ME 355: Compressible Flows, Spring 2016

Stanford University

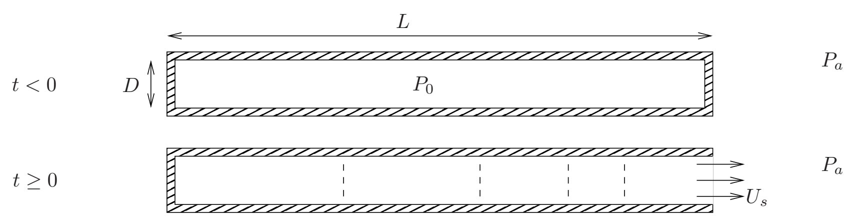
Homework 4: Wave motion

Due Tuesday, May 31, in class.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions. Assume $\gamma = 1.4$ and $c_p = 1$ KJ/KgK for all problems.

Student's Name:.. **Student's ID:**.............................

A cylinder of diameter D and length L initially closed at both ends is filled with an stagnant gas at pressure P_0 and density ρ_0. The cylinder is surrounded by air at pressure $P_a < P_0$. At $t = 0$, a pyrotechnic system breaks the right-end wall of the duct, so that the gas starts flowing out of the cylinder. This problem addresses the dynamics shortly after rupture of the right-end wall during times of order L/a_0, where a_0 is the speed of sound in the gas. Depending on the value of the overpressure $\epsilon = (P_0 - P_a)/P_0$, the disturbances produced in the gas may be small ($\epsilon \ll 1$), so that the linear-acoustics theory is applicable, or large ($\epsilon = O(1)$), in which case the linear approximation is no longer valid and the non-linear acoustics formulation needs to be employed.



Consider first the case in which the air pressure differs slightly from the pressure of the gas in the cylinder, $\epsilon = (P_0 - P_a)/P_0 \ll 1$.

a) Compute the velocity of the gas at the cylinder exit U_s.

b) Calculate the force on the cylinder.

c) Characterize the spatial profiles of velocity and pressure inside the cylinder during the time interval $0 < t < 2L/a_0$.

Secondly, consider the case in which the air pressure differs from the pressure of the gas by an amount of order unity, $\epsilon = (P_0 - P_a)/P_0 = O(1)$.

d) Compute the velocity of the gas at the cylinder exit U_s. Check that U_s becomes the one obtained in part a) in the limit $\epsilon \ll 1$.

e) Calculate the force on the cylinder.

f) Characterize the spatial profiles of velocity and pressure inside the cylinder during the time interval $0 < t < L/a_0$.