percent of GDP for every 1 percent of GDP expansion of the government debt stock. Due to these valuation effects the consumption gains following an open market sale are double those of the high exchange rate volatility case.

As can be seen by inspecting the reaction of money and prices in Foreign in Figure 5, the effects of open market operations in Home are only very marginally different if Foreign decided to sterilize their effects on its money supply, $\xi^* = 1$.

Figure 6 illustrates the same open market operations as Figure 5, but with a different baseline where both governments issue debt equal to 60 percent of GDP and hold foreign exchange reserves equal to 20 percent of GDP. This makes government foreign exchange positions much larger than private sector positions, which are not materially different in this new baseline. The key implication is of course that the valuation effects of open market sales are now negative, and more so for the case of high exchange rate volatility.

Our final result, which is implied by the foregoing, is harder to illustrate graphically. However it may be the most important in terms of its policy implications. This is that under volatile exchange rates a large aggregate negative net foreign asset position must have a large inter-governmental component because it is hard to rationalize as a result of private sector portfolio choice. To show this we compute equilibria, starting at the baseline, where the Home economy's net foreign assets to GDP ratio deteriorates accompanied by an equal sized increase in its government debt to GDP ratio, but without a concomitant increase in the foreign exchange reserves of Foreign's government. The additional government debt therefore needs to be absorbed by the Home and Foreign private sectors. The Home private sector can be shown to do this by switching its domestic currency lending from Foreign's private sector to its own government while maintaining its Foreign currency borrowing roughly unchanged. But the extent to which it is willing to do this is very limited especially for high exchange rate volatility, where equilibria can no longer be computed at around minus 0.5 percent of GDP negative net foreign assets. For low exchange rate volatility the limit is reached at around minus 10 percent of GDP. If this model is taken at face value, this would have significant implications for the ability of the U.S. government to attract private financing for its foreign indebtedness if foreign governments' desired holdings of U.S. government debt should decline.

6 Conclusions

This paper has developed a theory of international currency portfolios that holds in general equilibrium, and that is therefore not subject to the criticisms directed at the portfolio balance literature of the 1980s. The key ingredients needed to obtain this result are more plausible assumptions about fiscal policy than have so far been considered by the literature. The key implication is that the relationship between the rates of return of bonds denominated in different currencies is not correctly described by an arbitrage relationship but instead also depends on outstanding bond stocks.

This gives rise to several additional results. First, there is a monotonically increasing relationship between domestic nominal interest rates and the portfolio share of domestic currency denominated assets. This relationship is steep at low levels of government debt, and almost flat at high levels of government debt. The latter is consistent with the fact that traditional empirical tests of the portfolio balance model have only found small effects of bond stocks on risk premia in industrialized countries. Second, optimal private sector foreign currency positions are negative, and their size is decreasing in exchange rate volatility. Large negative aggregate net foreign asset positions can only be rationalized by assuming large public sector borrowing from foreign governments, especially when exchange rates are highly volatile. Third, for a baseline calibration with zero aggregate net foreign assets, unsterilized open market sales of domestic government debt appreciate the currency, increase output and consumption, and lead to valuation gains (losses) when the country as a whole has a negative (positive) exposure to foreign currency. Fourth, a fiscal theory of exchange rate determination is compatible with general equilibrium in a two-country world.

The focus of this paper is distinct from the growing literature on international portfolio diversification in DSGE models. That literature focuses on diversification among alternative private claims whose imperfect substitutability is given, and where the challenge is the computation rather than the existence of a portfolio equilibrium. The older portfolio balance literature was largely abandoned precisely because for government bonds imperfect substitutability could not be derived from first principles. This paper attempts to do so, and thereby to reconnect to that older literature. This is in fact an important task, because the currently existing global imbalances problems are not just, or even mainly, a question of private claims.

But of course private claims are also important, and this paper has abstracted from their equity component by assuming complete home bias in equity. In future work this model will therefore be extended to allow for internationally traded equity. That model will then connect to both the older and the more recent literatures on international portfolio choice.

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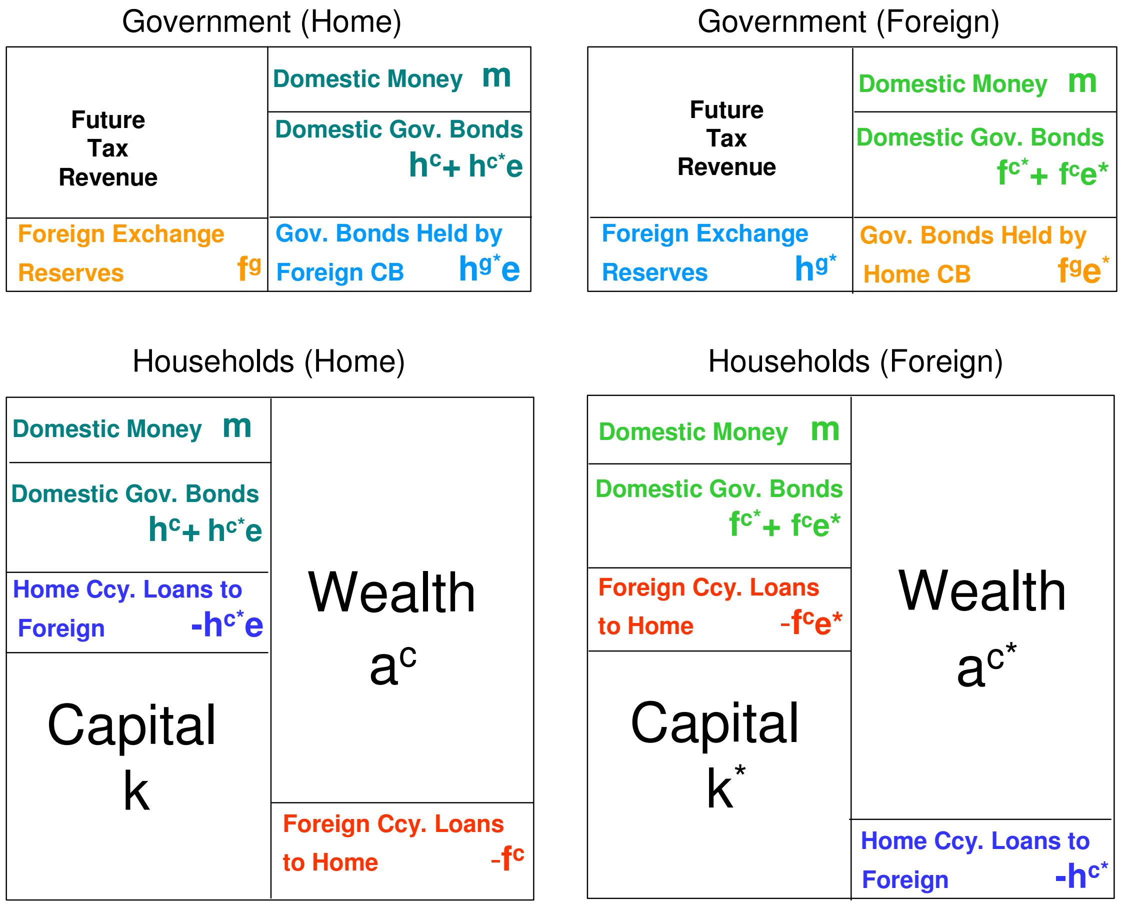


Figure 1: Household and Government Balance Sheets

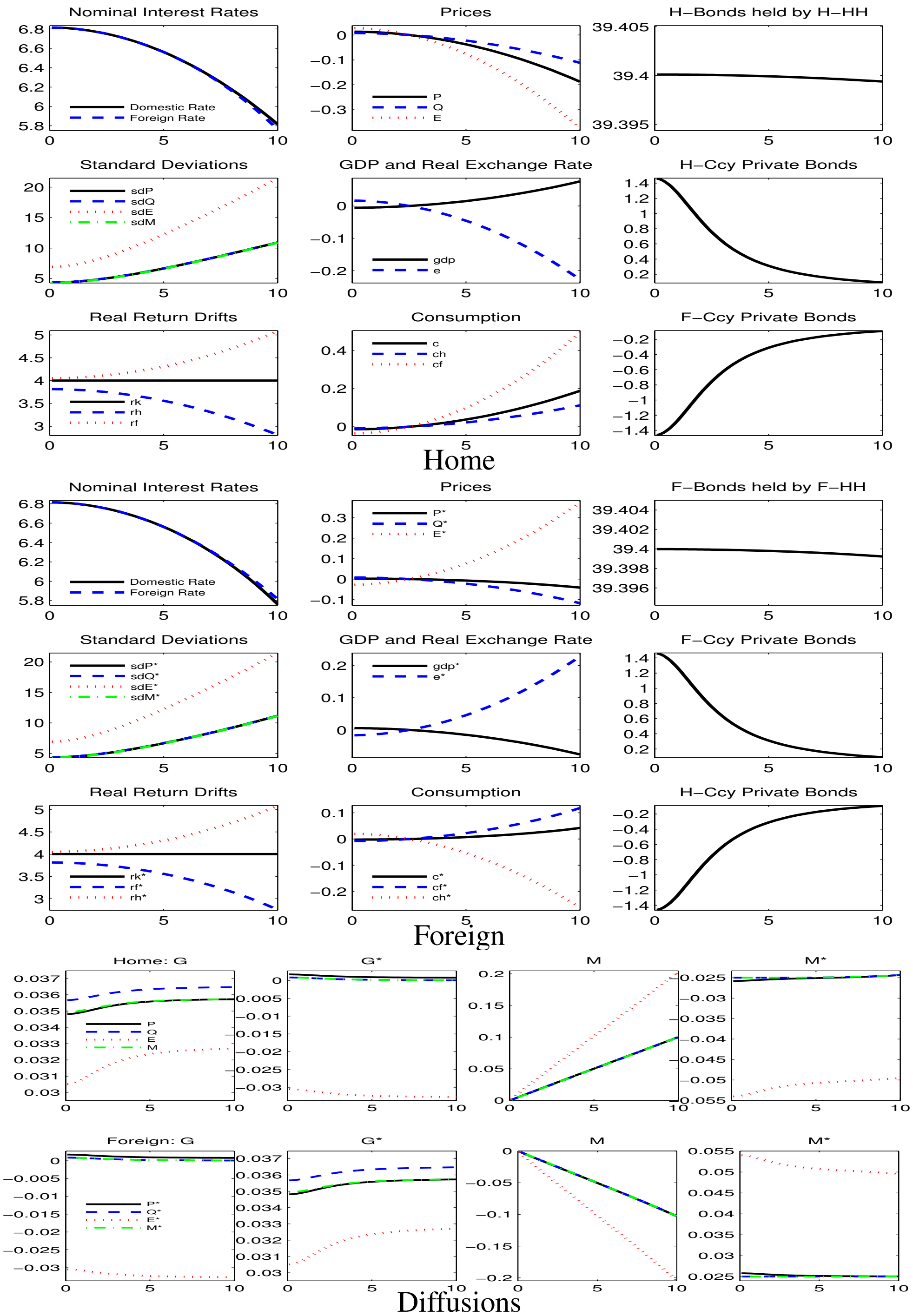


Figure 2a: Effects of Money Supply Volatility, $\phi = \phi^* = 1$

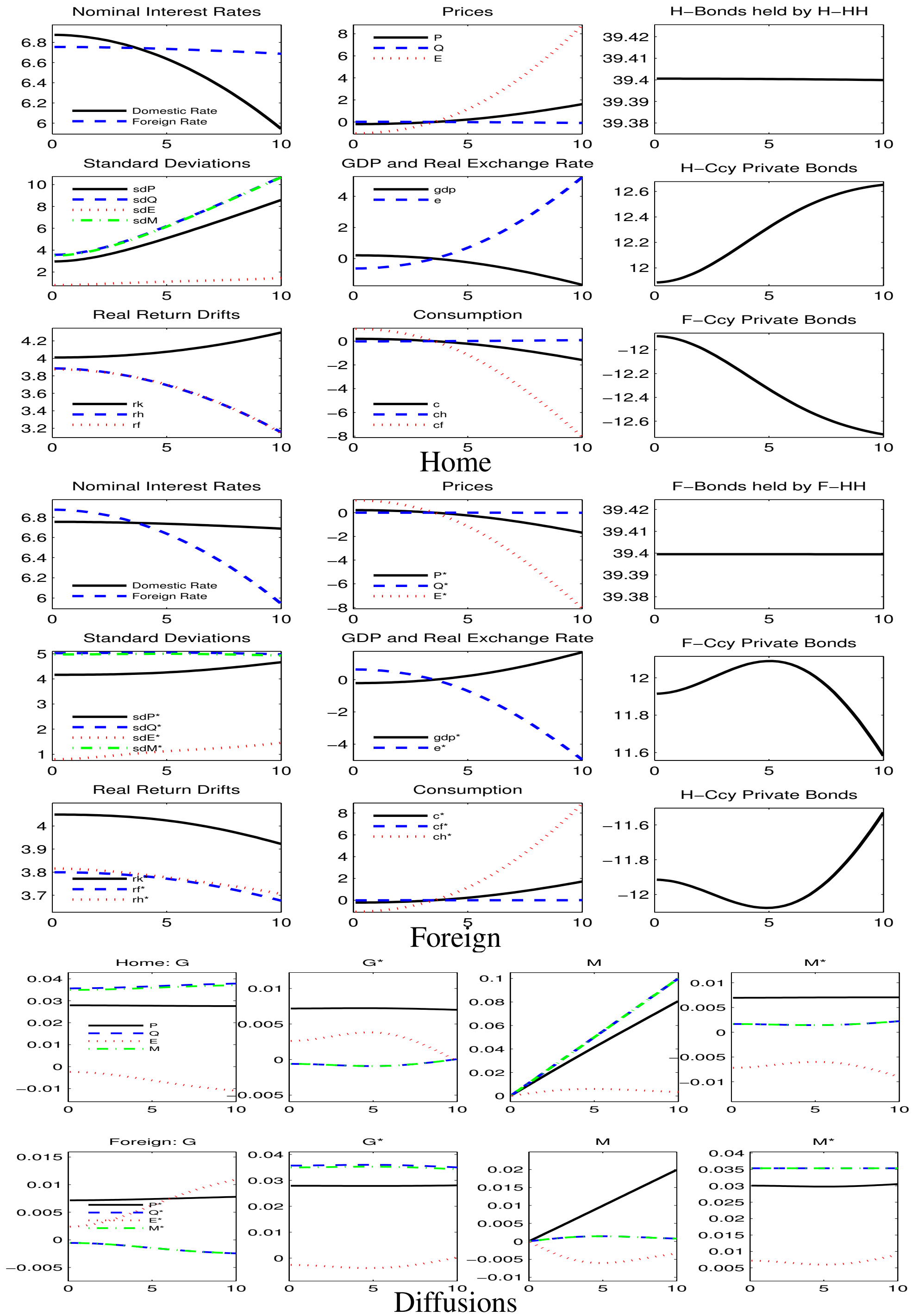


Figure 2b: Effects of Money Supply Volatility, $\phi = \phi^* = 0$

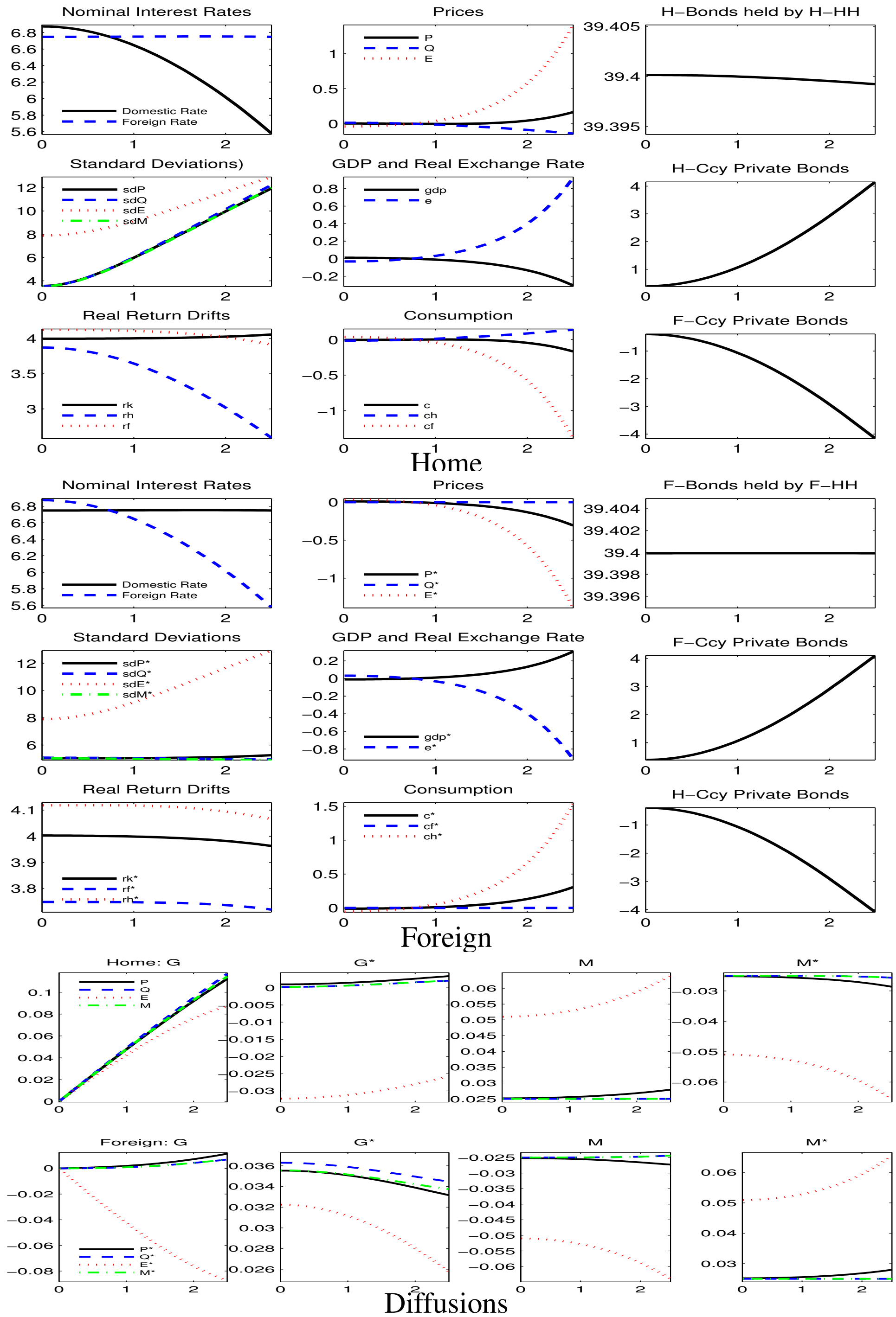


Figure 3: Effects of Government Spending Volatility, $\phi = \phi^* = 1$

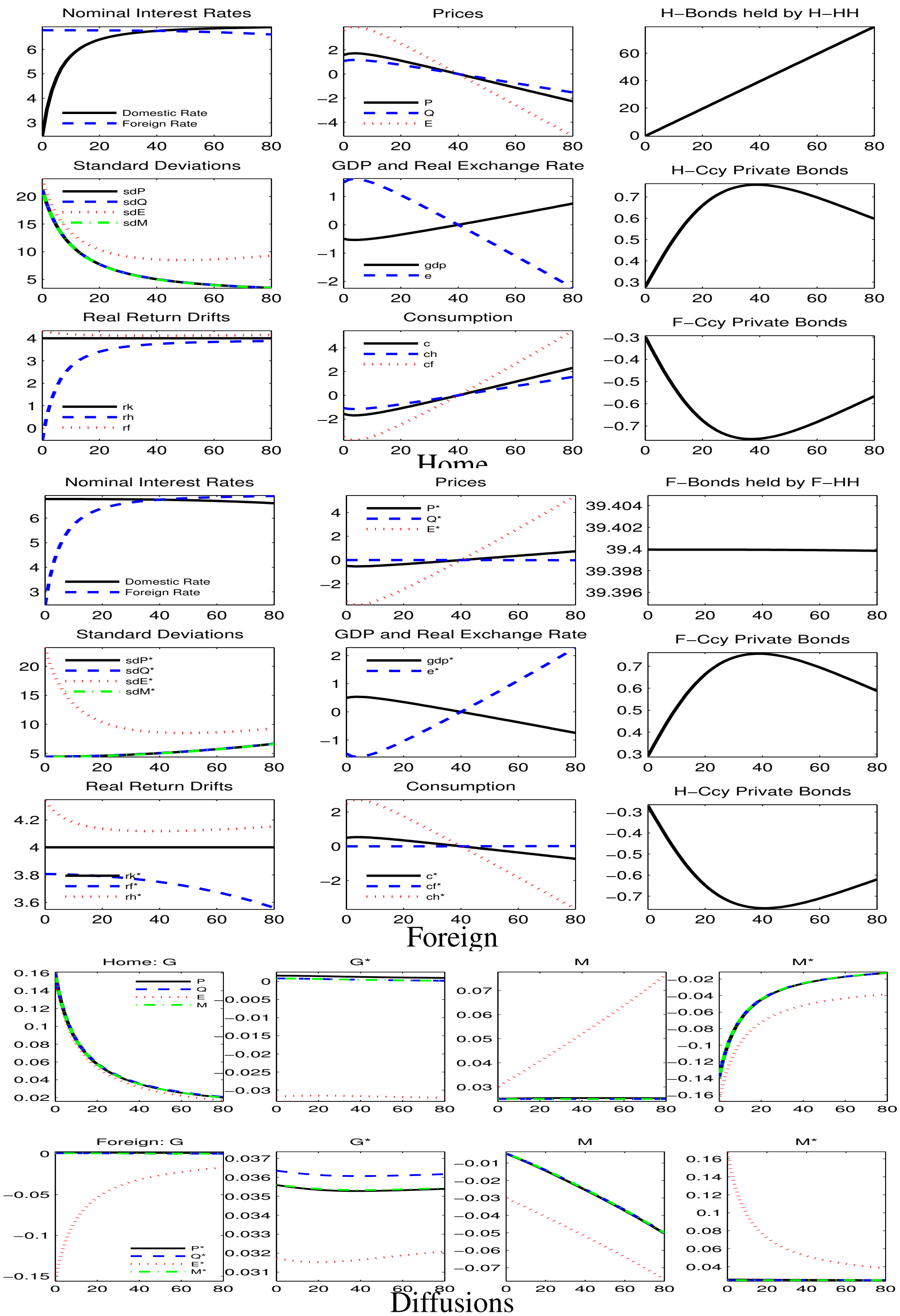


Figure 4: Effects of Government Debt, $\phi = \phi^* = 1$

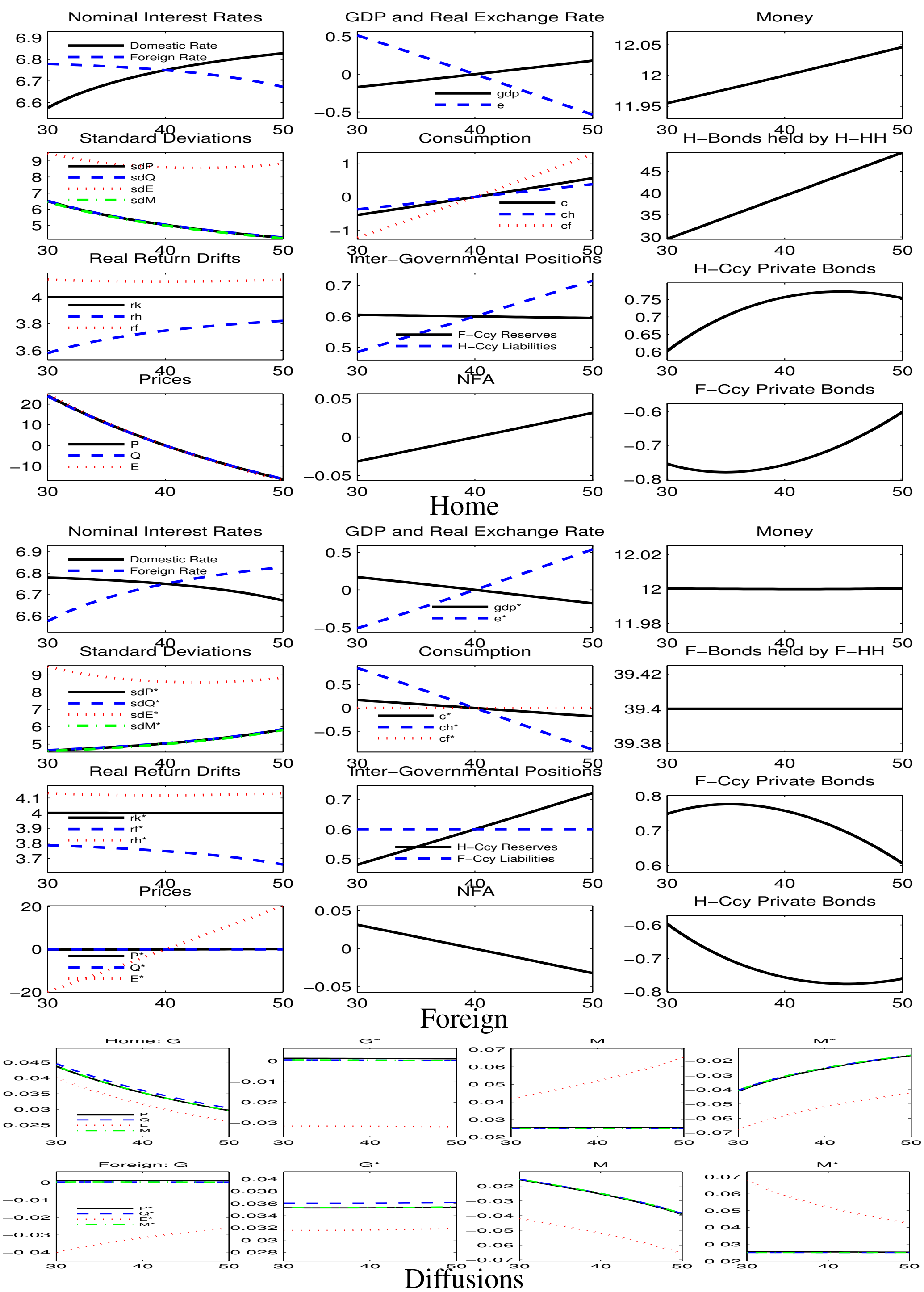


Figure 5a: Home Open Market Operations, $\phi = \phi^* = 1$

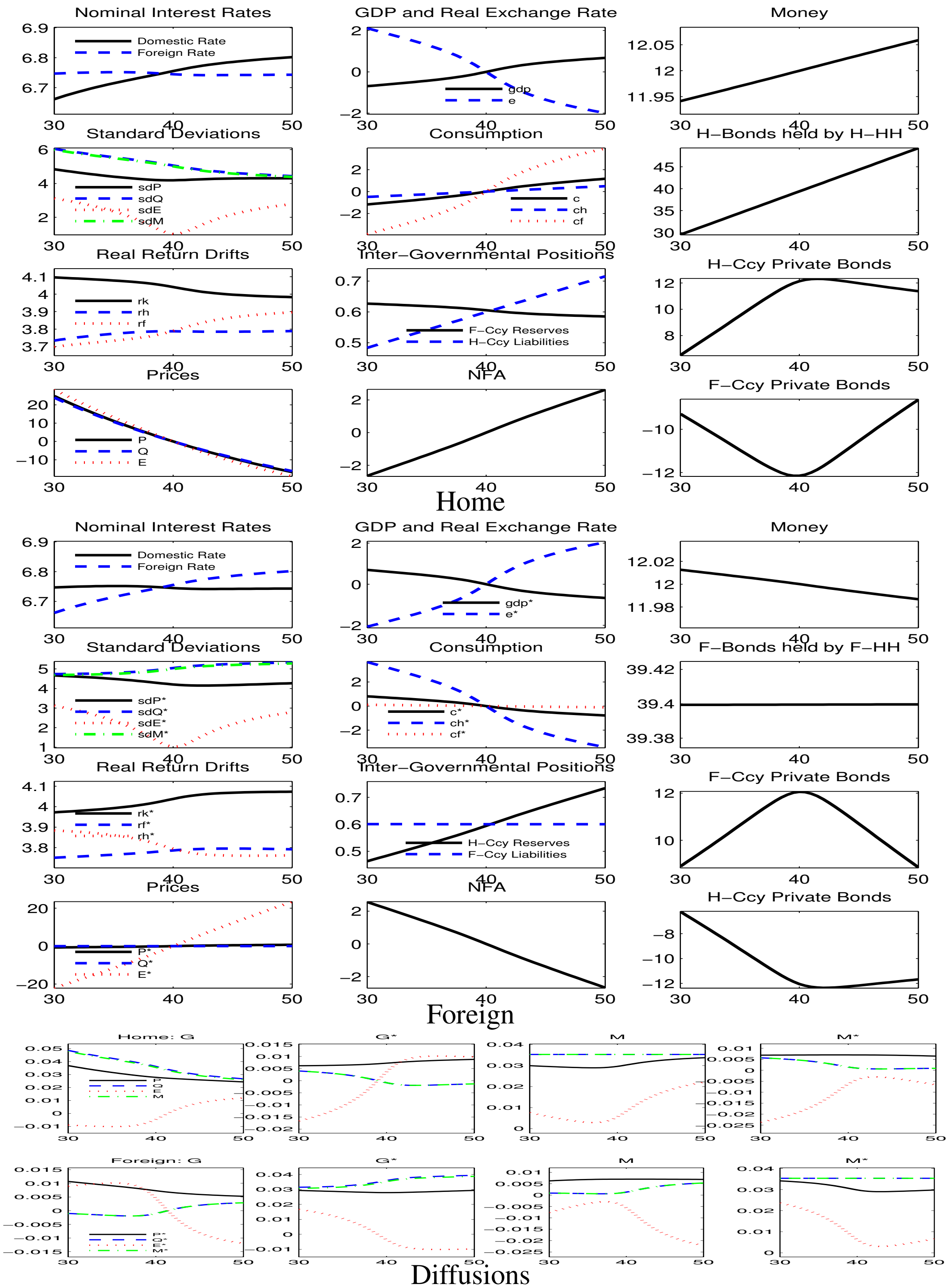
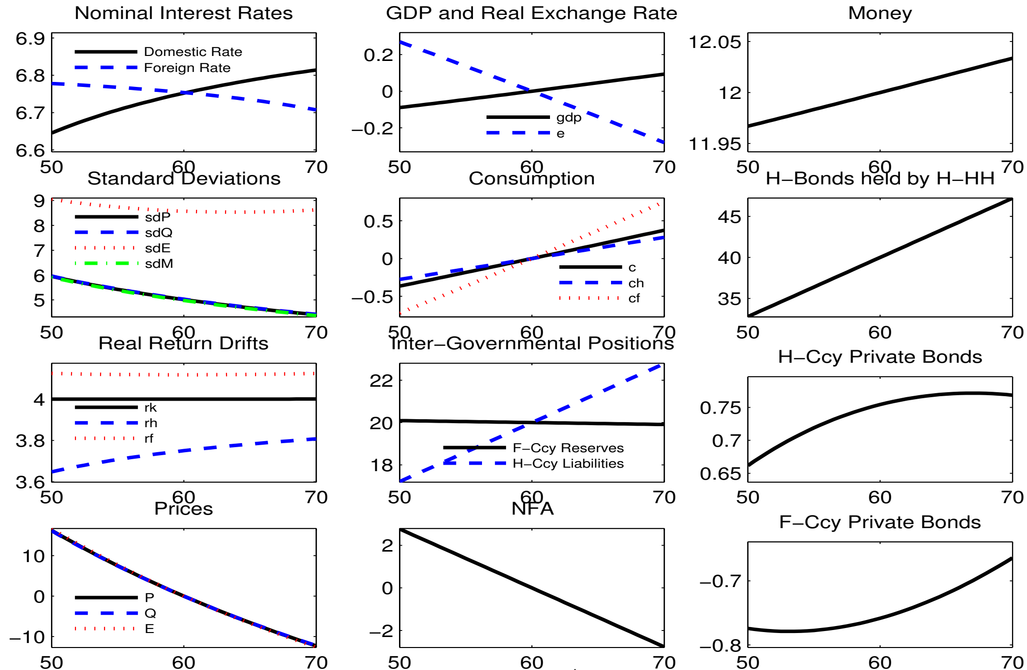
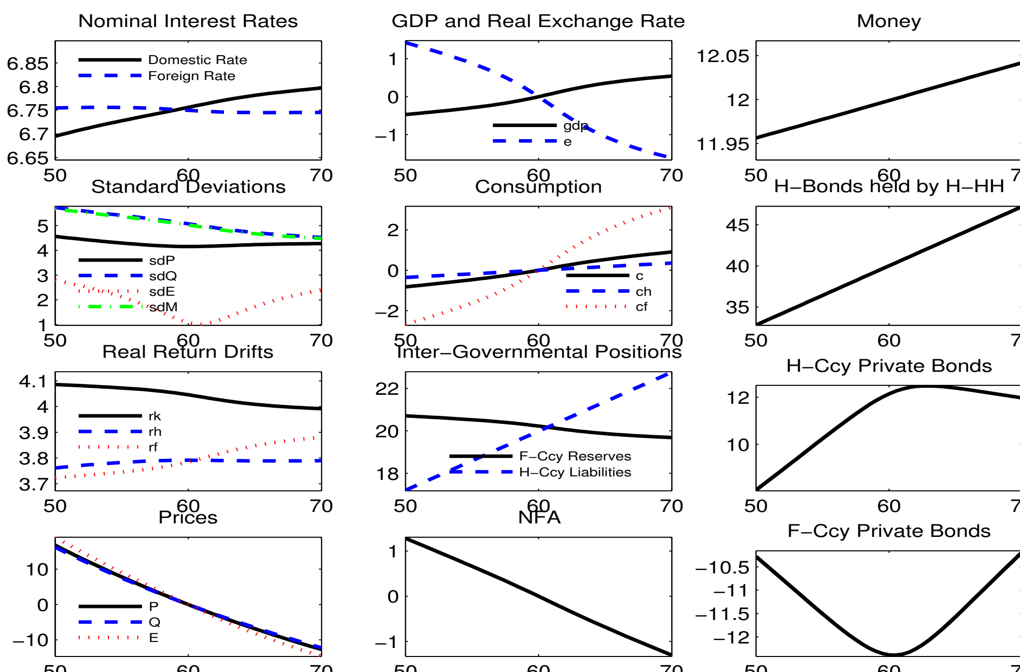


Figure 5b: Home Open Market Operations, $\phi = \phi^* = 0$



Home, $\phi = \phi^* = 1$



Home, $\phi = \phi^* = 0$

Figure 6: Home Open Market Operations, Large Gross FX Positions