ME 451C: Compressible Turbulence, Spring 2017

Stanford University

Homework 2: Fluctuation Modes in Compressible Turbulence

Due Thursday, May 18, in class.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions.

Student's Name:......JAVIER URZAY...................... **Student's ID:**...........................

In uniform mean flows with density $\overline{\rho}$, pressure $\overline{P}$ temperature $\overline{T}$, and velocity U in the $+x$ direction, the linearization of the equation of state, the entropy equation, and the entropy transport equation lead, respectively, to the relations [1]

$$\frac{P'}{\overline{P}} = \frac{\rho'}{\overline{\rho}} + \frac{T'}{\overline{T}}, \tag{1}$$

$$\frac{s'}{c_p} = \frac{T'}{\overline{T}} - \left(\frac{\gamma-1}{\gamma}\right)\frac{P'}{\overline{P}}, \tag{2}$$

$$\frac{D}{Dt}\left(\frac{s'}{c_p}\right) = \frac{4}{3}\overline{\nu}\nabla^2\left[\frac{s'}{c_p} + \left(\frac{\gamma-1}{\gamma}\right)\frac{P'}{\overline{P}}\right], \tag{3}$$

where $\overline{\nu}$ is the mean kinematic viscosity and $D/Dt = \partial/\partial t + U\partial/\partial x$ is the material derivative. Similarly, using the linearized momentum equation

$$\frac{D\boldsymbol{u}'}{Dt} = -\frac{\nabla P'}{\overline{\rho}} + \overline{\nu}\nabla^2\boldsymbol{u}' + \frac{\overline{\nu}}{3}\nabla(\nabla\cdot\boldsymbol{u}') \tag{4}$$

along with (1)-(3), the conservation equation for the pressure fluctuations

$$\frac{1}{\gamma}\frac{D^2}{Dt^2}\left(\frac{P'}{\overline{P}}\right) = \frac{1}{\overline{\rho}}\nabla^2 P' + \frac{4}{3}\overline{\nu}\nabla^2\left[\frac{D}{Dt}\left(\frac{P'}{\overline{P}}\right)\right] \tag{5}$$

is obtained for a Prandtl number equal to 3/4. Lastly, the curl of the momentum equation yields

$$\frac{D\boldsymbol{\omega}'}{Dt} = \overline{\nu}\nabla^2\boldsymbol{\omega}' \tag{6}$$

for the vorticity fluctuations. This homework problem focuses on the structure of Kovazsnay's fluctuation modes in Fourier space. Assume that $\overline{\nu} = 1.5\cdot 10^{-5}$ m^2/s, $\gamma = 1.4$, $U = 600$ m/s, and $\overline{a} = 340$ m/s for all your calculations. Additionally, assume that the disturbance wavelength is 1 mm, as being created for instance by a small grid in a supersonic wind tunnel (Fig. 1).

1. In Kovasznay's analysis, the pressure fluctuations P' are ascribed to an acoustic mode, which is denoted by a superindex $^{(p)}$ and corresponds to the solution of Eq. (5). In particular, consider P' expressed as a single Fourier wave as

$$\frac{P'}{\overline{P}} = \hat{P}^{(p)}\exp[j|\boldsymbol{\kappa}|(\boldsymbol{n}\cdot\boldsymbol{r} - ct)], \tag{7}$$

[1] See class notes for details.

where j is the imaginary unit, $\boldsymbol{\kappa}$ is the wavenumber vector, and c is generally a complex eigenvalue whose imaginary component represents the propagation velocity of the mode. Similarly, $\boldsymbol{n}$ is the unit vector in the direction of the wavenumber vector, and $\boldsymbol{r}$ is the position vector in the laboratory frame.

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a) Show that the modes (7) can be redefined in terms of $C = c - U\kappa_x/|\boldsymbol{\kappa}|$ when the problem is formulated in the reference frame moving at the mean velocity U in the $+x$ direction, where κ_x is the projection of $\boldsymbol{\kappa}$ in that direction.

In the remainder of this homework, we will assume for simplicity that $\boldsymbol{\kappa}$ is aligned with the x axis so that $C = c - U$. We will also use the reference frame moving at speed U, where the position vector is now $\boldsymbol{r}'$, so that all the material derivatives become local time variations $\partial/\partial t$.

8/8

b) Substituting (7) into (5), show that C is given by

$$C = -\frac{2}{3}\overline{\nu}\gamma|\boldsymbol{\kappa}|j \pm \overline{a}\sqrt{1 - \left(\frac{2\overline{\nu}\gamma|\boldsymbol{\kappa}|}{3\overline{a}}\right)^2}, \tag{8}$$

where $\overline{a} = \sqrt{\gamma\overline{P}/\overline{\rho}}$ is the mean speed of sound. Note that (8) indicates the presence of two modes of P' corresponding to the positive and negative signs preceding the square root.

8/8

c) How small must the characteristic disturbance wavelength be for the second (dispersive) term in the square root in Eq. (8) to become important?

8/8

d) What is the characteristic decay time of the acoustic pressure disturbance and how long have the two modes traveled along the x direction in the laboratory frame before significant decay by viscosity occurs?

8/8

e) Using Eq. (4), find an expression for the amplitude of the acoustic mode of the velocity fluctuation as a function of $\hat{P}^{(p)}$ and show that the field is irrotational. This indicates that the acoustic mode is a longitudinal wave, in that it is parallel to the wavenumber.

2. The entropy conservation equation (3) suggests that the Fourier representation of the entropy fluctuations can be written as

$$\frac{s'}{c_p} = \hat{s}^{(p)} \exp[j|\boldsymbol{\kappa}|(\boldsymbol{n}\cdot\boldsymbol{r}' - Ct)] + \hat{s}^{(s)} \exp[j|\boldsymbol{\kappa}|(\boldsymbol{n}\cdot\boldsymbol{r}' - C_s t)], \tag{9}$$

where the first and second terms on the right-hand side correspond, respectively, to the particular and homogeneous solutions of (3). In particular, the super-index $^{(s)}$ indicates the entropy mode.

8/8

a) The first term on the right-hand side of (9) represents an entropy disturbance generated by the the viscous diffusion of the acoustic pressure fluctuation (7) and can therefore be ascribed to the acoustic mode. Obtain a relation between the fluctuation amplitudes $\hat{s}^{(p)}$ and $\hat{P}^{(p)}$ by making use of Eq. (3), and show that $\hat{s}^{(p)}$ is proportional to the viscosity.

b) On the other hand, the second term on the right-hand side of (9) represents an entropy mode that does not have an acoustic origin, and which is necessarily related to temperature fluctuations as suggested by Eq. (2) and described below. Prove that the value of C_s is a purely imaginary one given by

$$C_s = -\frac{4}{3}\overline{\nu}|\boldsymbol{\kappa}|j. \tag{10}$$

8/8

At what speed relative to the laboratory frame does this entropy fluctuation propagate? Compute the characteristic decay time of this entropy disturbance and the distance it has traveled along the x direction in the laboratory frame before significant decay by viscosity occurs.

3. In view of Eqs. (1)-(2) and (9), it is convenient to express the temperature and density fluctuations as a sum of entropic and acoustic components, namely

$$\frac{T'}{\overline{T}} = \hat{T}^{(s)} \exp[j|\boldsymbol{\kappa}|(\boldsymbol{n}\cdot\boldsymbol{r}' - C_T t)] + \hat{T}^{(p)} \exp[j|\boldsymbol{\kappa}|(\boldsymbol{n}\cdot\boldsymbol{r}' - Ct)], \tag{11}$$

$$\frac{\rho'}{\overline{\rho}} = \hat{\rho}^{(s)} \exp[j|\boldsymbol{\kappa}|(\boldsymbol{n}\cdot\boldsymbol{r}' - C_\rho t)] + \hat{\rho}^{(p)} \exp[j|\boldsymbol{\kappa}|(\boldsymbol{n}\cdot\boldsymbol{r}' - Ct)]. \tag{12}$$

8/8 a) Compute C_T, the corresponding decay time of the entropic component of the temperature fluctuations, and the distance traveled by them along the x direction in the laboratory frame before significant decay by viscosity occurs. At what speed relative to the laboratory frame does this does this component of the temperature fluctuations propagate? Obtain relations between $\hat{T}^{(s)}$ and $\hat{s}^{(s)}$, and between $\hat{T}^{(p)}$ and $\hat{P}^{(p)}$.

8/8 b) Using Eq. (1), compute C_ρ, the corresponding decay time of the entropic component of the density fluctuations, and the distance traveled by them along the x direction in the laboratory frame before significant decay by viscosity occurs. At what speed relative to the laboratory frame does this component of the density fluctuations propagate? Derive relations between $\hat{\rho}^{(s)}$ and $\hat{s}^{(s)}$, and between $\hat{\rho}^{(p)}$ and $\hat{P}^{(p)}$.

8/8 c) The entropy mode does not include any pressure fluctuations. As a result, Eqs. (1)-(2) suggest that in this mode the entropy fluctuations are related to the density fluctuations,

$$\frac{s'}{c_p} = -\frac{\rho'}{\overline{\rho}}. \tag{13}$$

Using (13) along with continuity, $D\rho'/Dt = -\overline{\rho}\nabla\cdot\boldsymbol{u}'$, obtain an expression for the amplitude of the entropy mode of the velocity fluctuations as a function of $\hat{s}^{(s)}$, show that the resulting field is irrotational and that the velocities and their divergence induced by the entropy mode are proportional to $\overline{\nu}$.

8/8 4. As indicated above, the acoustic and entropy modes are irrotational. As a result, the vorticity conservation equation (6) suggests that a separate fluctuation mode for the vorticity exists, which in wave form may be written as

$$\boldsymbol{\omega}' = \hat{\omega}^{(\omega)} \exp[j|\boldsymbol{\kappa}|(\boldsymbol{n}\cdot\boldsymbol{r}' - C_\omega t)], \tag{14}$$

where the super-index $^{(\omega)}$ indicates the vortical mode.

a) Substituting (14) into (6) find an expression for C_ω, compute the corresponding decay time of the vorticity fluctuations, and the distance traveled by them along the x direction in the laboratory frame before significant decay by viscosity occurs. At what speed relative to the laboratory frame does the vorticity fluctuation propagate?

8 b) Since the vortical mode does not have any associated fluctuations in pressure, entropy or temperature, prove that the induced velocity field is incompressible and corresponds to a transversal wave (i.e., a wave that is orthogonal to the wavenumber vector).

8/8 5. The effect of viscosity can be neglected at distances from the grid much smaller than the smallest one of the characteristic streamwise distances computed above for decay of the fluctuations. Describe how the results obtained above change when $\overline{\nu} = 0$.

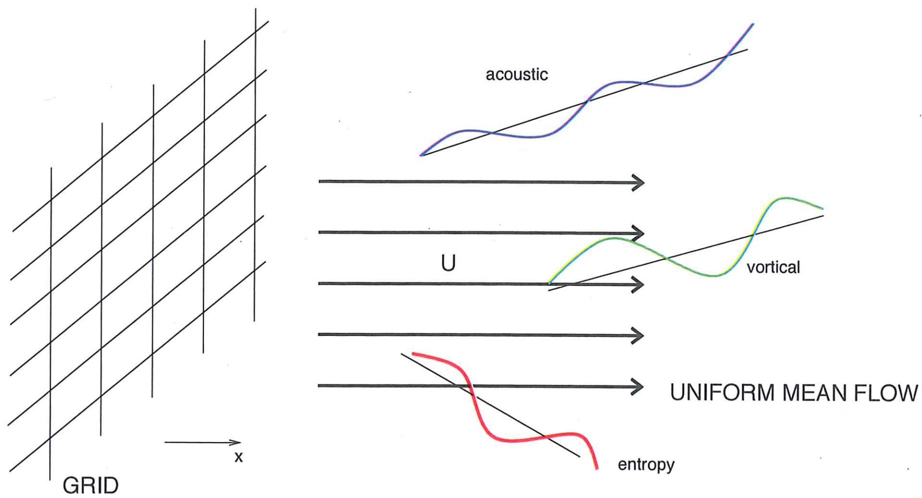
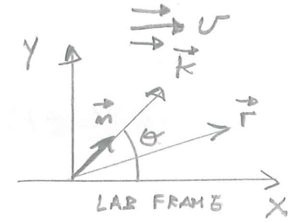


Figure 1: Schematics.

SOLUTIONS

1. $$\frac{p'^{(p)}}{\bar{p}} = \hat{p}^{(p)} e^{i|\vec{k}|(\vec{n}\cdot\vec{r} - ct)}$$



$\vec{r}$ = POSITION VECTOR
$\vec{k}$ = WAVENUMBER VECTOR

a) IN THE MOVING FRAME, $\vec{r}' = \vec{r} - Ut\vec{e}_x$, THEREFORE $i|\vec{k}|(\vec{n}\cdot\vec{r} - ct) = i|\vec{k}|(\vec{n}\cdot(\vec{r}' + Ut\vec{e}_x) - ct)$
$= i|\vec{k}|(\vec{n}\cdot\vec{r}' - \varsigma t)$, WITH $\varsigma = c - U\cos\theta = c - U\frac{k_x}{|\vec{k}|}$, WHICH BECOMES $\varsigma = c - U$ WHEN $\vec{k} \parallel \vec{e}_x$

b) NOTE THAT IN FOURIER SPACE,

$$\frac{\partial}{\partial t} \to -i|\vec{k}|\varsigma, \quad \frac{\partial^2}{\partial t^2} = -|\vec{k}|^2\varsigma^2, \quad \nabla^2 \to -|\vec{k}|^2, \quad \nabla \to -i\vec{k}$$

THEN (5) BECOMES $-|\vec{k}|^2\varsigma^2 = -\bar{a}^2|\vec{k}|^2 + |\vec{k}|^3\frac{4}{3}\bar{\nu}\gamma i\varsigma$, WITH $\bar{a}^2 = \frac{\gamma\bar{p}}{\bar{\rho}}$.

THIS CAN BE REWRITTEN AS $\varsigma^2 - \bar{a}^2 + \frac{4}{3}\bar{\nu}\gamma i|\vec{k}|\varsigma = 0$, WHICH HAS THE ROOTS CORRESPONDING TO

$$\varsigma = -\frac{2}{3}\bar{\nu}\gamma i|\vec{k}| \pm \bar{a}\sqrt{1 - \left(\frac{2\bar{\nu}\gamma|\vec{k}|}{3\bar{a}}\right)^2}$$

c) IT MUST SATISFY $\frac{2\bar{\nu}\gamma|\vec{k}|}{3\bar{a}} \sim 1 \Rightarrow |\vec{k}| \sim \frac{3\bar{a}}{2\bar{\nu}\gamma}$, SO THAT THE WAVELENGTH $\boxed{\lambda = \frac{2\pi}{|\vec{k}|} \lesssim \frac{4\pi\bar{\nu}\gamma}{3\bar{a}}}$
WHICH GIVES $\lambda \lesssim 2.58\cdot10^{-7}\,m$. FOR LARGER WAVELENGTHS, THE DISPERSIVE TERM IS NEGLIGIBLE. SINCE $\lambda = 1\,mm$ IN THIS PROBLEM, THEN THAT TERM CAN BE NEGLECTED.

d) SUBSTITUTING (8) INTO (7) GIVES

$$\frac{p'^{(p)}}{\bar{p}} = \hat{p}^{(p)} e^{i|\vec{k}|(\vec{n}\cdot\vec{r} - \frac{2}{3}\bar{\nu}\gamma i|\vec{k}|t \pm \bar{a}t)} = \hat{p}^{(p)} \underbrace{e^{|\vec{k}|^2\frac{2}{3}\bar{\nu}\gamma t}}_{\text{DECAYING}} \underbrace{e^{i|\vec{k}|(\vec{n}\cdot\vec{r} \pm at)}}_{\text{PROPAGATING}}$$

THE CHARACTERISTIC DECAY TIME OF THE ACOUSTIC MODE IS

$$t_\nu^{(p)} \sim \frac{3}{2}\frac{1}{|\vec{k}|^2\bar{\nu}\gamma} = \frac{3\lambda^2}{8\pi^2\bar{\nu}\gamma} = 1.8\cdot10^{-3}\,s$$

THE CORRESPONDING DISTANCES TRAVELED BY THE ACOUSTIC MODES BEFORE DECAY ARE

$$L_\nu^{(p)} \sim (U \pm \bar{a})\,t_\nu^{(p)} = 0.46\,m \text{ (LEFT RUNNING MODE)}$$
$$= 1.67\,m \text{ (RIGHT RUNNING MODE)}$$

e) SUBSTITUTING (7) INTO (4) AND ASSUMING THAT $\frac{\vec{U}^{(p)}}{U} = \hat{\vec{U}}^{(p)} e^{i|\vec{k}|(\vec{n}\cdot\vec{r} - \varsigma t)}$, THEN

$$-i|\vec{k}|\varsigma U\hat{\vec{U}}^{(p)} = -i\frac{\bar{p}}{\bar{\rho}}\vec{k}\hat{p}^{(p)} - \frac{4}{3}\bar{\nu}|\vec{k}|^2 U\hat{\vec{U}}^{(p)} \Rightarrow \boxed{\hat{\vec{U}}^{(p)} = \frac{\bar{a}^2}{\gamma U\varsigma}\frac{\vec{k}}{|\vec{k}|}\frac{\left(1 + i\frac{4\bar{\nu}|\vec{k}|}{3\varsigma}\right)\hat{p}^{(p)}}{\left(1 + \left(\frac{4}{3}\frac{\bar{\nu}|\vec{k}|}{\varsigma}\right)^2\right)}}$$

WHICH SHOWS THAT $\hat{\vec{U}}^{(p)} \parallel \vec{k}$, AND THEREFORE $\nabla\wedge\vec{U}^{(p)} = 0$

2. CONSIDER THE ENTROPY FLUCTUATIONS

$$\frac{s'}{c_p} = \underbrace{\hat{s}^{(p)} e^{i|\vec{k}|(\vec{n}\cdot\vec{r}' - \varsigma t)}}_{\text{ACOUSTIC MODE}} + \underbrace{\hat{s}^{(s)} e^{i|\vec{k}|(\vec{n}\cdot\vec{r}' - \varsigma_s t)}}_{\text{ENTROPY MODE}}$$

a) SUBSTITUTING THE FIRST TERM INTO (3) GIVES

$$-i|\vec{k}|\varsigma\,\hat{s}^{(p)} = -\frac{4}{3}\bar{\nu}|\vec{k}|^2\left(\hat{s}^{(p)} + \left(\frac{\gamma-1}{\gamma}\right)\hat{p}^{(p)}\right)$$

SO THAT

$$\boxed{\hat{s}^{(p)} = -\frac{\frac{4}{3}\frac{\bar{\nu}|\vec{k}|}{\varsigma}\left(\frac{4}{3}\frac{\bar{\nu}|\vec{k}|}{\varsigma} + i\right)\left(\frac{\gamma-1}{\gamma}\right)\hat{p}^{(p)}}{1+\left(\frac{4}{3}\frac{\bar{\nu}|\vec{k}|}{\varsigma}\right)^2}}$$

, WHICH SHOWS THAT $\hat{s}^{(p)}$ IS PROPORTIONAL TO THE VISCOSITY.

b) SUBSTITUTING THE SECOND TERM INTO (3) WITH $\Gamma' = 0$ GIVES

$$-i|\vec{k}|\varsigma_s\cancel{\hat{s}^{(s)}} = -\frac{4}{3}\bar{\nu}|\vec{k}|^2\cancel{\hat{s}^{(s)}} \Rightarrow \varsigma_s = -\frac{4}{3}\bar{\nu}|\vec{k}|\,i$$

ALSO SUBSTITUTING ς_s INTO THE ENTROPY MODE GIVES

$$\frac{s'^{(s)}}{c_p} = \hat{s}^{(s)}\underbrace{e^{-\frac{4}{3}\bar{\nu}|\vec{k}|^2 t}}_{\text{DECAYING}}\ \underbrace{e^{i|\vec{k}|(\vec{n}\cdot\vec{r})}}_{\text{PROPAGATING AT ZERO SPEED IN THE MOVING FRAME}}$$

PROPAGATING AT ZERO SPEED IN THE MOVING FRAME (U IN THE LAB FRAME)

THE CHARACTERISTIC DECAY TIME IS $t_D^{(s)} \sim \frac{3}{4}\frac{1}{\bar{\nu}|\vec{k}|^2} \overset{\lambda = 2\pi/|\vec{k}|}{=} \frac{3\lambda^2}{16\pi^2\bar{\nu}} = 1.2\cdot 10^{-3}\,\text{s}$

AND THE DISTANCE TRAVELLED BEFORE DECAY IS $L_D^{(s)} \sim U\cdot t_D^{(s)} = 0.76\,\text{m}$

3. THE TEMPERATURE AND DENSITY FLUCTUATIONS CAN BE WRITTEN AS

$$\left.\begin{aligned}\frac{T'}{\bar{T}} &= \hat{T}^{(s)} e^{i|\vec{k}|(\vec{n}\cdot\vec{r}' - \varsigma_T t)} + \hat{T}^{(p)} e^{i|\vec{k}|(\vec{n}\cdot\vec{r}' - \varsigma t)} \\ \frac{\rho'}{\bar{\rho}} &= \underbrace{\hat{\rho}^{(s)} e^{i|\vec{k}|(\vec{n}\cdot\vec{r}' - \varsigma_s t)}}_{\text{ENTROPY MODE}} + \underbrace{\hat{\rho}^{(p)} e^{i|\vec{k}|(\vec{n}\cdot\vec{r}' - \varsigma t)}}_{\text{ACOUSTIC MODE}}\end{aligned}\right\}$$

a) SUBSTITUTING THE ENTROPY MODE OF T' AND S' INTO (2) AND NOTICING THAT THE ENTROPIC PRESSURE DISTURBANCES ARE ZERO NECESSARILY LEADS TO $\hat{S}^{(s)} = \hat{T}^{(s)}$ AND $\varsigma_T = \varsigma_s$, SO THE CONCLUSIONS FROM 2b ARE APPLICABLE TO $T'^{(s)}$.

SUBSTITUTING THE ACOUSTIC MODE OF T' INTO (2) AND USING THE RESULT FROM 2a LEADS TO

$$\hat{s}^{(p)} = \hat{T}^{(p)} - \left(\frac{\gamma-1}{\gamma}\right)\hat{p}^{(p)} \Rightarrow \hat{T}^{(p)} = \left(\frac{-\frac{4}{3}\frac{\bar{\nu}|\vec{k}|}{\varsigma}\left(\frac{4}{3}\frac{\bar{\nu}|\vec{k}|}{\varsigma} + i\right)}{1+\left(\frac{4}{3}\frac{\bar{\nu}|\vec{k}|}{\varsigma}\right)^2} + 1\right)\left(\frac{\gamma-1}{\gamma}\right)\hat{p}^{(p)}$$

b) SUBSTITUTING THE ENTROPY MODE OF THE DENSITY FLUCTUATIONS INTO (1) AND NOTICING THAT THE ENTROPIC PRESSURE DISTURBANCES ARE ZERO NECESSARILY LEADS TO $c_g^{\not d} = c_s$ AND $\hat{\rho}^{(s)} = -\hat{T}^{(s)} = -\hat{s}^{(s)}$. THEREFORE ALL RESULTS FROM 2b ARE APPLICABLE TO $\rho'^{(s)}$

THE SAME SUBSTITUTION FOR THE ACOUSTIC MODE YIELDS

$$\hat{\rho}^{(P)} = \hat{p}^{(P)} - \hat{T}^{(P)}, \quad \text{WHERE } \hat{T}^{(P)} \text{ IS GIVEN IN 3a.}$$

c) SUBSTITUTING THE ENTROPY MODE OF ρ' INTO (1) AND ASSUMING THAT $\frac{\vec{U}'^{(s)}}{U} = \hat{\vec{U}}^{(s)} e^{i|\vec{k}|(\vec{n}\cdot\vec{r}' - c_s t)}$, THEN

$$-i|\vec{k}|c_s\hat{\rho}^{(s)} = -i\vec{k}\cdot\hat{\vec{U}}^{(s)}U$$

AND SINCE $\hat{\rho}^{(s)} = -\hat{s}^{(s)} \Rightarrow \vec{k}\cdot\hat{\vec{U}}^{(s)}U = -|\vec{k}|c_s\hat{s}^{(s)}$.

NOTE THAT THE ENTROPIC MODE OF THE VELOCITY FLUCTUATIONS IS IRROTATIONAL, AND AS A RESULT $\vec{U}^{(s)}$ DERIVES FROM A POTENTIAL, WHICH IN VIEW OF THE ABOVE EXPRESSION MUST BE GIVEN BY

$$\vec{k}\cdot\left(-i\frac{\vec{k}}{|\vec{k}|}\left(\frac{c_s\hat{s}^{(s)}}{i}\right)\right) = -|\vec{k}|c_s\hat{s}^{(s)} \Rightarrow \vec{U}^{(s)} = -iUc_s\hat{s}^{(s)}\frac{\vec{k}}{|\vec{k}|}$$

$$\text{WITH } c_s = -\frac{4}{3}\nu|\vec{k}|i$$

4. a) CONSIDER THE VORTICITY FLUCTUATION

$$\vec{\omega}' = \hat{\omega}^{(\omega)} e^{i|\vec{k}|(\vec{n}\cdot\vec{r}' - c_\omega t)}$$

AND SUBSTITUTE IT INTO (6), WHICH GIVES

$$-i|\vec{k}|c_\omega = -|\vec{k}|^2\bar{\nu} \Rightarrow c_\omega = -\bar{\nu}|\vec{k}|i$$

$$\text{SO THAT} \quad \vec{\omega}' = \hat{\omega}^{(\omega)} \underbrace{e^{+\nu|\vec{k}|^2 t}}_{\text{DECAYING}} \underbrace{e^{i|\vec{k}|(\vec{n}\cdot\vec{r}')}}_{\text{PROPAGATING AT ZERO SPEED WITH RESPECT TO THE MEAN FLOW}}$$

CHARACTERISTIC DECAY TIME: $t_D^{(\omega)} \sim \frac{1}{\nu|\vec{k}|^2} = \frac{\lambda^2}{4\pi^2\nu} = 1.7\cdot 10^{-3}$ s

LENGTH TRAVELLED BEFORE DECAY: $L_D^{(\omega)} \approx U t_D^{(\omega)} \sim 1$ m

b) SINCE $p^{(\omega)} = T^{(\omega)} = \rho^{(\omega)} = 0$ IN THE VORTICAL MODE, THE ASSOCIATED FLUCTUATION VELOCITY FIELD MUST SATISFY THE SOLENOIDAL CONDITION $\nabla\cdot\vec{U}'^{(\omega)} = 0$, WHICH IS EQUIVALENT TO $\vec{k}\cdot\hat{\vec{U}}^{(\omega)} = 0 \Rightarrow \hat{U}^{(\omega)} \perp \vec{k}$

5. VISCOSITY CAN BE NEGLECTED $X \ll \min\left(L_D^{(P)}, L_D^{(\omega)}, L_D^{(S)}\right) = 0.46\ m$.

SETTING $D = 0$ IN THE RESULTS LEADS TO

$$\varsigma = \pm a, \quad \frac{p'^{(P)}}{\bar{p}} = \hat{p}^{(P)} e^{j|\vec{k}|(\vec{m}\cdot\vec{r}' \pm a t)}, \quad \frac{\vec{U}'^{(P)}}{\bar{U}} = \frac{\bar{a}^2}{\gamma \bar{U} \varsigma} \frac{\vec{k}}{|\vec{k}|} \hat{p}^{(P)}$$

$$s'^{(P)} = 0, \quad \varsigma_s = 0, \quad \frac{s'^{(S)}}{c_p} = \hat{s}^{(S)} e^{j|\vec{k}|(\vec{m}\cdot\vec{r}')}, \quad \frac{T'}{\bar{T}} = \hat{T}^{(S)} e^{j|\vec{k}|(\vec{m}\cdot\vec{r}')} + \hat{T}^{(P)} e^{j|\vec{k}|(\vec{m}\cdot\vec{r}' \pm \bar{a} t)}$$

$$\frac{\rho'}{\bar{\rho}} = \hat{\rho}^{(S)} e^{j|\vec{k}|(\vec{m}\cdot\vec{r}')} + \hat{\rho}^{(P)} e^{j|\vec{k}|(\vec{m}\cdot\vec{r}' \pm \bar{a} t)}$$

$$\hat{T}^{(P)} = \left(\frac{\gamma - 1}{\gamma}\right) \hat{p}^{(P)}, \quad \vec{U}'^{(S)} = 0, \quad \vec{\omega}'^{(\omega)} = \hat{\omega}^{(\omega)} e^{j|\vec{k}|(\vec{m}\cdot\vec{r}')}.$$

THUS ILLUSTRATING THAT THE ENTROPY, VORTICITY AND PRESSURE EVOLVE INDEPENDENTLY AND NONE OF THE MODES IS SUBJECT TO VISCOUS DECAY.