

MATH 15: The educational philosophy behind the course

In addition to doing research in mathematics and its applications, I've spent the past 25 years trying to explain mathematics to "general audiences," from the enthusiastic amateur mathematician to the chronically math phobic. I've written general audience books, published articles in newspapers and popular magazines, worked on television and radio programs and series (including the acclaimed PBS series *Life by the Numbers*), and given public talks all over the world. I think I've done pretty well at this, and have won several awards, including the 2007 Carla Sagan Award for Science Popularization, the 2005 Pythagoras Prize, the 2003 Peano Prize, and the 2001 Communications Award of the Joint Policy Board for Mathematics.

In parallel with those activities, I've drawn upon my many years of experience interacting with the "more mathematically challenged" in society in order to develop what I believe is a fairly unique university-level course aimed primarily at nonscience majors in selective colleges and universities. I first gave such a course in 1990, and have been developing it and giving it ever since. The National Council of Teachers of Mathematics asked me to write a chapter about it in their *2000 Yearbook: Learning Mathematics for a New Century*.

I start from the assumption that if a young (or sometimes not so young) person is bright enough to get into a selective college, but feels insecure in, or threatened by, mathematics (and you find such students at places like Stanford and Harvard), then for many students there is little point in forcing that person to take one more course that resembles courses they have already taken and not done well in.

Given the masses of evidence that practically every one of us has the capacity to achieve competency in basic math (I catalog some of that evidence in my book *The Math Gene*), which we can switch on if and when the need arises, the focus of my course is not on developing mathematical competency but on the study of mathematics as a product of human culture: what it is, how it arose (and arises), how it is used in all walks of life, why mathematicians do it, and how it is done. The aim is to provide a broad, sophisticated knowledge of mathematics that I believe is important in an informed citizenry in today's world, and to prepare the ground so that, for those students that do subsequently find they need to acquire some particular mathematical skills, they will be able to approach that task with the confidence that comes from understanding the "big picture."

This is where Math 15 differs most significantly from most "non science majors math courses", which focus on what is generally called quantitative literacy. Many of the so-called "math for poets" courses have that as their main focus. The strategy is to dress up various bits of math in some attractive clothes, for example with number theory masquerading as the math of flowers and plants, coordinate geometry as an "exploration of the fourth dimension." That kind of

course, if taught well, can succeed with some students. But it's not what my course does.

In today's society, it is crucial for an informed citizen to (i) have a broad general awareness of mathematics, its true nature, its current role in society, its strengths and limitations, and its potential for good (and ill) in the future; and (ii) have working competency in quantitative reasoning (a good sense of number, shape and size, the ability to assess numerical, statistical, and probabilistic evidence, and a level of comfort with abstractly presented reasoning, such as in abstract algebra). While almost any nonscience majors mathematics course will address both of these needs (Math 15 certainly does), in a single term-long course a choice has to be made to focus primarily on one or the other.

My own investigations and experiences over many years have convinced me that, because almost everyone does have a latent capacity for mathematical thinking (read *The Math Gene* if you don't believe this), it is more effective to focus on the first goal, a broad awareness of, and appreciation for, mathematics. For one thing, this lays the groundwork for a subsequent acquisition of any needed quantitative skills, while the reverse is not the case. More importantly, the primary target audience for Math 15 comprises students whose school experience has left them with low confidence in mathematics, or even a fear or loathing of the subject. They need to form a more positive picture of, and appreciation for, mathematics, and hopefully even enjoy it, before they will be in a position to acquire further math skills.

The course makes no assumptions of mathematical ability on the part of the student. It does assume the student is intelligent, can read, can think reflectively, and is motivated to "do well" as a student. Because mathematics is primarily a way of thinking, part of the course does involve the students having to "do math," but this is not through anything like traditional "math problems," nor is the goal to produce "the right answer." Rather, the emphasis is on experiencing "what is involved in doing mathematics (as a professional mathematician does)."

There is no textbook, though there is a course reader, which is always a mass market "popular science" book. (Most recently I've been using my own book *The Language of Mathematics: Making the Invisible Visible*.) Reading the required book provides the mathematical backbone of the course, and a student who achieves a sufficient level of comprehension of the entire book is virtually guaranteed to pass the course with at least a B grade. Some lecture time is spent regularly on guiding students through the reading, but for the most part lectures are focused on building around the material in the chapter. This means that students can work through the core material at their own individual pace.

In the lectures I use videos and prepared visual materials to set the context for the topics discussed in the book, and to provide connections to contemporary

everyday life. One particular commitment I make to each student is that, during the course, at least one lecture will cover an application of mathematics of direct relevance to something important to him or her, either their major, hobby, or intended career. To facilitate this, they fill out a short questionnaire at the start of the course, on the basis of which I work out a detailed syllabus that meets this requirement.

The student's broad understanding of the material in the reader is assessed by means of a weekly quiz lasting 10 to 15 minutes. No, it's not a traditional "math quiz." The questions do not require that the student solves a math problem. They are more like the questions on general knowledge TV shows, designed to assess how well the student has understood the chapter. Students generally don't like the quizzes (who does? — well, some students do, as it turns out, but few students in my class like them), especially at first. But in their end-of-term course evaluation (read last year's for yourself, it's on the course website), they consistently say they recognize their value in helping them find their own pace through the material. (And some say they come to enjoy being able to demonstrate, to themselves, me, and their classmates, that they can do well in the dreaded M-subject!)

The next significant part of the final grade, after the reading, comes from a term paper, with each student choosing his or her own topic, where they research an application of mathematics in a domain that interests them. (The form and genre of the "paper" is up to the student; it can be a straight paper, a script for a TV news show, a short story, a magazine, a speech, etc.) At the end of the course, the students present their work to the entire class.

To make this approach work, the class is capped at 21 students. Many years of experience, using different variations of the basic format, have convinced me that such a course will not succeed (with the kind of student for which it is designed) if this cap is exceeded. I monitor the progress of each individual student closely (including grading most of their work myself), and help each person find their own path to success through the material.

In my experience, when a student gets to college with a feeling of inadequacy in math, or even worse math aversion, anxiety, or loathing, its origins can be traced back to a single bad experience with one teacher. In other words, the root problem is not math, it's a human experience with another person. The only way I know to counter that is to try to overcome that negative human experience with a positive one (with me). It's the nature of human interaction that I can't possibly succeed with every student, some years perhaps not even the majority in the class. But the class is built around that being *my* goal as the instructor.

Keith Devlin, August 2007