ME 355: Compressible Flows, Spring 2016

Stanford University

Homework 3: Convergent-divergent nozzles, and flows with heat addition

Due Tuesday, May 17, in class.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions. Assume $\gamma = 1.4$ and $c_p = 1$ KJ/KgK for all problems.

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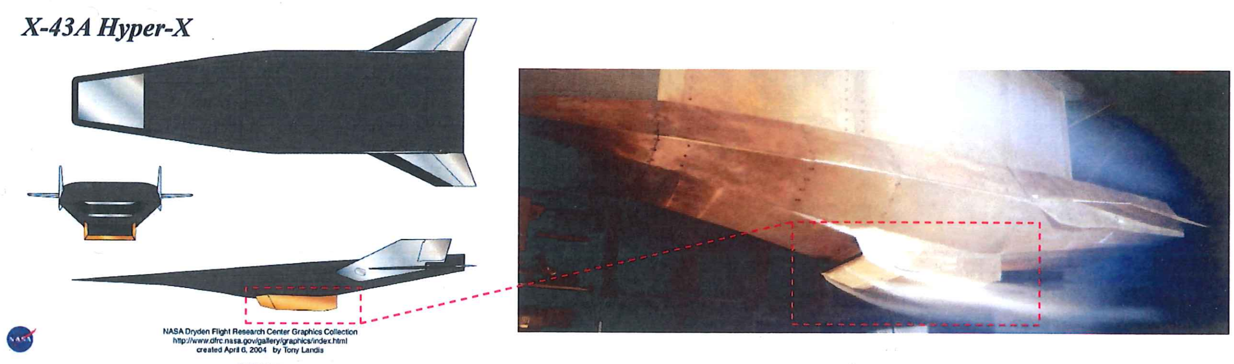
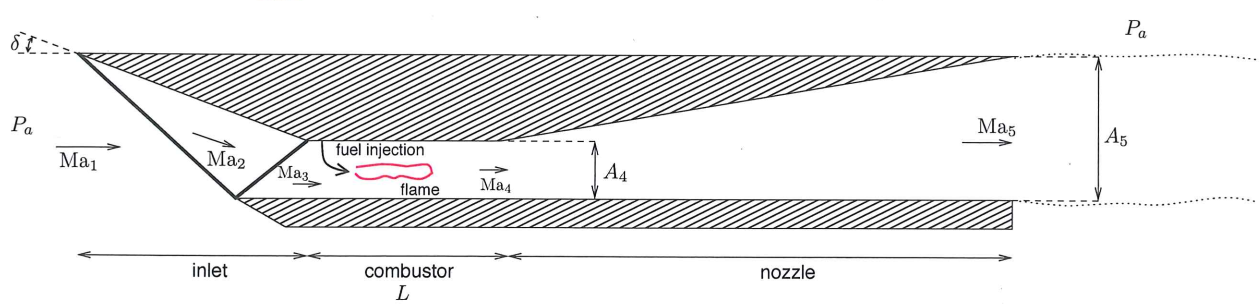
SCRAMJETS (supersonic combustion ramjets) are utilized as propulsion systems for suborbital hypersonic vehicles. SCRAMJET-powered vehicles carry the fuel on board, and obtain the oxidizer by ingestion of atmospheric oxygen. SCRAMJET engines are composed of three basic components: an inlet supersonic diffuser, where incoming air is compressed through a shock train; a combustor, where gaseous fuel burns supersonically with atmospheric oxygen to produce heat; and a diverging nozzle, where the hot combustion products are accelerated to produce thrust. A model description of the SCRAMJET power plant of the X-43A hypersonic aircraft is provided in the figure below, where $\mathrm{Ma}_1 = 5$, $\delta = 15°$, $P_a = 0.1$ bar, $T_a = 270$ K and $A_4/A_5 = 0.5$.





a) Compute the Mach number Ma_3, static pressure P_3 and static temperature T_3 at the combustor inlet.

b) Calculate the amount of heat release per unit mass that is required in the combustor to attain sonic conditions $\mathrm{Ma}_4 = 1.0$ at the nozzle inlet section (combustor exit). What are the associated static temperature T_4 and pressure P_4?

c) Determine the Mach number Ma_5 and static pressure P_5 at the nozzle exit.

d) Show that the thrust F produced by the engine can be written as

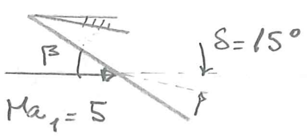
$$\frac{F}{P_1 A_5} = \frac{P_5}{P_1}(1+\gamma \mathrm{Ma}_5^2) - (1+\gamma \mathrm{Ma}_1^2), \tag{1}$$

and compute its numerical value for $A_5 = 0.03$ m^2.

e) Characterize the jet flow at the exhaust. Compute the streamline deflection angle and Mach number downstream from the first set of expansion fans.

f) The length of the combustor is $L = 10$ cm, and the autoignition time of the fuel+air mixture at temperature T_3 and pressure P_3 is $t_{ig} = 10\ \mu s$. Estimate the ratio of the characteristic residence time of a fluid particle in the combustor to the autoignition time. The ratio represents the Damköhler number Da. Based on the estimated value of Da, is the residence time in the combustor sufficiently large to warrant combustion of the reactants?

a) 1 → 2 : OBLIQUE SHOCK

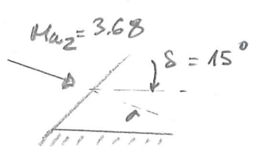


FROM CHART: $\beta = 24^\circ \Rightarrow Ma_{1n} = Ma_1 \sin\beta = 2.03$

TABLE II NORMAL SHOCK: $P_2/P_a = 4.5$, $T_2/T_a = 1.68$, $Ma_{2n} = 0.57$

THEN $Ma_2 = Ma_{2n}/\sin(\beta-\delta) = 3.68$

2 → 3 OBLIQUE SHOCK



FROM CHART: $\beta = 28^\circ \Rightarrow Ma_{2n} = Ma_2 \sin\beta = 1.73$

TABLE II NORMAL SHOCK: $P_3/P_2 = 3.30$, $T_3/T_2 = 1.48$, $Ma_{3n} = 0.63$

THEN $Ma_3 = Ma_{3n}/\sin(\beta-\delta) = \boxed{2.8}$

$\Rightarrow P_3 = \frac{P_3}{P_2}\frac{P_2}{P_a} P_a = \boxed{1.48 \text{ bar}}$, $T_3 = \frac{T_3}{T_2}\frac{T_2}{T_a} T_a = \boxed{671.3 \text{ K}}$

b) FOR SONIC CONDITIONS $Ma_4 = 1.0$, THE HEAT ADDED MUST BE

$$\frac{q_{MAX}}{h_{03}} = \frac{1}{2(\gamma+1)\mathcal{F}^2(Ma_3)} - 1 , \quad \text{WITH } \mathcal{F}(Ma_3) = \frac{Ma_3}{1+\gamma Ma_3^2}\left[1+\frac{(\gamma-1)}{2}Ma_3^2\right]^{1/2} = 0.37$$

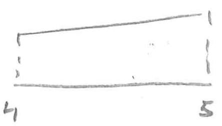
$$\Rightarrow \frac{q_{MAX}}{h_{03}} = 0.52 \Rightarrow q_{MAX} = 0.52\, c_p T_3\left(1+\frac{(\gamma-1)}{2}Ma_3^2\right) = \boxed{896.4\ \mathrm{kJ/kg}}$$

SINCE $q_{MAX} = c_p(T_{04}-T_{03}) \Rightarrow T_{04} = \frac{q_{MAX}}{c_p} + T_3\left(1+\frac{(\gamma-1)}{2}Ma_3^2\right) = 2620.2\text{ K} = T_4\left(1+\frac{(\gamma-1)}{2}Ma_4^2\right)$

$\Rightarrow T_4 = \boxed{2183.5 \text{ K}}$

AND $\dfrac{P_4}{P_3} = \dfrac{1+\gamma Ma_3^2}{1+\gamma \cancel{Ma_4^2}_{1}} = 4.99$ (CONS. OF MOMENTUM) $\Rightarrow P_4 = 4.99\, P_3 = \boxed{7.38 \text{ bar}}$

c) ISENTROPIC FLOW:



$Ma_4 = 1$ $\qquad Ma_5 = ?$

$A_4/A_5 = 0.5$

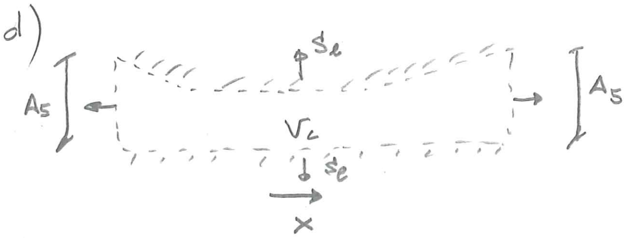
$$\dot{m} = \rho U A = \text{CONST} \Rightarrow A_5 Ma_5 \left(1+\frac{\gamma-1}{2} Ma_5^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}} = \left(\frac{1+\gamma}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} A_4$$

$$\frac{Ma_5\left(1+\frac{(\gamma-1)}{2} Ma_5^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}}{\left(\frac{1+\gamma}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}} = \frac{A_4}{A_5} = 0.5 = \frac{A^*}{A_5}$$

$\hookrightarrow$ FROM TABLE I (ISENTROPIC FLOW): $\boxed{Ma_5 = 2.18}$

$$\frac{P_5}{P_4} = \left(\frac{1+\frac{\gamma-1}{2}\cancel{Ma_4^2}^{1}}{1+\frac{(\gamma-1)}{2}Ma_5^2}\right)^{\frac{\gamma}{\gamma-1}} = 0.18 \Rightarrow \boxed{P_5 = 1.34 \text{ bar}}$$

d)



$$\int_{S_C} \rho \vec{v}\vec{v}\cdot d\vec{S} = -\int_{S_C} P\, d\vec{S}$$

$$(-\rho_1 U_1^2 A_5 + \rho_5 U_5^2 A_5)\vec{e}_x = \underbrace{-\int_{S_L} P\, d\vec{S}}_{-\vec{F}} + (P_1 A_5 - P_5 A_5)\vec{e}_x$$

$$\Rightarrow \vec{F} = (P_1 - P_5)A_5 \vec{e}_x + (\rho_1 U_1^2 - \rho_5 U_5^2)A_5 \vec{e}_x$$

$$\frac{\vec{F}}{P_1 A_5} = \left(1 - \frac{P_5}{P_1} + \frac{\rho_1 U_1^2}{P_1} - \frac{\rho_5 U_5^2}{P_1}\right)\vec{e}_x = \left(1 - \frac{P_5}{P_1} + \gamma Ma_1^2 - \gamma Ma_5^2 \frac{P_5}{P_1}\right)\vec{e}_x$$

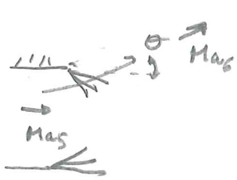
$$\Rightarrow \frac{\vec{F}}{P_1 A_5} = \left\{-\frac{P_5}{P_1}\left(1+\gamma Ma_5^2\right) + \left(1+\gamma Ma_1^2\right)\right\}\vec{e}_x$$

THEN THE THRUST (−x DIR) IS $\dfrac{F}{P_1 A_5} = \dfrac{P_5}{P_1}\left(1+\gamma Ma_5^2\right) - \left(1+\gamma Ma_1^2\right)$

SUBSTITUTING VALUES: $P_1 = 0.1$ bar, $P_5 = 1.34$ bar, $A_5 = 0.03\ m^2$, $Ma_5 = 2.18$, $Ma_1 = 5$ $\Big\}$ $\dfrac{F}{P_1 A} = 66.5 \Rightarrow \boxed{F \doteq 19.9 \text{ kN}}$

e) AT THE EXHAUST, THE FLOW IS SUPERSONIC $(Ma_5 = 2.18)$ AND UNDEREXPANDED $(P_5 > P_a)$

$\Rightarrow$ EXPANSION FANS (SEE FIG 27 IN PAGE 43 NOTES)



MACH OF DEFLECTED STREAM.

THE TURNING ANGLE IS $\theta = \nu(Ma_6) - \underbrace{\nu(Ma_5)}_{31.73^\circ \text{ TABLE III}}$

SINCE $\dfrac{P_6}{P_5} = \dfrac{P_a}{P_5} = 0.074 = \left(\dfrac{1+\left(\frac{\gamma-1}{2}\right)Ma_5^2}{1+\frac{(\gamma-1)}{2}Ma_6^2}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow Ma_6 = 3.94$

TABLE II: $\nu(Ma_6) = 65.1^\circ$

$\Rightarrow \boxed{\theta = 33.3^\circ}$

f) THE CHARACTERISTIC RESIDENCE TIME OF A FLUID PARTICLE IN THE COMBUSTOR IS $t_R \approx \frac{L}{U_3}$, WHERE $U_3 = Ma_3 \sqrt{\gamma R_g T_3} = 1451.6 \ m/s$ IS THE FLOW VELOCITY AT THE COMBUSTOR ENTRANCE. THEN $t_R \sim 68.9 \ \mu s$.

$\rightarrow$ DAMKÖHLER: $Da = \frac{t_R}{t_{ig}} = 6.89 > 1$, THE RESIDENCE TIME IS LONG ENOUGH TO WARRANT AUTOIGNITION.