ME 451C: Compressible Turbulence, Spring 2017

Stanford University

Homework 1: Fundamental Aspects of Compressible Flows and Shock Waves

Due Thursday, April 27, in class.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions.

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10pt 1. Air flows isentropically through a constant-area duct at Ma = 0.7. The stagnation pressure at point 1 is $P_0 = 2$ bar. What is the stagnation pressure at a downstream point 2?

10pt 2. In addition to the conditions described in the previous question, it is also known that the stagnation density at point 1 is $\rho_0 = 2.0$ kg/m^3. What is the static enthalpy at the downstream point 2?

10pt 3. Starting from Crocco's equation, derive Bernoulli's equation $h + |\mathbf{u}|^2/2 = C$, where h is the enthalpy, $\mathbf{u}$ is the velocity vector, and C is a constant that depends on the streamline chosen for integration.

10pt 4. Show that the entropy variation across two streamlines 1 and 2, i.e., $s_2 - s_1 = c_v \ln[(P_2/\rho_2^\gamma)/(P_1/\rho_1^\gamma)]$, can be written solely in terms of the ratio of stagnation pressures P_{02}/P_{01} if the stagnation enthalpy is uniform everywhere.

20pt 5. A supersonic air flow at $\mathrm{Ma}_1 = 4$ and $P_1 = 2$ bar encounters a normal shock wave. Compute the Mach number in the post-shock flow Ma_2 as well as the relative variation in the stagnation pressure $(P_{01} - P_{02})/P_{01}$ and specific entropy $(s_1 - s_2)/c_v$.

20pt 6. A supersonic inviscid mixture of H_2 and air flows parallel to a wall at $\mathrm{Ma}_1 = 5.0$, $T_1 = 300$ K and $P_1 = 1$ bar, and encounters a compression ramp that deflects the stream upwards at an angle δ creating an oblique shock wave that emanates from the ramp corner. Compute the angle δ required to increase the temperature of the gas to the crossover value $T_2 = \cancel{950}$ 1000 K required for autoignition. What is the associated post-shock Mach number Ma_2? Assume that the properties of the mixture are similar to those of air.

20pt 7. Sketch i) the inviscid supersonic flow at $\mathrm{Ma}_1 = 4.5$ around a symmetric wedge of semi-angle $\delta = 15°$ at pressure $P_1 = 1$ bar, ii) the same flow when the semi-angle is increased to $\delta = 70°$. Comment on whether the flow downstream of the shock in both cases is rotational or irrotational and give appropriate analytical justifications to support your explanations.

SOLUTIONS:

① : ISENTROPIC FLOW $\Rightarrow P_{02} = 2$ bar $= P_{01}$

② : $h_{01} = h_{02} = \dfrac{P_{01}}{\rho_{01}}\dfrac{\gamma}{\gamma-1} = h_2 + \dfrac{U_2^2}{2} \Rightarrow h_2 = \dfrac{h_{01}}{1+\frac{(\gamma-1)}{2}\mathrm{Ma}_1^2} = \dfrac{\frac{P_{01}}{\rho_{01}}\frac{\gamma}{\gamma-1}}{1+\frac{(\gamma-1)}{2}\mathrm{Ma}_1^2} = \underline{\underline{318\ \mathrm{kJ/kg}}}$

CONSTANT AREA, ISENTROPIC

③ $\vec{\omega} \wedge \vec{U} = T\nabla s - \nabla\left(h + \frac{|\vec{U}|^2}{2}\right)$ CROCCO'S EQUATION

MULTIPLYING THIS EQUATION BY A UNIT VECTOR $\vec{e}_\ell$ TANGENT TO THE STREAMLINES:

ISENTROPIC $(\vec{U} \cdot \nabla s = 0)$

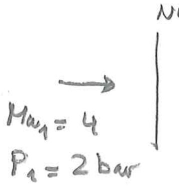
$$\cancel{(\vec{\omega}\wedge\vec{U})\cdot\vec{e}_\ell} = \cancel{T\vec{e}_\ell \nabla s} - \vec{e}_\ell \cdot \nabla\left(h + \frac{|\vec{U}|^2}{2}\right) \Rightarrow \frac{\partial}{\partial \ell}\left(h + \frac{|\vec{U}|^2}{2}\right) = 0$$

0 SINCE $\vec{\omega}\wedge\vec{U} \perp \vec{U}$ AND $\perp \vec{e}_\ell$

AND THEREFORE $h + \frac{|\vec{U}|^2}{2} = C$

④ SEE PAGE 18 OF CLASS NOTES.

⑤ NORMAL SHOCK



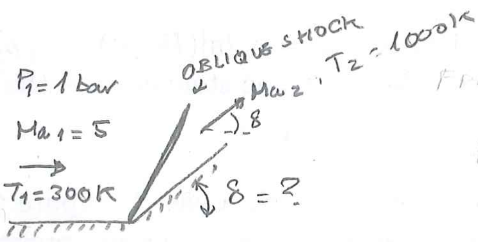
- POST-SHOCK MACH: $Ma_2 = \left(\frac{2 + (\gamma-1)Ma_1^2}{2\gamma Ma_1^2 + 1 - \gamma}\right)^{1/2} = \underline{0.43}$
- STAGNATION PRESSURE: $\frac{P_{02}}{P_{01}} = \left(\frac{1 + \frac{\gamma-1}{2}Ma_2^2}{1 + \frac{\gamma-1}{2}Ma_1^2}\right)^{\frac{\gamma}{\gamma-1}}\left[1 + \frac{2\gamma}{\gamma+1}(Ma_1^2 - 1)\right] = 0.138$

  $\Rightarrow \frac{P_{02} - P_{01}}{P_{01}} = 0.862$
- ENTROPY INCREASES THROUGH THE SHOCK

$$\frac{s_2 - s_1}{c_v} = \ln\left(\frac{P_2/\rho_2^\gamma}{P_1/\rho_1^\gamma}\right) = \ln\left(\frac{P_2}{P_1}\left(\frac{\rho_1}{\rho_2}\right)^\gamma\right) = \ln\left(\frac{18.5}{4.57^{1.4}}\right) = \underline{0.79}$$

JUMP CONDITIONS (64)-(66) IN CLASS NOTES

⑥



TEMPERATURE JUMP: $\frac{T_2}{T_1} = \frac{1000}{300} = 3.33$,

WHICH CORRESPONDS TO A NORMAL MACH NUMBER THAT HAS TO BE OBTAINED FROM THE JUMP CONDITION

$$\frac{T_2}{T_1} = \frac{[2\gamma Ma_{1n}^2 - (\gamma-1)][2 + (\gamma-1)Ma_{1n}^2]}{(\gamma+1)^2 Ma_{1n}^2} = 3.33$$

$\Rightarrow Ma_{1n} \sim 3.5$, THEREFORE $\beta = \arcsin\left(\frac{Ma_{1n}}{Ma_1}\right) = 44.4^\circ$

ENTERING IN THE β-δ DIAGRAM WITH $\beta = 44.4^\circ$ AND $Ma_1 = 5$ ONE OBTAINS $\underline{\delta \sim 33^\circ}$ (WEAK SOLUTION)

⑦ NOTE THAT δ_{MAX} AT $Ma_1 = 4.5$ IS $\underline{40^\circ}$. FOR $\delta < \delta_{MAX}$ THE SHOCK IS ATTACHED. FOR $\delta > \delta_{MAX}$ THE SHOCK IS DETACHED AND FORMS A BOW SHOCK. SKETCHES OF THE CORRESPONDING FLOWS ARE PROVIDED IN PAGE 26 OF THE CLASS NOTES. THE POST-SHOCK GAS IS IRROTATIONAL WHEN $\delta < \delta_{MAX}$, AND ROTATIONAL WHEN $\delta > \delta_{MAX}$, THE REASONS BEING EXPLAINED IN PAGES 18-19 OF THE CLASS NOTES.

FROM THE δ-β CHART