Pricing Policies and Inflation Dynamics

TECHNICAL APPENDIX

(Not for Publication)

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1 Unit Root in the Inflation Target

- Unit root shocks to the gross inflation target π_t^* are a key feature of the model.
- As we will normalize by this process throughout this Technical Appendix rather than at the end, we present it here first.
- To make the model stationary, all nominal variables must be normalized by the target price level P_t^*. The unnormalized growth rate of prices (inflation) is a non-stationary variable and therefore needs to be eliminated from the equation system by suitable normalization.
- Inflation target shocks are given by:
  - In Levels:
$$\pi_t^* = \pi_{t-1}^* \varepsilon_t^{\pi^*} \ .$$
  - Linearized:
$$\hat{\pi}_t^* = \hat{\pi}_{t-1}^* + \hat{\varepsilon}_t^{\pi^*} \ .$$

2 Households

- Objective Function:
$$Max \quad E_t \sum_{k=0}^{\infty} \beta^k \left\{ \ln\left(C_{t+k}\right) - \kappa \frac{L_{t+k}^{1+\gamma}}{1+\gamma} + \frac{a}{1-\epsilon} \left(\frac{M_{t+k}}{P_{t+k}} \right)^{1-\epsilon} \right\} \ ,$$
$$C_t = c_t - \nu c_{t-1} \ .$$
- Budget Constraint (multiplier = Λ_t):
$$B_t = i_{t-1} B_{t-1} + M_{t-1} - M_t + W_t L_t + \int_0^1 \Pi_t(j) dj + P_t \tau_t - P_t c_t \ .$$
- Note 1: Linearization for any variable x around its steady state $\bar{x}$ uses the following notation: $\hat{x} = (x_t - \bar{x})/\bar{x}$.
- Note 2: All interest rates and inflation rates are gross rates.

- FOC for c_t ($\lambda_t = \Lambda_t P_t$):

$$C_t^{-1} - \beta\nu E_t C_{t+1}^{-1} = \lambda_t \ .$$

  - Steady state:

$$\bar{C}^{-1}(1-\beta\nu) = \bar{\lambda} \ .$$

  - Linearization:

$$\beta v E_t \hat{C}_{t+1} = \hat{C}_t + (1-\beta v)\hat{\lambda}_t \ .$$

  - Linearization of habit relationship:

$$\hat{C}_t = \frac{1}{1-v}\hat{c}_t - \frac{v}{1-v}\hat{c}_{t-1} \ .$$

  - Combine the two ($\hat{q}_t^c = \hat{c}_{t-1}$):

$$\frac{\beta\nu}{1-v}E_t\hat{c}_{t+1} = \frac{1+\beta\nu^2}{1-v}\hat{c}_t - \frac{\nu}{1-v}\hat{q}_t^c + (1-\beta\nu)\hat{\lambda}_t \ . \tag{1}$$

- FOC for B_t:

$$\lambda_t = \beta i_t E_t\left(\frac{\lambda_{t+1}}{\pi_{t+1}}\right) \ .$$

  - Normalization ($\check{i}_t = i_t/\pi_t^*$, $\check{\pi}_{t+1} = \pi_{t+1}/\pi_{t+1}^*$):

$$\lambda_t = \beta\check{i}_t E_t\left(\frac{\lambda_{t+1}}{\check{\pi}_{t+1}\varepsilon_{t+1}^{\pi^*}}\right) \ .$$

  - Linearization (remember that $E_t\hat{\varepsilon}_{t+1}^{\pi^*} = 0$):

$$\hat{\lambda}_t - \hat{\imath}_t = E_t\left(\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\right) \ . \tag{2}$$

- FOC for L_t:

$$\kappa L_t^\gamma = \lambda_t w_t \ .$$

  - Linearization:

$$\hat{w}_t + \hat{\lambda}_t = \gamma\hat{L}_t \ . \tag{3}$$

- Consumption -labor relationship:

$$\hat{c}_t = \hat{z}_t + \hat{L}_t \ . \tag{4}$$

3 Firms

3.1 Maximization Problem

- Discounted nominal profits:
  - Nominal revenue $P_t(j)y_t(j)$, where $y_t(j)$ is output.
  - Nominal wage bill $W_t\ell_t(j)$.
  - Nominal k-period ahead gross interest rate i_t^k.
- Pricing policy of firm j that reoptimizes at t, choosing V_t and v_t (a gross inflation rate):

$$P_{t+k}(j) = V_t^j \left(v_t^j\right)^k \ .$$

- Profit maximization:

$$\underset{V_t^j, v_t^j}{Max} \quad E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} \left[\frac{P_{t+k}(j) y_{t+k}(j)}{P_{t+k}} - w_{t+k}\ell_{t+k}(j)\right] \quad , \ s.t.$$

$$y_{t+k}(j) = z_{t+k}\ell_{t+k}(j) \ ,$$

$$y_{t+k}(j) = c_{t+k}\left(\frac{V_t^j (v_t^j)^k}{P_{t+k}}\right)^{-\sigma} \ .$$

- Substitute constraints:

$$\underset{V_t^j, v_t^j}{Max} E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} \left[\left(\frac{V_t^j (v_t^j)^k}{P_{t+k}} - \frac{w_{t+k}}{z_{t+k}}\right)\left(c_{t+k}\left(\frac{V_t^j (v_t^j)^k}{P_{t+k}}\right)^{-\sigma}\right)\right] \ .$$

- Can drop superscript j.
- Define terms:
  - Front-loading term:

$$p_t \equiv \frac{V_t}{P_t} \ .$$

  - Inflation rates re-scaled by the inflation target, with $\check{v}_t = v_t/\pi_t^*$, $\check{\pi}_t = \pi_t/\pi_t^*$.
  - Cumulative aggregate normalized inflation:

$$\check{\Pi}_{t,k} = \Pi_{j=1}^k \check{\pi}_{t+j} \text{ for } k \geq 1 \ \ (\equiv 1 \text{ for } k = 0) \ .$$

  - Cumulative aggregate inflation deviation:

$$\hat{\Pi}_{t,k} = \Sigma_{j=1}^k \hat{\pi}_{t+j} \text{ for } k \geq 1 \ \ (\equiv 0 \text{ for } k = 0) \ .$$

3.2 First-Order Conditions

- FOC w.r.t. V_t:

$$E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} \left[\left(\frac{V_t {v_t}^k}{P_{t+k}} (1-\sigma) + \sigma \frac{w_{t+k}}{z_{t+k}} \right) \left(c_{t+k} \left(\frac{V_t {v_t}^k}{P_{t+k}} \right)^{-\sigma} \right) \right] = 0 \ .$$

  - Equivalently:

$$E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} \left[\left(p_t \frac{{v_t}^k}{\Pi_{t,k}} (\sigma-1) - \sigma \frac{w_{t+k}}{z_{t+k}} \right) \left(c_{t+k} \left(p_t \frac{{v_t}^k}{\Pi_{t,k}} \right)^{-\sigma} \right) \right] = 0 \ .$$

  - Equivalently:

$$p_t = \mu \frac{E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \frac{w_{t+k}}{z_{t+k}}}{E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \left(\frac{(\check{v}_t)^k}{\check{\Pi}_{t,k}} \right)} \ , \tag{5}$$

  where $\mu = \frac{\sigma}{\sigma-1}$.

- FOC w.r.t. v_t:

$$p_t = \mu \frac{E_t \sum_{k=0}^{\infty} (\delta\beta)^k k \lambda_{t+k} y_{t+k}(j) \frac{w_{t+k}}{z_{t+k}}}{E_t \sum_{k=0}^{\infty} (\delta\beta)^k k \lambda_{t+k} y_{t+k}(j) \left(\frac{(\check{v}_t)^k}{\check{\Pi}_{t,k}} \right)} \ . \tag{6}$$

- Re-scaling by the inflation target - comments:
  - This takes the form of dividing all price level rates of change by the inflation target π_t^*.
  - In the preceding formulas this only affects $E_t \frac{(v_t)^k}{\Pi_{t,k}}$, i.e. future firm-specific and aggregate cumulative inflation rates have to be deflated by future cumulative target inflation rates.
  - However, under unit roots all **expected** future quarterly target inflation rates are simply equal to the current target rate.
  - In all **forward-looking** conditions like (5) and (6) we can therefore simply linearize around the current inflation target π_t^*.
  - Example: Our final conditions will contain **expected** inflation terms for periods t and $t+1$. For actual future inflation the correct definition is $\hat{\pi}_{t+1} = \ln(\pi_{t+1}) - \ln(\pi_{t+1}^*)$. But it is OK to linearize around π_t^* instead because only the **expected** future inflation deviation enters the equations, and for this we have:

$$E_t \hat{\pi}_{t+1} = E_t \left(\ln(\pi_{t+1}) - \ln(\pi_{t+1}^*) \right) = E_t \left(\ln(\pi_{t+1}) - \ln(\pi_t^*) \right)$$

  - The same is not true for **backward-looking** conditions. See the section on “The Price Index”.

3.3 Linearization

- Geometric distribution formulas:

$$\sum_{k=0}^{\infty} (\delta\beta)^k k = \frac{\delta\beta}{(1-\delta\beta)^2} , \tag{7}$$

$$\sum_{k=0}^{\infty} (\delta\beta)^k k^2 = \frac{\delta\beta(1+\delta\beta)}{(1-\delta\beta)^3} . \tag{8}$$

- Linearization for V_t:
  - Rewrite (5):

$$p_t E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \left(\frac{\check{v}_t^{\,k}}{\check{\Pi}_{t,k}} \right) = \mu E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \frac{w_{t+k}}{z_{t+k}} .$$

  - Linearization of the common terms $\lambda_{t+k} y_{t+k}(j)$ cancels.
  - Remaining terms are:

$$\frac{\hat{p}_t}{1-\delta\beta} + E_t \sum_{k=0}^{\infty} (\delta\beta)^k k \hat{v}_t = E_t \sum_{k=0}^{\infty} (\delta\beta)^k \left[\hat{\Pi}_{t,k} + \hat{w}_{t+k} - \hat{z}_{t+k} \right] .$$

  - Equivalently:

$$\frac{\hat{p}_t}{1-\delta\beta} + \frac{\hat{v}_t \delta\beta}{(1-\delta\beta)^2} = E_t \sum_{k=0}^{\infty} (\delta\beta)^k \left[\hat{\Pi}_{t,k} + \hat{w}_{t+k} - \hat{z}_{t+k} \right] . \tag{9}$$

- Linearization for v_t:
  - Rewrite (6):

$$p_t E_t \sum_{k=0}^{\infty} (\delta\beta)^k k \lambda_{t+k} y_{t+k}(j) \left(\frac{\check{v}_t^{\,k}}{\check{\Pi}_{t,k}} \right) = \mu E_t \sum_{k=0}^{\infty} (\delta\beta)^k k \lambda_{t+k} y_{t+k}(j) \frac{w_{t+k}}{z_{t+k}} .$$

  - Linearization of the common terms $\lambda_{t+k} y_{t+k}(j)$ cancels.
  - Applying (7) to the $\hat{p}_t$-term, the remaining terms are:

$$\frac{\hat{p}_t \delta\beta}{(1-\delta\beta)^2} + E_t \sum_{k=0}^{\infty} (\delta\beta)^k k^2 \hat{v}_t = E_t \sum_{k=0}^{\infty} (\delta\beta)^k k \left[\hat{\Pi}_{t,k} + \hat{w}_{t+k} - \hat{z}_{t+k} \right] .$$

  - Applying (8) to the $\hat{v}_t$-term and simplifying further, we get:

$$\frac{\hat{p}_t \delta\beta}{(1-\delta\beta)^2} + \frac{\hat{v}_t \delta\beta(1+\delta\beta)}{(1-\delta\beta)^3} = E_t \sum_{k=0}^{\infty} (\delta\beta)^k k \left[\hat{\Pi}_{t,k} + \hat{w}_{t+k} - \hat{z}_{t+k} \right] . \tag{10}$$

3.4 Quasi-Differencing

3.4.1 Quasi-Differencing for (9)

- Note the following for $\hat{\Pi}_{t+1,k}$:

$$\begin{aligned}\hat{\Pi}_{t+1,k} &= 0 \text{ for } k=0, \\ &= \hat{\pi}_{t+2} \text{ for k=1} \\ &= \hat{\pi}_{t+2} + ... + \hat{\pi}_{t+k+1} \text{ for k=2,3,4,...}\end{aligned} \tag{11}$$

- Rewrite (9) as:

$$\hat{p}_t + \frac{\hat{v}_t \delta\beta}{(1-\delta\beta)} = (1-\delta\beta) E_t \sum_{k=0}^{\infty} (\delta\beta)^k \left[\hat{\Pi}_{t,k} + \hat{w}_{t+k} - \hat{z}_{t+k}\right] . \tag{12}$$

- For future reference, lead this by one period and multiply by $(1-\delta\beta)$:

$$(1-\delta\beta) E_t \hat{p}_{t+1} + \delta\beta E_t \hat{v}_{t+1} = (1-\delta\beta)^2 E_t \left[\sum_{k=0}^{\infty} (\delta\beta)^k \left[\hat{w}_{t+1+k} - \hat{z}_{t+1+k}\right] + \sum_{k=1}^{\infty} (\delta\beta)^k \hat{\Pi}_{t+1,k}\right] . \tag{13}$$

For the last term the subscript runs from 1 because $\hat{\Pi}_{t+1,0} = 0$.

- Write out terms in (12):

$$\hat{p}_t + \frac{\hat{v}_t \delta\beta}{(1-\delta\beta)} = (1-\delta\beta) *$$

$$E_t \left[(\hat{w}_t - \hat{z}_t) + (\delta\beta)(\hat{w}_{t+1} - \hat{z}_{t+1} + \hat{\pi}_{t+1}) + (\delta\beta)^2 (\hat{w}_{t+2} - \hat{z}_{t+2} + \hat{\pi}_{t+1} + \hat{\pi}_{t+2}) \right.$$
$$\left. + (\delta\beta)^3 (\hat{w}_{t+3} - \hat{z}_{t+3} + \hat{\pi}_{t+1} + \hat{\pi}_{t+2} + \hat{\pi}_{t+3}) + ...\right] .$$

- Multiply the last equation by $\delta\beta$ and lead by one period:

$$\delta\beta E_t \hat{p}_{t+1} + \frac{(\delta\beta)^2}{(1-\delta\beta)} E_t \hat{v}_{t+1} = (1-\delta\beta) *$$

$$E_t \left[(\delta\beta)(\hat{w}_{t+1} - \hat{z}_{t+1}) + (\delta\beta)^2 (\hat{w}_{t+2} - \hat{z}_{t+2} + \hat{\pi}_{t+2}) \right.$$
$$\left. + (\delta\beta)^3 (\hat{w}_{t+3} - \hat{z}_{t+3} + \hat{\pi}_{t+2} + \hat{\pi}_{t+3}) + ...\right] .$$

- Deduct the last equation from the preceding one:

$$[\hat{p}_t - \delta\beta E_t \hat{p}_{t+1}] + \frac{\delta\beta}{(1-\delta\beta)} [\hat{v}_t - \delta\beta E_t \hat{v}_{t+1}]$$

$$= (1-\delta\beta) \left[(\hat{w}_t - \hat{z}_t) + E_t \hat{\pi}_{t+1} \left(\delta\beta + (\delta\beta)^2 + + (\delta\beta)^3 + ...\right)\right] .$$

- Equivalently:

$$\left[\hat{p}_t - E_t\hat{p}_{t+1} + (1-\delta\beta)\,E_t\hat{p}_{t+1}\right] + \frac{\delta\beta}{(1-\delta\beta)}\left[\hat{v}_t - E_t\hat{v}_{t+1} + (1-\delta\beta)\,E_t\hat{v}_{t+1}\right]$$
$$= (1-\delta\beta)\,(\hat{w}_t - \hat{z}_t) + \delta\beta E_t\hat{\pi}_{t+1}\ .$$

- Equivalently:

$$\left[\hat{p}_t - E_t\hat{p}_{t+1}\right] + \frac{\delta\beta}{(1-\delta\beta)}\left[\hat{v}_t - E_t\hat{v}_{t+1}\right] = (1-\delta\beta)\,(\hat{w}_t - \hat{z}_t - E_t\hat{p}_{t+1}) + \delta\beta\left(E_t\hat{\pi}_{t+1} - E_t\hat{v}_{t+1}\right)\ .$$

- Equivalently:

$$\left[E_t\hat{p}_{t+1} - \hat{p}_t\right] + \frac{\delta\beta}{(1-\delta\beta)}\left[E_t\hat{v}_{t+1} - \hat{v}_t\right] = -(1-\delta\beta)\,(\hat{w}_t - \hat{z}_t - E_t\hat{p}_{t+1}) + \delta\beta\left(E_t\hat{v}_{t+1} - E_t\hat{\pi}_{t+1}\right)\ .$$

- Equivalently:

$$\delta\beta E_t\hat{p}_{t+1} - \hat{p}_t + \frac{\delta\beta}{(1-\delta\beta)}\left[E_t\hat{v}_{t+1} - \hat{v}_t\right] = -(1-\delta\beta)\,(\hat{w}_t - \hat{z}_t) + \delta\beta E_t\hat{v}_{t+1} - \delta\beta E_t\hat{\pi}_{t+1}\ . \tag{14}$$

3.4.2 Quasi-Differencing for (10)

- Rewrite (10) as:

$$\hat{p}_t + \frac{(1+\delta\beta)}{(1-\delta\beta)}\hat{v}_t = \frac{(1-\delta\beta)^2}{\delta\beta}E_t\sum_{k=0}^{\infty}(\delta\beta)^k\,k\left[\hat{\Pi}_{t,k} + \hat{w}_{t+k} - \hat{z}_{t+k}\right]\ .$$

- Write out terms:

$$\hat{p}_t + \frac{(1+\delta\beta)}{(1-\delta\beta)}\hat{v}_t = \left(\frac{(1-\delta\beta)^2}{\delta\beta}\right) *$$
$$E_t\left[(\delta\beta)\,(\hat{w}_{t+1} - \hat{z}_{t+1} + \hat{\pi}_{t+1}) + 2\,(\delta\beta)^2\,(\hat{w}_{t+2} - \hat{z}_{t+2} + \hat{\pi}_{t+1} + \hat{\pi}_{t+2})\right.$$
$$\left. +3\,(\delta\beta)^3\,(\hat{w}_{t+3} - \hat{z}_{t+3} + \hat{\pi}_{t+1} + \hat{\pi}_{t+2} + \hat{\pi}_{t+3}) + ...\right]\ .$$

- Multiply the last equation by $\delta\beta$ and lead by one period:

$$\delta\beta E_t\hat{p}_{t+1} + \frac{\delta\beta(1+\delta\beta)}{(1-\delta\beta)}E_t\hat{v}_{t+1} = \left(\frac{(1-\delta\beta)^2}{\delta\beta}\right) *$$
$$E_t\left[(\delta\beta)^2\,(\hat{w}_{t+2} - \hat{z}_{t+2} + \hat{\pi}_{t+2}) + 2\,(\delta\beta)^3\,(\hat{w}_{t+3} - \hat{z}_{t+3} + \hat{\pi}_{t+2} + \hat{\pi}_{t+3}) + ...\right]\ .$$

- Deduct the last equation from the preceding one:

$$\left[\hat{p}_t - \delta\beta E_t\hat{p}_{t+1}\right] + \frac{(1+\delta\beta)}{(1-\delta\beta)}\left[\hat{v}_t - \delta\beta E_t\hat{v}_{t+1}\right] = \left(\frac{(1-\delta\beta)^2}{\delta\beta}\right) *$$

$$E_t\left[(\delta\beta)\left(\hat{w}_{t+1} - \hat{z}_{t+1}\right) + (\delta\beta)^2\left(\hat{w}_{t+2} - \hat{z}_{t+2} + \hat{\pi}_{t+2}\right) + (\delta\beta)^3\left(\hat{w}_{t+3} - \hat{z}_{t+3} + \hat{\pi}_{t+2} + \hat{\pi}_{t+3}\right) + \ldots \right.$$
$$\left. + \hat{\pi}_{t+1}\left(\delta\beta + 2(\delta\beta)^2 + 3(\delta\beta)^3 + \ldots\right)\right] \ .$$

- Use (7) for the final term:

$$E_t\hat{\pi}_{t+1}\left(\frac{(1-\delta\beta)^2}{\delta\beta}\right)\left(\delta\beta + 2(\delta\beta)^2 + 3(\delta\beta)^3 + \ldots\right) = E_t\hat{\pi}_{t+1} \ .$$

- Simplify:

$$\left[\hat{p}_t - \delta\beta E_t\hat{p}_{t+1}\right] + \frac{(1+\delta\beta)}{(1-\delta\beta)}\left[\hat{v}_t - \delta\beta E_t\hat{v}_{t+1}\right] = E_t\hat{\pi}_{t+1} + (1-\delta\beta)^2 *$$

$$E_t\left[\left(\hat{w}_{t+1} - \hat{z}_{t+1}\right) + (\delta\beta)\left(\hat{w}_{t+2} - \hat{z}_{t+2} + \hat{\pi}_{t+2}\right) + (\delta\beta)^2\left(\hat{w}_{t+3} - \hat{z}_{t+3} + \hat{\pi}_{t+2} + \hat{\pi}_{t+3}\right) + \ldots\right] \ .$$

- Rewrite:

$$\left[\hat{p}_t - \delta\beta E_t\hat{p}_{t+1}\right] + \frac{(1+\delta\beta)}{(1-\delta\beta)}\left[\hat{v}_t - \delta\beta E_t\hat{v}_{t+1}\right] = E_t\hat{\pi}_{t+1} + (1-\delta\beta)^2 *$$

$$E_t\left[\sum_{k=0}^{\infty}(\delta\beta)^k\left(\hat{w}_{t+1+k} - \hat{z}_{t+1+k}\right) + E_t\left((\delta\beta)\hat{\pi}_{t+2} + (\delta\beta)^2\left(\hat{\pi}_{t+2} + \hat{\pi}_{t+3}\right) + \ldots\right)\right] \ .$$

- The final term can be rewritten using (11):

$$E_t\left[(\delta\beta)\hat{\pi}_{t+2} + (\delta\beta)^2\left(\hat{\pi}_{t+2} + \hat{\pi}_{t+3}\right) + \ldots\right] = \sum_{k=1}^{\infty}(\delta\beta)^k\hat{\Pi}_{t+1,k} \ .$$

- Then we can simplify further as:

$$\left[\hat{p}_t - \delta\beta E_t\hat{p}_{t+1}\right] + \frac{(1+\delta\beta)}{(1-\delta\beta)}\left[\hat{v}_t - \delta\beta E_t\hat{v}_{t+1}\right] = E_t\hat{\pi}_{t+1} +$$

$$(1-\delta\beta)^2 E_t\left[\sum_{k=0}^{\infty}(\delta\beta)^k\left(\hat{w}_{t+1+k} - \hat{z}_{t+1+k}\right) + \sum_{k=1}^{\infty}(\delta\beta)^k\hat{\Pi}_{t+1,k}\right] \ .$$

- Now we can simplify the right-hand side using (13):

$$\left[\hat{p}_t - \delta\beta E_t\hat{p}_{t+1}\right] + \frac{(1+\delta\beta)}{(1-\delta\beta)}\left[\hat{v}_t - \delta\beta E_t\hat{v}_{t+1}\right] = E_t\hat{\pi}_{t+1} + (1-\delta\beta)E_t\hat{p}_{t+1} + \delta\beta E_t\hat{v}_{t+1} \ .$$

- Further simplification:

$$\left[\hat{p}_t - E_t\hat{p}_{t+1} + (1-\delta\beta)\, E_t\hat{p}_{t+1}\right] + \frac{(1+\delta\beta)}{(1-\delta\beta)}\left[\hat{v}_t - E_t\hat{v}_{t+1} + (1-\delta\beta)E_t\hat{v}_{t+1}\right]$$
$$= E_t\hat{\pi}_{t+1} + (1-\delta\beta)\, E_t\hat{p}_{t+1} + \delta\beta E_t\hat{v}_{t+1}\ .$$

- Cancel terms:

$$\left[\hat{p}_t - E_t\hat{p}_{t+1}\right] + \frac{(1+\delta\beta)}{(1-\delta\beta)}\left[\hat{v}_t - E_t\hat{v}_{t+1}\right] = E_t\hat{\pi}_{t+1} - E_t\hat{v}_{t+1}\ .$$

- Equivalently:

$$E_t\hat{p}_{t+1} = \hat{p}_t + E_t\hat{v}_{t+1} - E_t\hat{\pi}_{t+1} - \frac{(1+\delta\beta)}{(1-\delta\beta)}\left[E_t\hat{v}_{t+1} - \hat{v}_t\right]\ . \tag{15}$$

- Equivalently:

$$E_t\hat{p}_{t+1} - \hat{p}_t = \frac{-2\delta\beta}{(1-\delta\beta)}E_t\hat{v}_{t+1} - E_t\hat{\pi}_{t+1} + \frac{(1+\delta\beta)}{(1-\delta\beta)}\hat{v}_t\ . \tag{16}$$

3.4.3 Combine (14) and (15)

- (14) for ease of reference:

$$\delta\beta E_t\hat{p}_{t+1} - \hat{p}_t + \frac{\delta\beta}{(1-\delta\beta)}\left[E_t\hat{v}_{t+1} - \hat{v}_t\right] = -\left(1-\delta\beta\right)\left(\hat{w}_t - \hat{z}_t\right) + \delta\beta E_t\hat{v}_{t+1} - \delta\beta E_t\hat{\pi}_{t+1}\ .$$

- Plug in (15):

$$\delta\beta\left[\hat{p}_t + E_t\hat{v}_{t+1} - E_t\hat{\pi}_{t+1} - \frac{(1+\delta\beta)}{(1-\delta\beta)}\left[E_t\hat{v}_{t+1} - \hat{v}_t\right]\right] - \hat{p}_t + \frac{\delta\beta}{(1-\delta\beta)}\left[E_t\hat{v}_{t+1} - \hat{v}_t\right]$$
$$= -\left(1-\delta\beta\right)\left(\hat{w}_t - \hat{z}_t\right) + \delta\beta E_t\hat{v}_{t+1} - \delta\beta E_t\hat{\pi}_{t+1}\ .$$

- Equivalently:

$$\left(\delta\beta - 1\right)\hat{p}_t + \delta\beta E_t\hat{v}_{t+1} + \frac{\delta\beta}{(1-\delta\beta)}\left(1 - 1 - \delta\beta\right)\left(E_t\hat{v}_{t+1} - \hat{v}_t\right) - \delta\beta E_t\hat{\pi}_{t+1}$$
$$= -\left(1-\delta\beta\right)\left(\hat{w}_t - \hat{z}_t\right) + \delta\beta E_t\hat{v}_{t+1} - \delta\beta E_t\hat{\pi}_{t+1}\ .$$

- Cancel terms to get a preliminary difference equation for $\hat{v}_t$:

$$\frac{(\delta\beta)^2}{(1-\delta\beta)}\left(E_t\hat{v}_{t+1} - \hat{v}_t\right) = \left(1-\delta\beta\right)\left(\hat{w}_t - \hat{z}_t - \hat{p}_t\right)\ .$$

- Equivalently:

$$\left(E_t\hat{v}_{t+1} - \hat{v}_t\right) = \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\left(\hat{w}_t - \hat{z}_t - \hat{p}_t\right)\ . \tag{17}$$

4 The Price Index

4.1 Formula for the Index

- Price Level:

$$P_t = \left[(1-\delta) \sum_{s=0}^{\infty} \delta^s \left[V_{t-s} (v_{t-s})^s \right]^{1-\sigma} \right]^{\frac{1}{1-\sigma}} .$$

- Deflate by current target price level P_t^* and write out terms ($\check{P}_t = P_t / P_t^*$, $\check{V}_t = V_t / P_t^*$):

$$\left(\check{P}_t\right)^{1-\sigma} = (1-\delta)\check{V}_t^{1-\sigma} + (1-\delta)\delta \check{V}_{t-1}^{1-\sigma} \left(\frac{v_{t-1}}{\pi_t^*} \right)^{1-\sigma}$$

$$+(1-\delta)\delta^2 \check{V}_{t-2}^{1-\sigma} \left(\frac{v_{t-2}^2}{\pi_t^* \pi_{t-1}^*} \right)^{(1-\sigma)} + (1-\delta)\delta^3 \check{V}_{t-3}^{1-\sigma} \left(\frac{v_{t-3}^3}{\pi_t^* \pi_{t-1}^* \pi_{t-2}^*} \right)^{(1-\sigma)} + ...$$

- Divide by $\check{P}_{t-1}$:

$$\begin{aligned} \left(\frac{\check{P}_t}{\check{P}_{t-1}} \right)^{1-\sigma} &= (1-\delta) \left(\frac{\check{V}_t}{\check{P}_t} \right)^{1-\sigma} \left(\frac{\check{P}_t}{\check{P}_{t-1}} \right)^{1-\sigma} \\ &+ (1-\delta)\delta \left(\frac{\check{V}_{t-1}}{\check{P}_{t-1}} \right)^{1-\sigma} \left(\frac{v_{t-1}}{\pi_t^*} \right)^{1-\sigma} \\ &+ (1-\delta)\delta^2 \left(\frac{\check{V}_{t-2}}{\check{P}_{t-2}} \right)^{1-\sigma} \left(\frac{\check{P}_{t-2}}{\check{P}_{t-1}} \right)^{1-\sigma} \left(\frac{v_{t-2}^2}{\pi_t^* \pi_{t-1}^*} \right)^{1-\sigma} \\ &+ (1-\delta)\delta^3 \left(\frac{\check{V}_{t-3}}{\check{P}_{t-3}} \right)^{1-\sigma} \left(\frac{\check{P}_{t-3}}{\check{P}_{t-2}} \right)^{1-\sigma} \left(\frac{\check{P}_{t-2}}{\check{P}_{t-1}} \right)^{1-\sigma} \left(\frac{v_{t-3}^3}{\pi_t^* \pi_{t-1}^* \pi_{t-2}^*} \right)^{1-\sigma} + ... \end{aligned}$$

- Use $\check{P}_t / \check{P}_{t-1} = \check{\pi}_t = \frac{\pi_t}{\pi_t^*}$ (this is the deviation of gross inflation from its target):

$$\begin{aligned} (\check{\pi}_t)^{1-\sigma} &= (1-\delta) (p_t)^{1-\sigma} (\check{\pi}_t)^{1-\sigma} \\ &+ (1-\delta)\delta (p_{t-1})^{1-\sigma} \left(\frac{v_{t-1}}{\pi_t^*} \right)^{1-\sigma} \\ &+ (1-\delta)\delta^2 (p_{t-2})^{1-\sigma} \left(\frac{1}{\check{\pi}_{t-1}} \right)^{1-\sigma} \left(\frac{v_{t-2}^2}{\pi_t^* \pi_{t-1}^*} \right)^{1-\sigma} \\ &+ (1-\delta)\delta^3 (p_{t-3})^{1-\sigma} \left(\frac{1}{\check{\pi}_{t-2}} \right)^{1-\sigma} \left(\frac{1}{\check{\pi}_{t-1}} \right)^{1-\sigma} \left(\frac{v_{t-3}^3}{\pi_t^* \pi_{t-1}^* \pi_{t-2}^*} \right)^{1-\sigma} + ... \end{aligned}$$

- Divide through by $(\breve{\pi}_t)^{1-\sigma}$:

$$\begin{aligned} 1 &= (1-\delta)p_t^{1-\sigma} \\ &+(1-\delta)\delta p_{t-1}^{1-\sigma}\left(\frac{v_{t-1}}{\breve{\pi}_t\pi_t^*}\right)^{1-\sigma} \\ &+(1-\delta)\delta^2 p_{t-2}^{1-\sigma}\left(\frac{(v_{t-2})^2}{\breve{\pi}_{t-1}\breve{\pi}_t\pi_{t-1}^*\pi_t^*}\right)^{1-\sigma} \\ &+(1-\delta)\delta^3 p_{t-3}^{1-\sigma}\left(\frac{(v_{t-3})^3}{\breve{\pi}_{t-2}\breve{\pi}_{t-1}\breve{\pi}_t\pi_{t-2}^*\pi_{t-1}^*\pi_t^*}\right)^{1-\sigma} + ... \end{aligned}$$

- Let $\breve{v}_{t-1} = v_{t-1}/\pi_{t-1}^*$, the deviation of firm-specific gross inflation from the inflation target, and use $\pi_t^*/\pi_{t-1}^* = \varepsilon_t^{\pi^*}$:

$$\begin{aligned} 1 &= (1-\delta)p_t^{1-\sigma} \\ &+(1-\delta)\delta p_{t-1}^{1-\sigma}\left(\frac{\breve{v}_{t-1}}{\breve{\pi}_t\varepsilon_t^{\pi^*}}\right)^{1-\sigma} \\ &+(1-\delta)\delta^2 p_{t-2}^{1-\sigma}\left(\frac{(\breve{v}_{t-2})^2}{\breve{\pi}_{t-1}\breve{\pi}_t\left(\varepsilon_{t-1}^{\pi^*}\right)^2\varepsilon_t^{\pi^*}}\right)^{1-\sigma} \\ &+(1-\delta)\delta^3 p_{t-3}^{1-\sigma}\left(\frac{(\breve{v}_{t-3})^3}{\breve{\pi}_{t-2}\breve{\pi}_{t-1}\breve{\pi}_t\left(\varepsilon_{t-2}^{\pi^*}\right)^3\left(\varepsilon_{t-1}^{\pi^*}\right)^2\varepsilon_t^{\pi^*}}\right)^{1-\sigma} + ... \end{aligned} \quad (18)$$

4.2 Linearize (18)

- Linearize:

$$\begin{aligned} 0 &= (1-\delta)(1-\sigma)\hat{p}_t \\ &+(1-\delta)(1-\sigma)\delta\left(\hat{p}_{t-1}+\hat{v}_{t-1}-\hat{\pi}_t-\hat{\varepsilon}_t^{\pi^*}\right) \\ &+(1-\delta)(1-\sigma)\delta^2\left(\hat{p}_{t-2}+2\hat{v}_{t-2}-\hat{\pi}_{t-1}-\hat{\pi}_t-2\hat{\varepsilon}_{t-1}^{\pi^*}-\hat{\varepsilon}_t^{\pi^*}\right) \\ &+(1-\delta)(1-\sigma)\delta^3\left(\hat{p}_{t-3}+3\hat{v}_{t-3}-\hat{\pi}_{t-2}-\hat{\pi}_{t-1}-\hat{\pi}_t-3\hat{\varepsilon}_{t-2}^{\pi^*}-2\hat{\varepsilon}_{t-1}^{\pi^*}-\hat{\varepsilon}_t^{\pi^*}\right)+... \end{aligned}$$

- Cancel terms and bring $\hat{\pi}_t$ onto left-hand side:

$$\hat{\pi}_t\left(\delta+\delta^2+\delta^3+...\right) = \hat{p}_t+\delta\left(\hat{p}_{t-1}+\hat{v}_{t-1}-\hat{\varepsilon}_t^{\pi^*}\right)$$
$$+\delta^2\left(\hat{p}_{t-2}+2\hat{v}_{t-2}-\hat{\pi}_{t-1}-2\hat{\varepsilon}_{t-1}^{\pi^*}-\hat{\varepsilon}_t^{\pi^*}\right)+\delta^3\left(\hat{p}_{t-3}+3\hat{v}_{t-3}-\hat{\pi}_{t-2}-\hat{\pi}_{t-1}-3\hat{\varepsilon}_{t-2}^{\pi^*}-2\hat{\varepsilon}_{t-1}^{\pi^*}-\hat{\varepsilon}_t^{\pi^*}\right)+...$$

- Equivalently:

$$\hat{\pi}_t = \frac{1-\delta}{\delta}\hat{p}_t+(1-\delta)\left(\hat{p}_{t-1}+\hat{v}_{t-1}-\hat{\varepsilon}_t^{\pi^*}\right)+(1-\delta)\delta\left(\hat{p}_{t-2}+2\hat{v}_{t-2}-\hat{\pi}_{t-1}-2\hat{\varepsilon}_{t-1}^{\pi^*}-\hat{\varepsilon}_t^{\pi^*}\right)$$
$$+(1-\delta)\delta^2\left(\hat{p}_{t-3}+3\hat{v}_{t-3}-\hat{\pi}_{t-2}-\hat{\pi}_{t-1}-3\hat{\varepsilon}_{t-2}^{\pi^*}-2\hat{\varepsilon}_{t-1}^{\pi^*}-\hat{\varepsilon}_t^{\pi^*}\right)+...$$

4.3 Quasi-Differencing

- Lag the last equation and multiply by δ:

$$\begin{aligned}\delta\hat{\pi}_{t-1} &= (1-\delta)\,\hat{p}_{t-1} + (1-\delta)\,\delta\left(\hat{p}_{t-2} + \hat{v}_{t-2} - \hat{\varepsilon}^{\pi^*}_{t-1}\right) \\ &\quad + (1-\delta)\,\delta^2\left(\hat{p}_{t-3} + 2\hat{v}_{t-3} - \hat{\pi}_{t-2} - 2\hat{\varepsilon}^{\pi^*}_{t-2} - \hat{\varepsilon}^{\pi^*}_{t-1}\right) + \ldots\end{aligned}$$

- Deduct the last equation from the previous one:

$$\begin{aligned}\hat{\pi}_t - \delta\hat{\pi}_{t-1} &= \frac{1-\delta}{\delta}\hat{p}_t + (1-\delta)\left(\hat{v}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) + (1-\delta)\,\delta\left(\hat{v}_{t-2} - \hat{\pi}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) \\ &\quad + (1-\delta)\,\delta^2\left(\hat{v}_{t-3} - \hat{\pi}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t-2} - \hat{\varepsilon}^{\pi^*}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) + \ldots\end{aligned}$$

- Equivalently:

$$\begin{aligned}\hat{\pi}_t - \delta\hat{\pi}_{t-1} &= \frac{1-\delta}{\delta}\hat{p}_t - \delta\hat{\pi}_{t-1} + (1-\delta)\left(\hat{v}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) + (1-\delta)\,\delta\left(\hat{v}_{t-2} - \hat{\varepsilon}^{\pi^*}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) \\ &\quad + (1-\delta)\,\delta^2\left(\hat{v}_{t-3} - \hat{\varepsilon}^{\pi^*}_{t-2} - \hat{\varepsilon}^{\pi^*}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) + \ldots\end{aligned}$$

- Equivalently:

$$\begin{aligned}\hat{\pi}_t &= \frac{1-\delta}{\delta}\hat{p}_t + (1-\delta)\left(\hat{v}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) + (1-\delta)\,\delta\left(\hat{v}_{t-2} - \hat{\varepsilon}^{\pi^*}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) \\ &\quad + (1-\delta)\,\delta^2\left(\hat{v}_{t-3} - \hat{\varepsilon}^{\pi^*}_{t-2} - \hat{\varepsilon}^{\pi^*}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) + \ldots\end{aligned}$$

- Equivalently:

$$\hat{\pi}_t = \frac{1-\delta}{\delta}\hat{p}_t + \hat{\psi}_t\ , \tag{19}$$

$$\hat{\psi}_t = (1-\delta)\left(\hat{v}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) + (1-\delta)\,\delta\left(\hat{v}_{t-2} - \hat{\varepsilon}^{\pi^*}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) + (1-\delta)\,\delta^2\left(\hat{v}_{t-3} - \hat{\varepsilon}^{\pi^*}_{t-2} - \hat{\varepsilon}^{\pi^*}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\right) + \ldots$$

Equation (19) is the key expression discussed at some length in the paper.

4.4 The Auxiliary Variable $\hat{\psi}_t$

- Lag the last equation for $\hat{\psi}_t$ and multiply by δ:

$$\delta\hat{\psi}_{t-1} = (1-\delta)\delta\left(\hat{v}_{t-2} - \hat{\varepsilon}^{\pi^*}_{t-1}\right) + (1-\delta)\delta^2\left(\hat{v}_{t-3} - \hat{\varepsilon}^{\pi^*}_{t-2} - \hat{\varepsilon}^{\pi^*}_{t-1}\right) + \ldots$$

- Deduct the last equation from the previous one:

$$\hat{\psi}_t = \delta\hat{\psi}_{t-1} + (1-\delta)\hat{v}_{t-1} - \hat{\varepsilon}^{\pi^*}_{t}\ .$$

- Create auxiliary variables for the King/Watson dynamic system formulation:

Auxiliary Variable III: $\hat{q}^{\psi}_t = \hat{\psi}_{t-1}$.

Auxiliary Variable IV: $\hat{q}^{v}_t = \hat{v}_{t-1}$.

- Use these in the equation for $\hat{\psi}_t$:

$$\hat{\psi}_t = \delta \hat{q}_t^{\psi} + (1-\delta)\hat{q}_t^{v} - \hat{\varepsilon}_t^{\pi^*} \ . \tag{20}$$

- Also, for future reference:

$$\left(E_t \hat{\psi}_{t+1} - \hat{\psi}_t\right) = (1-\delta)\hat{v}_t - (1-\delta)\,\hat{\psi}_t \ . \tag{21}$$

- Finally we will also be able to use (19) to substitute $\hat{p}_t$ out of equation (17):

$$\hat{p}_t = \frac{\delta}{1-\delta}\left(\hat{\pi}_t - \hat{\psi}_t\right) \ . \tag{22}$$

4.5 More Intuition for (20)

- Assume that we incorrectly linearize (18) around the period t inflation target π_t^* regardless of the time subscript of the variables. We get the following, after changing notation to distinguish this linearization from (20):

$$\widehat{\psi_t} = \delta \widehat{\psi_{t-1}} + (1-\delta)\widehat{v_{t-1}} \ . \tag{23}$$

- Note that for $\widehat{v_{t-1}}$ we have (the same holds for $\widehat{\psi_{t-1}}$)

$$\begin{aligned} \widehat{v_{t-1}} &= \ln(v_{t-1}) - \ln(\pi_t^*) \quad , \text{ while} \\ \hat{v}_{t-1} &= \ln(v_{t-1}) - \ln(\pi_{t-1}^*) \ . \end{aligned}$$

- This implies that (again similarly for $\widehat{\psi_{t-1}}$)

$$\widehat{v_{t-1}} = \hat{v}_{t-1} - \hat{\varepsilon}_t^{\pi^*} \ .$$

- Also

$$\widehat{\psi_t} = \hat{\psi}_t \ .$$

- Substituting the foregoing into (23) we obtain (20):

$$\hat{\psi}_t = \delta \hat{\psi}_{t-1} + (1-\delta)\hat{v}_{t-1} - \hat{\varepsilon}_t^{\pi^*} \ .$$

4.6 Final Equation for $\hat{v}_t$

- Equation (17) reproduced for ease of reference:

$$\left(E_t \hat{v}_{t+1} - \hat{v}_t\right) = \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\left(\hat{w}_t - \hat{z}_t - \hat{p}_t\right) \ .$$

- Plug (22) into this:

$$E_t \hat{v}_{t+1} = \hat{v}_t + \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\frac{\delta}{1-\delta}\hat{\psi}_t - \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\frac{\delta}{1-\delta}\hat{\pi}_t + \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\hat{w}_t - \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\hat{z}_t \ . \tag{24}$$

4.7 Final Equation for $\hat{\pi}_t$

- Deduct (19) from its once led version to get:

$$(E_t\hat{\pi}_{t+1} - \hat{\pi}_t) = \frac{1-\delta}{\delta}\left(E_t\hat{p}_{t+1} - \hat{p}_t\right) + \left(E_t\hat{\psi}_{t+1} - \hat{\psi}_t\right) \ .$$

- To substitute for the first term on the right-hand side, use (16), which we reproduce here:

$$E_t\hat{p}_{t+1} - \hat{p}_t = \frac{-2\delta\beta}{(1-\delta\beta)}E_t\hat{v}_{t+1} - E_t\hat{\pi}_{t+1} + \frac{(1+\delta\beta)}{(1-\delta\beta)}\hat{v}_t \ .$$

- To substitute for the second term on the right-hand side, use (21), which we reproduce here:

$$\left(E_t\hat{\psi}_{t+1} - \hat{\psi}_t\right) = (1-\delta)\hat{v}_t - (1-\delta)\,\hat{\psi}_t \ .$$

- Substitute:

$$(E_t\hat{\pi}_{t+1} - \hat{\pi}_t) = (1-\delta)\hat{v}_t - (1-\delta)\,\hat{\psi}_t$$
$$+\frac{1-\delta}{\delta}\left[\frac{-2\delta\beta}{(1-\delta\beta)}E_t\hat{v}_{t+1} - E_t\hat{\pi}_{t+1} + \frac{(1+\delta\beta)}{(1-\delta\beta)}\hat{v}_t\right] \ .$$

- Equivalently:

$$E_t\hat{\pi}_{t+1}\left(1+\frac{1-\delta}{\delta}\right) = \hat{\pi}_t - \frac{1-\delta}{\delta}\frac{2\delta\beta}{(1-\delta\beta)}E_t\hat{v}_{t+1} + \left(\frac{1-\delta}{\delta}\frac{(1+\delta\beta)}{(1-\delta\beta)} + (1-\delta)\right)\hat{v}_t - (1-\delta)\,\hat{\psi}_t \ .$$

- Simplify and substitute from (24) for $E_t\hat{v}_{t+1}$:

$$E_t\hat{\pi}_{t+1}\frac{1}{\delta} = \hat{\pi}_t + \left(\frac{1-\delta}{\delta}\frac{(1+\delta\beta)}{(1-\delta\beta)} + (1-\delta)\right)\hat{v}_t - (1-\delta)\,\hat{\psi}_t$$
$$-\frac{1-\delta}{\delta}\frac{2\delta\beta}{(1-\delta\beta)}\left[\hat{v}_t + \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\frac{\delta}{1-\delta}\hat{\psi}_t - \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\frac{\delta}{1-\delta}\hat{\pi}_t + \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\left(\hat{w}_t - \hat{z}_t\right)\right] \ .$$

- Equivalently:

$$E_t\hat{\pi}_{t+1}\frac{1}{\delta} = \hat{\pi}_t\left(1+\frac{2\,(1-\delta\beta)}{(\delta\beta)}\right) + \hat{v}_t\left(\frac{1-\delta}{\delta}\frac{(1+\delta\beta)}{(1-\delta\beta)} + (1-\delta) - \frac{1-\delta}{\delta}\frac{2\delta\beta}{(1-\delta\beta)}\right)$$
$$-\hat{\psi}_t\left(\frac{2\,(1-\delta\beta)}{(\delta\beta)} + (1-\delta)\right) - \frac{1-\delta}{\delta}\frac{2\,(1-\delta\beta)}{(\delta\beta)}\left(\hat{w}_t - \hat{z}_t\right) \ .$$

- Equivalently:

$$E_t\hat{\pi}_{t+1}\frac{1}{\delta}=\hat{\pi}_t\left(\frac{\delta\beta+2-2\delta\beta}{\delta\beta}\right)+\hat{v}_t\left(\frac{1-\delta}{\delta}\frac{(1+\delta\beta-2\delta\beta)}{(1-\delta\beta)}+(1-\delta)\right)$$
$$-\hat{\psi}_t\left(\frac{2-2\delta\beta+\delta\beta-\delta^2\beta}{\delta\beta}\right)-\frac{1-\delta}{\delta}\frac{2\,(1-\delta\beta)}{(\delta\beta)}\left(\hat{w}_t-\hat{z}_t\right)\ .$$

- Equivalently:

$$E_t\hat{\pi}_{t+1}\frac{1}{\delta}=\hat{\pi}_t\left(\frac{2-\delta\beta}{\delta\beta}\right)+\hat{v}_t\left(\frac{(1-\delta)(1+\delta)}{\delta}\right)$$
$$-\hat{\psi}_t\left(\frac{2-\delta\beta-\delta^2\beta}{\delta\beta}\right)-\frac{1-\delta}{\delta}\frac{2\,(1-\delta\beta)}{(\delta\beta)}\left(\hat{w}_t-\hat{z}_t\right)\ .$$

- Equivalently:

$$E_t\hat{\pi}_{t+1}=\hat{\pi}_t\left(\frac{2}{\beta}-\delta\right)+\hat{v}_t\left((1-\delta)\,(1+\delta)\right)+\hat{\psi}_t\left(\delta(1+\delta)-\frac{2}{\beta}\right) \tag{25}$$
$$-\hat{w}_t\left(\frac{2(1-\delta)\,(1-\delta\beta)}{(\delta\beta)}\right)+\hat{z}_t\left(\frac{2(1-\delta)\,(1-\delta\beta)}{(\delta\beta)}\right)\ .$$

5 The Backward-Indexation Case

5.1 Profit Maximization

- Pricing policy of firm j that reoptimizes at t, choosing V_t and indexing to π_{t-1} (the lagged aggregate inflation rate):

$$P_{t+k}(j) = V_t \tilde{\Pi}_{t,k} \ .$$

- Define additional terms:
  - Cumulative lagged aggregate inflation:

$$\tilde{\Pi}_{t,k} \equiv E_t \Pi_{j=1}^{k} \pi_{t+j-1} \text{ for } k \geq 1 \ \ (\equiv 1 \text{ for } k = 0)$$

  - Cumulative lagged aggregate normalized inflation:

$$\widetilde{\check{\Pi}}_{t,k} \equiv E_t \Pi_{j=1}^{k} \check{\pi}_{t+j-1} \text{ for } k \geq 1 \ \ (\equiv 1 \text{ for } k = 0)$$

  - Cumulative lagged aggregate inflation deviation:

$$\widehat{\tilde{\Pi}}_{t,k} \equiv E_t \Sigma_{j=1}^{k} \hat{\pi}_{t+j-1} \text{ for } k \geq 1 \ \ (\equiv 0 \text{ for } k = 0)$$

- Profit maximization:

$$\underset{V_t}{Max} \quad E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} \left[\left(\frac{V_t \tilde{\Pi}_{t,k}}{P_t \Pi_{t,k}} - \frac{w_{t+k}}{z_{t+k}} \right) y_{t+k}(j) \right] \quad , \ s.t.$$

$$y_{t+k}(j) = c_{t+k} \left(\frac{V_t \tilde{\Pi}_{t,k}}{P_t \Pi_{t,k}} \right)^{-\sigma} \ .$$

- Substitute constraints:

$$\underset{V_t}{Max} E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} \left[\left(\frac{V_t \tilde{\Pi}_{t,k}}{P_t \Pi_{t,k}} \right)^{1-\sigma} c_{t+k} - \frac{w_{t+k}}{z_{t+k}} \left(\frac{V_t \tilde{\Pi}_{t,k}}{P_t \Pi_{t,k}} \right)^{-\sigma} c_{t+k} \right] \ .$$

- FOC for V_t normalized by inflation target:

$$p_t = \mu \frac{E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \frac{w_{t+k}}{z_{t+k}}}{E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \left(\frac{\widetilde{\check{\Pi}}_{t,k}}{\check{\Pi}_{t,k}} \right)} \ . \tag{26}$$

- Rewrite (26):

$$p_t E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \left(\frac{\widetilde{\check{\Pi}}_{t,k}}{\check{\Pi}_{t,k}} \right) = \mu E_t \sum_{k=0}^{\infty} (\delta\beta) \lambda_{t+k} y_{t+k}(j) \frac{w_{t+k}}{z_{t+k}} \ .$$

- Linearization of (26):

$$\frac{\hat{p}_t}{1-\delta\beta} + E_t \sum_{k=0}^{\infty} (\delta\beta)^k \left(\widehat{\tilde{\Pi}}_{t,k} - \hat{\Pi}_{t,k}\right) = E_t \sum_{k=0}^{\infty} (\delta\beta)^k \left(\hat{w}_{t+k} - \hat{z}_{t+k}\right) \ .$$

- Equivalently:

$$\frac{\hat{p}_t}{1-\delta\beta} + E_t \sum_{k=0}^{\infty} (\delta\beta)^k \left(\hat{\pi}_t - \hat{\pi}_{t+k}\right) = E_t \sum_{k=0}^{\infty} (\delta\beta)^k \left(\hat{w}_{t+k} - \hat{z}_{t+k}\right) \ .$$

- Equivalently:

$$\frac{\hat{p}_t + \hat{\pi}_t}{1-\delta\beta} = E_t \sum_{k=0}^{\infty} (\delta\beta)^k \left(\hat{w}_{t+k} - \hat{z}_{t+k} + \hat{\pi}_{t+k}\right) \ .$$

- Quasi-Differencing:

$$\left(\hat{p}_t + \hat{\pi}_t\right) - \delta\beta E_t \left(\hat{p}_{t+1} + \hat{\pi}_{t+1}\right) = (1-\delta\beta)\left(\hat{w}_{t+k} - \hat{z}_{t+k} + \hat{\pi}_t\right) \ . \tag{27}$$

5.2 The Price Index

- Price Level:

$$P_t = \left[(1-\delta)\sum_{s=0}^{\infty} \delta^s \left[V_{t-s}\tilde{\Pi}_{t-s,s}\right]^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \ .$$

- Deflate by current target price level P_t^* and write out terms ($\check{P}_t = P_t/P_t^*$, $\check{V}_t = V_t/P_t^*$):

$$\left(\check{P}_t\right)^{1-\sigma} = (1-\delta)\check{V}_t^{1-\sigma} + (1-\delta)\delta\check{V}_{t-1}^{1-\sigma}\left(\frac{\pi_{t-1}}{\pi_t^*}\right)^{1-\sigma}$$
$$+(1-\delta)\delta^2\check{V}_{t-2}^{1-\sigma}\left(\frac{\pi_{t-2}\pi_{t-1}}{\pi_t^*\pi_{t-1}^*}\right)^{(1-\sigma)} + (1-\delta)\delta^3\check{V}_{t-3}^{1-\sigma}\left(\frac{\pi_{t-3}\pi_{t-2}\pi_{t-1}}{\pi_t^*\pi_{t-1}^*\pi_{t-2}^*}\right)^{(1-\sigma)} + \ldots$$

- Divide by $\check{P}_{t-1}$:

$$\begin{aligned}
\left(\frac{\check{P}_t}{\check{P}_{t-1}}\right)^{1-\sigma} &= (1-\delta)\left(\frac{\check{V}_t}{\check{P}_t}\right)^{1-\sigma}\left(\frac{\check{P}_t}{\check{P}_{t-1}}\right)^{1-\sigma} \\
&+(1-\delta)\delta\left(\frac{\check{V}_{t-1}}{\check{P}_{t-1}}\right)^{1-\sigma}\left(\frac{\pi_{t-1}}{\pi_t^*}\right)^{1-\sigma} \\
&+(1-\delta)\delta^2\left(\frac{\check{V}_{t-2}}{\check{P}_{t-2}}\right)^{1-\sigma}\left(\frac{\check{P}_{t-2}}{\check{P}_{t-1}}\right)^{1-\sigma}\left(\frac{\pi_{t-2}\pi_{t-1}}{\pi_t^*\pi_{t-1}^*}\right)^{(1-\sigma)} \\
&+(1-\delta)\delta^3\left(\frac{\check{V}_{t-3}}{\check{P}_{t-3}}\right)^{1-\sigma}\left(\frac{\check{P}_{t-3}}{\check{P}_{t-2}}\right)^{1-\sigma}\left(\frac{\check{P}_{t-2}}{\check{P}_{t-1}}\right)^{1-\sigma}\left(\frac{\pi_{t-3}\pi_{t-2}\pi_{t-1}}{\pi_t^*\pi_{t-1}^*\pi_{t-2}^*}\right)^{(1-\sigma)} + \ldots
\end{aligned}$$

- Use $\check{P}_t/\check{P}_{t-1} = \check{\pi}_t = \frac{\pi_t}{\pi_t^*}$:

$$
\begin{aligned}
\left(\check{\pi}_t\right)^{1-\sigma} &= (1-\delta)\left(p_t\right)^{1-\sigma}\left(\check{\pi}_t\right)^{1-\sigma} \\
&+(1-\delta)\delta\left(p_{t-1}\right)^{1-\sigma}\left(\frac{\pi_{t-1}}{\pi_t^*}\right)^{1-\sigma} \\
&+(1-\delta)\delta^2\left(p_{t-2}\right)^{1-\sigma}\left(\frac{1}{\check{\pi}_{t-1}}\right)^{1-\sigma}\left(\frac{\pi_{t-2}\pi_{t-1}}{\pi_t^*\pi_{t-1}^*}\right)^{(1-\sigma)} \\
&+(1-\delta)\delta^3\left(p_{t-3}\right)^{1-\sigma}\left(\frac{1}{\check{\pi}_{t-2}}\right)^{1-\sigma}\left(\frac{1}{\check{\pi}_{t-1}}\right)^{1-\sigma}\left(\frac{\pi_{t-3}\pi_{t-2}\pi_{t-1}}{\pi_t^*\pi_{t-1}^*\pi_{t-2}^*}\right)^{(1-\sigma)} + ...
\end{aligned}
$$

- Divide through by $\left(\check{\pi}_t\right)^{1-\sigma}$:

$$
\begin{aligned}
1 &= (1-\delta)p_t^{1-\sigma} \\
&+(1-\delta)\delta p_{t-1}^{1-\sigma}\left(\frac{\pi_{t-1}}{\check{\pi}_t\pi_t^*}\right)^{1-\sigma} \\
&+(1-\delta)\delta^2 p_{t-2}^{1-\sigma}\left(\frac{\pi_{t-2}\pi_{t-1}}{\check{\pi}_{t-1}\check{\pi}_t\pi_{t-1}^*\pi_t^*}\right)^{1-\sigma} \\
&+(1-\delta)\delta^3 p_{t-3}^{1-\sigma}\left(\frac{\pi_{t-3}\pi_{t-2}\pi_{t-1}}{\check{\pi}_{t-2}\check{\pi}_{t-1}\check{\pi}_t\pi_{t-2}^*\pi_{t-1}^*\pi_t^*}\right)^{1-\sigma} + ...
\end{aligned}
$$

- Use $\pi_t^*/\pi_{t-1}^* = \varepsilon_t^{\pi^*}$:

$$
\begin{aligned}
1 &= (1-\delta)p_t^{1-\sigma} \\
&+(1-\delta)\delta p_{t-1}^{1-\sigma}\left(\frac{\check{\pi}_{t-1}}{\check{\pi}_t\varepsilon_t^{\pi^*}}\right)^{1-\sigma} \\
&+(1-\delta)\delta^2 p_{t-2}^{1-\sigma}\left(\frac{\check{\pi}_{t-2}}{\check{\pi}_t\varepsilon_{t-1}^{\pi^*}\varepsilon_t^{\pi^*}}\right)^{1-\sigma} \\
&+(1-\delta)\delta^3 p_{t-3}^{1-\sigma}\left(\frac{\check{\pi}_{t-3}}{\check{\pi}_t\varepsilon_{t-2}^{\pi^*}\varepsilon_{t-1}^{\pi^*}\varepsilon_t^{\pi^*}}\right)^{1-\sigma} + ...
\end{aligned}
$$

- Linearize:

$$
\begin{aligned}
0 &= \hat{p}_t \\
&+\delta\left(\hat{p}_{t-1} + \hat{\pi}_{t-1} - \hat{\pi}_t - \hat{\varepsilon}_t^{\pi^*}\right) \\
&+\delta^2\left(\hat{p}_{t-2} + \hat{\pi}_{t-2} - \hat{\pi}_t - \hat{\varepsilon}_{t-1}^{\pi^*} - \hat{\varepsilon}_t^{\pi^*}\right) \\
&+\delta^3\left(\hat{p}_{t-3} + \hat{\pi}_{t-3} - \hat{\pi}_t - \hat{\varepsilon}_{t-2}^{\pi^*} - \hat{\varepsilon}_{t-1}^{\pi^*} - \hat{\varepsilon}_t^{\pi^*}\right) + ...
\end{aligned}
$$

- Cancel terms and bring $\hat{\pi}_t$ onto left-hand side:

$$\hat{\pi}_t\left(\delta+\delta^2+\delta^3+...\right)=\hat{p}_t+\delta\left(\hat{p}_{t-1}+\hat{\pi}_{t-1}-\hat{\varepsilon}_t^{\pi^*}\right)$$
$$+\delta^2\left(\hat{p}_{t-2}+\hat{\pi}_{t-2}-\hat{\varepsilon}_{t-1}^{\pi^*}-\hat{\varepsilon}_t^{\pi^*}\right)+\delta^3\left(\hat{p}_{t-3}+\hat{\pi}_{t-3}-\hat{\varepsilon}_{t-2}^{\pi^*}-\hat{\varepsilon}_{t-1}^{\pi^*}-\hat{\varepsilon}_t^{\pi^*}\right)+...$$

- Equivalently:

$$\hat{\pi}_t=\frac{1-\delta}{\delta}\hat{p}_t+(1-\delta)\left(\hat{p}_{t-1}+\hat{\pi}_{t-1}-\hat{\varepsilon}_t^{\pi^*}\right)+(1-\delta)\,\delta\left(\hat{p}_{t-2}+\hat{\pi}_{t-2}-\hat{\varepsilon}_{t-1}^{\pi^*}-\hat{\varepsilon}_t^{\pi^*}\right)$$
$$+(1-\delta)\,\delta^2\left(\hat{p}_{t-3}+\hat{\pi}_{t-3}-\hat{\varepsilon}_{t-2}^{\pi^*}-\hat{\varepsilon}_{t-1}^{\pi^*}-\hat{\varepsilon}_t^{\pi^*}\right)+...$$

- Lag the last equation and multiply by δ:

$$\begin{aligned}\delta\hat{\pi}_{t-1} &= (1-\delta)\,\hat{p}_{t-1}+(1-\delta)\,\delta\left(\hat{p}_{t-2}+\hat{\pi}_{t-2}-\hat{\varepsilon}_{t-1}^{\pi^*}\right)\\ &\quad+(1-\delta)\,\delta^2\left(\hat{p}_{t-3}+\hat{\pi}_{t-3}-\hat{\varepsilon}_{t-2}^{\pi^*}-\hat{\varepsilon}_{t-1}^{\pi^*}\right)+...\end{aligned}$$

- Deduct the last equation from the one before:

$$\begin{aligned}\hat{\pi}_t-\delta\hat{\pi}_{t-1} &= \frac{1-\delta}{\delta}\hat{p}_t+(1-\delta)\,\hat{\pi}_{t-1}\\ &\quad+(1-\delta)\left(-\hat{\varepsilon}_t^{\pi^*}\right)+(1-\delta)\,\delta\left(-\hat{\varepsilon}_t^{\pi^*}\right)+(1-\delta)\,\delta^2\left(-\hat{\varepsilon}_t^{\pi^*}\right)+...\end{aligned}$$

- Equivalently:

$$\hat{\pi}_t-\hat{\pi}_{t-1}=\frac{1-\delta}{\delta}\hat{p}_t-\hat{\varepsilon}_t^{\pi^*}\ . \tag{28}$$

5.3 Final Inflation Dynamics

- Rewrite (27):

$$\hat{p}_t-\delta\beta E_t\hat{p}_{t+1}-\delta\beta\left(E_t\hat{\pi}_{t+1}-\hat{\pi}_t\right)=(1-\delta\beta)\left(\hat{w}_t-\hat{z}_t\right)\ .$$

- Use (28):

$$\hat{p}_t-\frac{\delta^2\beta}{1-\delta}\left(E_t\hat{\pi}_{t+1}-\hat{\pi}_t\right)-\delta\beta\left(E_t\hat{\pi}_{t+1}-\hat{\pi}_t\right)=(1-\delta\beta)\left(\hat{w}_t-\hat{z}_t\right)\ .$$

- Equivalently:

$$\hat{p}_t-\frac{\delta^2\beta}{1-\delta}\left(E_t\hat{\pi}_{t+1}-\hat{\pi}_t\right)-\delta\beta\left(E_t\hat{\pi}_{t+1}-\hat{\pi}_t\right)=(1-\delta\beta)\left(\hat{w}_t-\hat{z}_t\right)\ .$$

- Use (28) again:

$$\hat{p}_t-\beta E_t\hat{p}_{t+1}=(1-\delta\beta)\left(\hat{w}_t-\hat{z}_t\right)\ .$$

- Use (28) again:

$$\left(\hat{\pi}_t-\hat{\pi}_{t-1}\right)=\beta E_t\left(\hat{\pi}_{t+1}-\hat{\pi}_t\right)+\frac{(1-\delta\beta)(1-\delta)}{\delta}\left(\hat{w}_t-\hat{z}_t\right)-\hat{\varepsilon}_t^{\pi^*}\ . \tag{29}$$

- This is the New Keynesian Phillips Curve for the backward-indexation case, which replaces the three equation pricing block of the optimal indexation system.

6 THE CALVO-YUN CASE

6.1 Profit Maximization

- Pricing policy of firm j that reoptimizes at t, choosing V_t and indexing to π_t^* (the current inflation target):
$$P_{t+k}(j) = V_t \Pi_{t,k}^* \ .$$
- Define additional terms:
  - Cumulative inflation targets:
$$\Pi_{t,k}^* \equiv E_t \prod_{j=1}^{k} \pi_{t+j}^* \text{ for } k \geq 1 \ \ (\equiv 1 \text{ for } k = 0)$$
  - Cumulative aggregate inflation target deviation:
$$\hat{\Pi}_{t,k}^* \equiv E_t \sum_{j=1}^{k} \hat{\pi}_{t+j}^* \text{ for } k \geq 1 \ \ (\equiv 0 \text{ for } k = 0)$$
- Profit maximization:
$$\underset{V_t}{Max} \quad E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} \left[\left(\frac{V_t \Pi_{t,k}^*}{P_t \Pi_{t,k}} - \frac{w_{t+k}}{z_{t+k}} \right) y_{t+k}(j) \right] \quad , \ s.t.$$
$$y_{t+k}(j) = c_{t+k} \left(\frac{V_t \Pi_{t,k}^*}{P_t \Pi_{t,k}} \right)^{-\sigma} \ .$$
- Substitute constraints:
$$\underset{V_t}{Max} E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} \left[\left(\frac{V_t \Pi_{t,k}^*}{P_t \Pi_{t,k}} \right)^{1-\sigma} c_{t+k} - \frac{w_{t+k}}{z_{t+k}} \left(\frac{V_t \Pi_{t,k}^*}{P_t \Pi_{t,k}} \right)^{-\sigma} c_{t+k} \right] \ .$$
- FOC for V_t, normalized by inflation target:
$$p_t = \mu \frac{E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \frac{w_{t+k}}{z_{t+k}}}{E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \left(\frac{1}{\check{\Pi}_{t,k}} \right)} \ . \tag{30}$$
- Rewrite (30):
$$p_t E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \left(\frac{1}{\check{\Pi}_{t,k}} \right) = \mu E_t \sum_{k=0}^{\infty} (\delta\beta)^k \lambda_{t+k} y_{t+k}(j) \frac{w_{t+k}}{z_{t+k}} \ .$$

- Linearization of (30):

$$\frac{\hat{p}_t}{1-\delta\beta} = E_t \sum_{k=0}^{\infty} (\delta\beta)^k \left(\hat{w}_{t+k} - \hat{z}_{t+k} + \hat{\Pi}_{t,k} \right) \ .$$

- Quasi-Differencing:

$$\hat{p}_t - \delta\beta E_t \hat{p}_{t+1} = (1-\delta\beta)\left(\hat{w}_t - \hat{z}_t\right) + \delta\beta E_t \hat{\pi}_{t+1} \ . \qquad (31)$$

6.2 The Price Index

- Price Level:

$$P_t = \left[(1-\delta) \sum_{s=0}^{\infty} \delta^s \left[V_{t-s} \Pi^*_{t-s,s} \right]^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \ .$$

- Deflate by current target price level P_t^* and write out terms ($\check{P}_t = P_t / P_t^*$, $\check{V}_t = V_t / P_t^*$):

$$\left(\check{P}_t\right)^{1-\sigma} = (1-\delta)\check{V}_t^{1-\sigma} + (1-\delta)\delta\check{V}_{t-1}^{1-\sigma} + (1-\delta)\delta^2\check{V}_{t-2}^{1-\sigma} + (1-\delta)\delta^3\check{V}_{t-3}^{1-\sigma} + \ldots$$

- Divide by $\check{P}_{t-1}$:

$$\begin{aligned} \left(\frac{\check{P}_t}{\check{P}_{t-1}}\right)^{1-\sigma} &= (1-\delta)\left(\frac{\check{V}_t}{\check{P}_t}\right)^{1-\sigma}\left(\frac{\check{P}_t}{\check{P}_{t-1}}\right)^{1-\sigma} \\ &\quad + (1-\delta)\delta\left(\frac{\check{V}_{t-1}}{\check{P}_{t-1}}\right)^{1-\sigma} \\ &\quad + (1-\delta)\delta^2\left(\frac{\check{V}_{t-2}}{\check{P}_{t-2}}\right)^{1-\sigma}\left(\frac{\check{P}_{t-2}}{\check{P}_{t-1}}\right)^{1-\sigma} \\ &\quad + (1-\delta)\delta^3\left(\frac{\check{V}_{t-3}}{\check{P}_{t-3}}\right)^{1-\sigma}\left(\frac{\check{P}_{t-3}}{\check{P}_{t-2}}\right)^{1-\sigma}\left(\frac{\check{P}_{t-2}}{\check{P}_{t-1}}\right)^{1-\sigma} + \ldots \end{aligned}$$

- Use $\check{P}_t / \check{P}_{t-1} = \check{\pi}_t = \frac{\pi_t}{\pi_t^*}$:

$$\begin{aligned} \left(\check{\pi}_t\right)^{1-\sigma} &= (1-\delta)\left(p_t\right)^{1-\sigma}\left(\check{\pi}_t\right)^{1-\sigma} \\ &\quad + (1-\delta)\delta\left(p_{t-1}\right)^{1-\sigma} \\ &\quad + (1-\delta)\delta^2\left(p_{t-2}\right)^{1-\sigma}\left(\frac{1}{\check{\pi}_{t-1}}\right)^{1-\sigma} \\ &\quad + (1-\delta)\delta^3\left(p_{t-3}\right)^{1-\sigma}\left(\frac{1}{\check{\pi}_{t-2}}\right)^{1-\sigma}\left(\frac{1}{\check{\pi}_{t-1}}\right)^{1-\sigma} + \ldots \end{aligned}$$

- Divide through by $(\breve{\pi}_t)^{1-\sigma}$:

$$\begin{aligned} 1 &= (1-\delta)p_t^{1-\sigma} \\ &+(1-\delta)\delta p_{t-1}^{1-\sigma}\left(\frac{1}{\breve{\pi}_t}\right)^{1-\sigma} \\ &+(1-\delta)\delta^2 p_{t-2}^{1-\sigma}\left(\frac{1}{\breve{\pi}_{t-1}\breve{\pi}_t}\right)^{1-\sigma} \\ &+(1-\delta)\delta^3 p_{t-3}^{1-\sigma}\left(\frac{1}{\breve{\pi}_{t-2}\breve{\pi}_{t-1}\breve{\pi}_t}\right)^{1-\sigma} + ... \end{aligned}$$

- Linearize:

$$0 = \hat{p}_t + \delta\left(\hat{p}_{t-1} - \hat{\pi}_t\right) + \delta^2\left(\hat{p}_{t-2} - \hat{\pi}_{t-1} - \hat{\pi}_t\right) + \delta^3\left(\hat{p}_{t-3} - \hat{\pi}_{t-2} - \hat{\pi}_{t-1} - \hat{\pi}_t\right) + ...$$

- Cancel terms and bring $\hat{\pi}_t$ onto left-hand side:

$$\hat{\pi}_t\left(\delta + \delta^2 + \delta^3 + ...\right) = \hat{p}_t + \delta\hat{p}_{t-1} + \delta^2\left(\hat{p}_{t-2} - \hat{\pi}_{t-1}\right) + \delta^3\left(\hat{p}_{t-3} - \hat{\pi}_{t-2} - \hat{\pi}_{t-1}\right) + ...$$

- Equivalently:

$$\hat{\pi}_t = \frac{1-\delta}{\delta}\hat{p}_t + (1-\delta)\,\hat{p}_{t-1} + (1-\delta)\,\delta\left(\hat{p}_{t-2} - \hat{\pi}_{t-1}\right) + (1-\delta)\,\delta^2\left(\hat{p}_{t-3} - \hat{\pi}_{t-2} - \hat{\pi}_{t-1}\right) + ...$$

- Lag the last equation and multiply by δ:

$$\delta\hat{\pi}_{t-1} = (1-\delta)\,\hat{p}_{t-1} + (1-\delta)\,\delta\hat{p}_{t-2} + (1-\delta)\,\delta^2\left(\hat{p}_{t-3} - \hat{\pi}_{t-2}\right) + ...$$

- Deduct the last equation from the preceding one:

$$\hat{\pi}_t = \frac{1-\delta}{\delta}\hat{p}_t \ . \tag{32}$$

6.3 Final Inflation Dynamics

- Plug (32) into (31):

$$\frac{\delta}{1-\delta}\hat{\pi}_t - \delta\beta\frac{\delta}{1-\delta}E_t\hat{\pi}_{t+1} = (1-\delta\beta)\left(\hat{w}_t - \hat{z}_t\right) + \delta\beta E_t\hat{\pi}_{t+1} \ .$$

- Simplify:

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{(1-\delta\beta)(1-\delta)}{\delta}\left(\hat{w}_t - \hat{z}_t\right) \ . \tag{33}$$

- This is the New Keynesian Phillips Curve for the Calvo-Yun case, which replaces the three equation pricing block of the optimal indexation system.

7 Household Wage Setting

7.1 Labor Demand

- $\ell_t(i,j) =$ demand for labor variety i by firm j.
- $L_t(i) =$ aggregate demand for labor variety i:

$$L_{t+k}(i) = \int_0^1 \ell_t(i,j)dj \ .$$

- $L_t =$ aggregate demand for composite labor:

$$L_t = \int_0^1 \left(\int_0^1 \ell_t(i,j)^{\frac{\sigma_w - 1}{\sigma_w}} di \right)^{\frac{\sigma_w}{\sigma_w - 1}} dj \ .$$

- Labor demand equation for labor variety i:

$$L_{t+k}(i) = L_{t+k} \left(\frac{V_t^w \left(v_t^w\right)^k}{W_{t+k}} \right)^{-\sigma_w} \ . \tag{34}$$

- Key parameter: $\sigma_w =$ elasticity of substitution between labor types.

7.2 Utility Maximization

- Wage setting policy of a worker i that reoptimizes at t, choosing V_t^w and v_t^w (a gross wage inflation rate):

$$W_{t+k}(i) = V_t^w \left(v_t^w\right)^k$$

- Define terms:
  - Front-loading term:

$$p_t^w \equiv \frac{V_t^w}{W_t}$$

  - Real wage:

$$w_t = \frac{W_t}{P_t}$$

  - Wage inflation rates re-scaled by the inflation target, with $\breve{\pi}_t^w = \pi_t^w / \pi_t^*$.
  - Cumulative aggregate rescaled wage inflation:

$$\breve{\Pi}_{t,k}^w = \Pi_{j=1}^k \breve{\pi}_{t+j}^w \text{ for } k \geq 1 \ \ (\equiv 1 \text{ for } k = 0)$$

  - Cumulative aggregate rescaled wage inflation deviation:

$$\hat{\Pi}_{t,k}^w = \Sigma_{j=1}^k \hat{\pi}_{t+j}^w \text{ for } k \geq 1 \ \ (\equiv 0 \text{ for } k = 0)$$

- Utility maximization - relevant part of the problem:

$$\underset{V_t^w, v_t^w}{Max} \quad E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k \left[-\kappa \frac{(L_{t+k}(i))^{1+\gamma}}{1+\gamma} + \lambda_{t+k} \frac{V_t^w (v_t^w)^k}{W_{t+k}} \frac{W_{t+k}}{P_{t+k}} L_{t+k}(i) \right] \ s.t.(34) \ .$$

- Key parameter: δ_w = Calvo delta for wage setters.

7.3 First-Order Conditions

- Firm specific wage inflation rescaled by the inflation target $\check{v}_t^w = v_t^w / \pi_t^*$.
  - By the unit root property we can ignore future changes in the inflation target (see discussion above for firms):

$$E_t \hat{\check{v}}_{t+k}^w = E_t \left(\ln \left(v_{t+k}^w \right) - \ln(\pi_{t+k}^*) \right) = E_t \left(\ln \left(v_{t+k}^w \right) - \ln(\pi_t^*) \right)$$

- FOC for V_t^w:

$$p_t^w E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k \lambda_{t+k} L_{t+k}(i) \left(w_{t+k} \frac{(\check{v}_t^w)^k}{\check{\Pi}_{t,k}^w} \right) = \mu^w E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k \kappa \left(L_{t+k}(i) \right)^{1+\gamma} \tag{35}$$

- FOC w.r.t. v_t^w:

$$p_t^w E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k k \lambda_{t+k} L_{t+k}(i) \left(w_{t+k} \frac{(\check{v}_t^w)^k}{\check{\Pi}_{t,k}^w} \right) = \mu^w E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k k \kappa \left(L_{t+k}(i) \right)^{1+\gamma} \tag{36}$$

7.4 Linearization

7.4.1 Linearization for V_t^w

- First step:

$$E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k \left[\hat{\lambda}_{t+k} - \gamma \hat{L}_{t+k}(i) + \hat{w}_{t+k} + \left(\hat{p}_t^w + k \hat{\check{v}}_t^w - \hat{\check{\Pi}}_{t,k}^w \right) \right] = 0$$

- Linearization of labor demand:

$$\left(\hat{L}_{t+k}(i) - \hat{L}_{t+k} \right) = -\sigma_w \left(\hat{p}_t^w + k \hat{\check{v}}_t^w - \hat{\check{\Pi}}_{t,k}^w \right)$$

- Marginal rate of substitution:
  - MRS in levels:

$$mrs_t = \frac{\kappa L_t(i)^\gamma}{\lambda_t}$$

- Log-linearized:

$$\widehat{mrs}_t = \gamma \hat{L}_t(i) - \hat{\lambda}_t$$

- Combine with the expression for labor demand (note that, for contemporaneous terms, $k = 0$):

$$\widehat{mrs}_t = \gamma \hat{L}_t - \gamma \sigma_w \hat{p}_t^w - \hat{\lambda}_t$$

- Combine the above:

$$E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k \left(\hat{p}_t^w + k\hat{v}_t^w - \hat{\Pi}_{t,k}^w \right) = E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k \left(\widehat{mrs}_{t+k} - \hat{w}_{t+k} \right)$$

- Apply formulas (7) and (8):

$$\frac{\hat{p}_t^w}{1 - \delta_w \beta} + \frac{\hat{v}_t^w \delta_w \beta}{(1 - \delta_w \beta)^2} = E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k \left[\widehat{mrs}_{t+k} - \hat{w}_{t+k} + \hat{\Pi}_{t,k}^w \right] \tag{37}$$

- With the appropriate change in notation this is exactly identical to equation (9) for price setting, after replacing $\hat{w}_{t+k} - \hat{z}_{t+k}$ with $\widehat{mrs}_{t+k} - \hat{w}_{t+k}$.

7.4.2 Linearization for v_t^w

- First step:

$$E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k k \left[\hat{\lambda}_{t+k} - \gamma \hat{L}_{t+k}(i) + \hat{w}_{t+k} + \left(\hat{p}_t^w + k\hat{v}_t^w - \hat{\Pi}_{t,k}^w \right) \right] = 0$$

- Combine with the above:

$$E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k k \left(\hat{p}_t^w + k\hat{v}_t^w - \hat{\Pi}_{t,k}^w \right) = E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k k \left(\widehat{mrs}_{t+k} - \hat{w}_{t+k} \right)$$

- Apply formulas (7) and (8):

$$\frac{\hat{p}_t^w \delta_w \beta}{(1 - \delta_w \beta)^2} + \frac{\hat{v}_t^w \delta_w \beta (1 + \delta_w \beta)}{(1 - \delta_w \beta)^3} = E_t \sum_{k=0}^{\infty} (\delta_w \beta)^k k \left[\widehat{mrs}_{t+k} - \hat{w}_{t+k} + \hat{\Pi}_{t,k}^w \right] \tag{38}$$

- With the appropriate change in notation this is exactly identical to equation (10) for price setting, after replacing $\hat{w}_{t+k} - \hat{z}_{t+k}$ with $\widehat{mrs}_{t+k} - \hat{w}_{t+k}$.

7.5 Final Wage Inflation Dynamics

- Given the identical forms of (37)/(9) and (38)/(10), we obtain after quasi-differencing as the end result an equation analogous to (17):

$$\left(E_t\hat{v}^w_{t+1} - \hat{v}^w_t\right) = \frac{\left(1-\delta_w\beta\right)^2}{\left(\delta_w\beta\right)^2}\left(\widehat{mrs}_t - \hat{w}_t - \hat{p}^w_t\right) \tag{39}$$

- Furthermore, the derivation of the wage index is identical to that for the price index above. The correct expression for wage inflation is:

$$\hat{\pi}^w_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \tag{40}$$

- Furthermore, the derivation of the wage index is identical to that for the price index above. We get:

$$\hat{\pi}^w_t = \frac{1-\delta_w}{\delta_w}\hat{p}^w_t + \hat{\psi}^w_t \tag{41}$$

$$\hat{\psi}^w_t = \delta_w\hat{\psi}^w_{t-1} + (1-\delta_w)\hat{v}^w_{t-1} - \hat{\varepsilon}^{\pi^*}_t \tag{42}$$

- Combining this with the results of quasi-differencing we obtain the equations for $E_t\hat{v}^w_{t+1}$ and $E_t\hat{\pi}^w_{t+1}$:

$$\begin{aligned} E_t\hat{v}^w_{t+1} &= \hat{v}^w_t + \frac{\left(1-\delta_w\beta\right)^2}{\left(\delta_w\beta\right)^2}\frac{\delta_w}{1-\delta_w}\hat{\psi}^w_t - \frac{\left(1-\delta_w\beta\right)^2}{\left(\delta_w\beta\right)^2}\frac{\delta_w}{1-\delta_w}\hat{\pi}^w_t \\ &\quad + \frac{\left(1-\delta_w\beta\right)^2}{\left(\delta_w\beta\right)^2}\left(\widehat{mrs}_t - \hat{w}_t\right) \end{aligned} \tag{43}$$

$$\begin{aligned} E_t\hat{\pi}^w_{t+1} &= \hat{\pi}^w_t\left(\frac{2}{\beta} - \delta_w\right) + \hat{v}^w_t\left(\left(1-\delta_w\right)\left(1+\delta_w\right)\right) \\ &\quad + \hat{\psi}^w_t\left(\delta_w(1+\delta_w) - \frac{2}{\beta}\right) - \frac{2(1-\delta_w)\left(1-\delta_w\beta\right)}{\left(\delta_w\beta\right)}\left(\widehat{mrs}_t - \hat{w}_t\right) \end{aligned} \tag{44}$$

- We can also use (41) to get a final expression for the marginal rate of substitution:

$$\widehat{mrs}_t = \gamma\hat{L}_t - \gamma\sigma_w\frac{\delta_w}{1-\delta_w}\hat{\pi}^w_t + \gamma\sigma_w\frac{\delta_w}{1-\delta_w}\hat{\psi}^w_t - \hat{\lambda}_t \tag{45}$$

7.6 The Backward Indexation Case

By analogy with the derivation under price setting we obtain the following for wage setting:

$$\left(\hat{\pi}_t^w - \hat{\pi}_{t-1}^w\right) = \beta E_t \left(\hat{\pi}_{t+1}^w - \hat{\pi}_t^w\right) + \frac{(1-\delta_w\beta)(1-\delta_w)}{\delta_w} \left(\widehat{mrs}_t - \hat{w}_t\right) - \hat{\varepsilon}_t^{\pi^*} \tag{46}$$

$$\widehat{mrs}_t = \gamma \hat{L}_t - \gamma\sigma_w \frac{\delta_w}{1-\delta_w}\hat{\pi}_t^w + \gamma\sigma_w \frac{\delta_w}{1-\delta_w}\hat{\pi}_{t-1}^w - \hat{\lambda}_t \tag{47}$$

7.7 The Calvo-Yun Case

By analogy with the derivation under price setting we obtain the following for wage setting:

$$\hat{\pi}_t^w = \beta E_t \hat{\pi}_{t+1}^w + \frac{(1-\delta_w\beta)(1-\delta_w)}{\delta_w} \left(\widehat{mrs}_t - \hat{w}_t\right) \tag{48}$$

$$\widehat{mrs}_t = \gamma \hat{L}_t - \gamma\sigma_w \frac{\delta_w}{1-\delta_w}\hat{\pi}_t^w - \hat{\lambda}_t \tag{49}$$

8 MONETARY POLICY

- Policy rule in levels ($\rho^i = 0$ in the baseline case):

$$i_t = (i_{t-1})^{\rho^i} \left(\frac{\pi_t^*}{\beta} E_t \left(\frac{\pi_{t+1}}{\pi_t^*}\right)^{\phi} \left(\frac{y_t}{y_t^n}\right)^{\theta} h_t^{\phi-1} \right)^{1-\rho^i}$$

- Normalization (remember that $E_t \hat{\varepsilon}_{t+1}^{\pi^*} = 0$ and $\hat{y}_t = \hat{c}_t$):

$$\check{i}_t = \left(\frac{\check{i}_{t-1}}{\varepsilon_t^{\pi^*}}\right)^{\rho^i} \left(\frac{1}{\beta} E_t \left(\check{\pi}_{t+1}\right)^{\phi} \left(\frac{y_t}{y_t^n}\right)^{\theta} h_t^{\phi-1} \right)^{1-\rho^i}$$

- Linearization:

$$\hat{\imath}_t = \rho^i \left(\hat{\imath}_{t-1} - \hat{\varepsilon}_t^{\pi^*}\right) + (1-\rho^i) \left(\phi E_t \hat{\pi}_{t+1} + \theta \left(\hat{y}_t - \hat{y}_t^n\right) + (\phi - 1)\hat{h}_t\right) \tag{50}$$

9 COMPLETE DYNAMIC SYSTEM

9.1 Calibration

$$\beta = 0.99$$

$$\text{Habit: } \ v = 0.7$$

$$\text{Inverse Frisch: } \ \gamma = 1$$

$$\text{Calvo Delta Prices: } \ \delta = 0.8$$

$$\text{Calvo Delta Wages: } \ \delta_w = 0.8$$

$$\text{EoS between Goods Varieties: } \ \sigma = 6$$

$$\text{EoS between Labor Varieties: } \ \sigma_w = 6$$

$$\text{Monetary Policy Rule - Inflation Feedback: } \ \phi = 1.5$$

$$\text{Monetary Policy Rule - Output Feedback: } \ \theta = 0.5$$

9.2 Steady State

$$\bar{y} = \bar{c} = \bar{L} = \bar{\ell} = 1 \quad \text{(normalization)}$$

$$\bar{C} = 1 - v$$

$$\bar{\lambda} = \frac{1 - \beta v}{1 - v}$$

$$\bar{w} = \frac{\sigma - 1}{\sigma}$$

$$\kappa = \bar{\lambda}\bar{w}$$

9.3 Aggregate Demand Block

- Equation (1):

$$\frac{\beta\nu}{1 - v} E_t \hat{c}_{t+1} = \frac{1 + \beta\nu^2}{1 - v}\hat{c}_t - \frac{\nu}{1 - v}\hat{c}_{t-1} + (1 - \beta\nu)\hat{\lambda}_t \tag{OPT1}$$

- Equation (2):

$$E_t \hat{\lambda}_{t+1} - E_t \hat{\pi}_{t+1} = \hat{\lambda}_t - \hat{\imath}_t \tag{OPT2}$$

9.4 Market Clearing

- Equation (4):

$$\hat{c}_t = \hat{z}_t + \hat{L}_t \tag{OPT3}$$

9.5 Monetary Policy

- Equation (50):

$$\hat{\imath}_t = \rho^i \left(\hat{\imath}_{t-1} - \hat{\varepsilon}_t^{\pi^*}\right) + (1-\rho^i)\left(\phi E_t \hat{\pi}_{t+1} + \theta(\hat{c}_t - \hat{c}_t^n) + (\phi - 1)\hat{h}_t\right) \tag{OPT4}$$

9.6 Exogenous Shocks

- Monetary Policy Shock:

$$\hat{h}_t = \rho^h \hat{h}_{t-1} + \hat{\varepsilon}_t^h \tag{OPT5}$$

- Technology Shock:

$$\hat{z}_t = \rho^z \hat{z}_{t-1} + \hat{\varepsilon}_t^z \tag{OPT6}$$

- Inflation Target Shock:

$$\hat{\varepsilon}_t^{\pi^*} \tag{OPT7}$$

9.7 Natural Rate of Output Block

- Consumption Euler Equation:

$$\frac{\beta\nu}{1-v} E_t \hat{c}_{t+1}^n = \frac{1+\beta\nu^2}{1-v}\hat{c}_t^n - \frac{\nu}{1-v}\hat{c}_{t-1}^n + (1-\beta\nu)\hat{\lambda}_t^n \tag{OPT8}$$

- Labor Supply:

$$\hat{w}_t^n + \hat{\lambda}_t^n - \gamma \hat{L}_t^n = 0 \tag{OPT9}$$

- Labor Demand:

$$\hat{w}_t^n = \hat{z}_t \tag{OPT10}$$

- Market Clearing:

$$\hat{c}_t^n = \hat{z}_t + \hat{L}_t^n \tag{OPT11}$$

9.8 Price Setting Block

9.8.1 Optimal Indexation

- Equation (20):

$$\hat{\psi}_t = \delta \hat{q}_t^{\psi} + (1-\delta)\hat{q}_t^{v} - \hat{\varepsilon}_t^{\pi^*} \tag{OPT12}$$

- Equation (24):

$$E_t \hat{v}_{t+1} = \hat{v}_t + \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\frac{\delta}{1-\delta}\left(\hat{\psi}_t - \hat{\pi}_t\right) + \frac{(1-\delta\beta)^2}{(\delta\beta)^2}\left(\hat{w}_t - \hat{z}_t\right) \tag{OPT13}$$

- Equation (25):

$$\begin{aligned} E_t \hat{\pi}_{t+1} &= \left(\frac{2}{\beta} - \delta\right)\hat{\pi}_t + (1-\delta)(1+\delta)\hat{v}_t + \left(\delta(1+\delta) - \frac{2}{\beta}\right)\hat{\psi}_t \\ &\quad - \left(2\left(1-\delta\right)\frac{(1-\delta\beta)}{(\delta\beta)}\right)\left(\hat{w}_t - \hat{z}_t\right) \end{aligned} \tag{OPT14}$$

9.8.2 Backward Indexation

$$\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) = \beta E_t\left(\hat{\pi}_{t+1} - \hat{\pi}_t\right) + \frac{(1-\delta\beta)(1-\delta)}{\delta}\left(\hat{w}_t - \hat{z}_t\right) - \hat{\varepsilon}_t^{\pi^*} \tag{BWI12}$$

9.8.3 Calvo-Yun

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\delta\beta)(1-\delta)}{\delta}\left(\hat{w}_t - \hat{z}_t\right) \tag{CY12}$$

9.9 Wage Setting Block

9.9.1 Flexible Wages

- Equation (3):

$$\hat{w}_t + \hat{\lambda}_t - \gamma \hat{L}_t = 0 \tag{OPT15/BWI13/CY13}$$

9.9.2 Optimal Indexation

- Equation (42):

$$\hat{\psi}_t^{w} = \delta_w \hat{\psi}_{t-1}^{w} + (1-\delta_w)\hat{v}_{t-1}^{w} - \hat{\varepsilon}_t^{\pi^*} \tag{OPT15-w}$$

- Equation (43):

$$E_t\hat{v}^w_{t+1} = \hat{v}^w_t + \frac{(1-\delta_w\beta)^2}{(\delta_w\beta)^2}\frac{\delta_w}{1-\delta_w}\left(\hat{\psi}^w_t - \hat{\pi}^w_t\right) + \frac{(1-\delta_w\beta)^2}{(\delta_w\beta)^2}\left(\widehat{mrs}_t - \hat{w}_t\right) \quad \text{(OPT16-w)}$$

- Equation (44):

$$\begin{aligned} E_t\hat{\pi}^w_{t+1} &= \hat{\pi}^w_t\left(\frac{2}{\beta} - \delta_w\right) + \hat{v}^w_t(1-\delta_w)(1+\delta_w) + \hat{\psi}^w_t\left(\delta_w(1+\delta_w) - \frac{2}{\beta}\right) \\ &\quad - \frac{2(1-\delta_w)(1-\delta_w\beta)}{(\delta_w\beta)}\left(\widehat{mrs}_t - \hat{w}_t\right) \end{aligned} \quad \text{(OPT17-w)}$$

- Equation (45):

$$\widehat{mrs}_t = \gamma\hat{L}_t - \gamma\sigma_w\frac{\delta_w}{1-\delta_w}\hat{\pi}^w_t + \gamma\sigma_w\frac{\delta_w}{1-\delta_w}\hat{\psi}^w_t - \hat{\lambda}_t \quad \text{(OPT18-w)}$$

- Equation (40):

$$\hat{\pi}^w_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \quad \text{(OPT19-w)}$$

9.9.3 Backward Indexation

- Equation (46):

$$\left(\hat{\pi}^w_t - \hat{\pi}^w_{t-1}\right) = \beta E_t\left(\hat{\pi}^w_{t+1} - \hat{\pi}^w_t\right) + \frac{(1-\delta_w\beta)(1-\delta_w)}{\delta_w}\left(\widehat{mrs}_t - \hat{w}_t\right) - \hat{\varepsilon}^{\pi^*}_t \quad \text{(BWI13-w)}$$

- Equation (47):

$$\widehat{mrs}_t = \gamma\hat{L}_t - \gamma\sigma_w\frac{\delta_w}{1-\delta_w}\hat{\pi}^w_t + \gamma\sigma_w\frac{\delta_w}{1-\delta_w}\hat{\pi}^w_{t-1} - \hat{\lambda}_t \quad \text{(BWI14-w)}$$

- Equation (40):

$$\hat{\pi}^w_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \quad \text{(BWI15-w)}$$

9.9.4 Calvo-Yun

- Equation (48):

$$\hat{\pi}^w_t = \beta E_t\hat{\pi}^w_{t+1} + \frac{(1-\delta_w\beta)(1-\delta_w)}{\delta_w}\left(\widehat{mrs}_t - \hat{w}_t\right) \quad \text{(CY13-w)}$$

- Equation (49):

$$\widehat{mrs}_t = \gamma\hat{L}_t - \gamma\sigma_w\frac{\delta_w}{1-\delta_w}\hat{\pi}^w_t - \hat{\lambda}_t \quad \text{(CY14-w)}$$

- Equation (40):

$$\hat{\pi}^w_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \quad \text{(CY15-w)}$$