ME 451C: Compressible Turbulence, Spring 2017

Stanford University

Homework 1: Fundamental Aspects of Compressible Flows and Shock Waves

Due Thursday, April 27, in class.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions.

Student's Name:.. **Student's ID:**............................

1. Air flows isentropically through a constant-area duct at $\mathrm{Ma} = 0.7$. The stagnation pressure at point 1 is $P_0 = 2$ bar. What is the stagnation pressure at a downstream point 2?

2. In addition to the conditions described in the previous question, it is also known that the stagnation density at point 1 is $\rho_0 = 2.0\ \mathrm{kg/m^3}$. What is the static enthalpy at the downstream point 2?

3. Starting from Crocco's equation, derive Bernoulli's equation $h + |\mathbf{u}|^2/2 = C$, where h is the enthalpy, $\mathbf{u}$ is the velocity vector, and C is a constant that depends on the streamline chosen for integration.

4. Show that the entropy variation across two streamlines 1 and 2, i.e., $s_2 - s_1 = c_v \ln[(P_2/\rho_2^\gamma)/(P_1/\rho_1^\gamma)]$, can be written solely in terms of the ratio of stagnation pressures P_{02}/P_{01} if the stagnation enthalpy is uniform everywhere.

5. A supersonic air flow at $\mathrm{Ma}_1 = 4$ and $P_1 = 2$ bar encounters a normal shock wave. Compute the Mach number in the post-shock flow Ma_2 as well as the relative variation in the stagnation pressure $(P_{01} - P_{02})/P_{01}$ and specific entropy $(s_1 - s_2)/c_v$.

6. A supersonic inviscid mixture of H_2 and air flows parallel to a wall at $\mathrm{Ma}_1 = 5.0$, $T_1 = 300$ K and $P_1 = 1$ bar, and encounters a compression ramp that deflects the stream upwards at an angle δ creating an oblique shock wave that emanates from the ramp corner. Compute the angle δ required to increase the temperature of the gas to the crossover value $T_2 = 1000$ K required for autoignition. What is the associated post-shock Mach number Ma_2? Assume that the properties of the mixture are similar to those of air.

7. Sketch i) the inviscid supersonic flow at $\mathrm{Ma}_1 = 4.5$ around a symmetric wedge of semi-angle $\delta = 15°$ at pressure $P_1 = 1$ bar, ii) the same flow when the semi-angle is increased to $\delta = 70°$. Comment on whether the flow downstream of the shock in both cases is rotational or irrotational and give appropriate analytical justifications to support your explanations.