ME 355: Compressible Flows, Spring 2016

Stanford University

Homework 2: Shocks and expansion waves

Due Thursday, May 5, in class.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions. Assume $\gamma = 1.4$ for all problems.

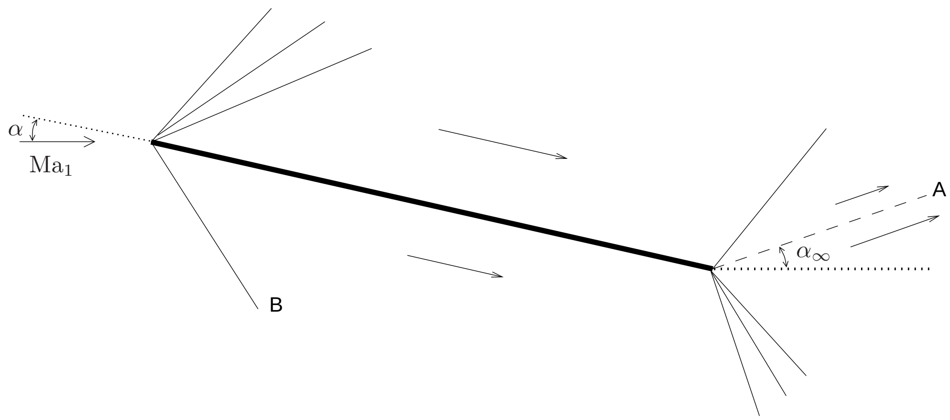
Student's Name:... **Student's ID:**............................

Questions (40 pts)

1. A supersonic air flow at $\mathrm{Ma}_1 = 2.4$ and $P_1 = 1$ bar encounters a normal shock wave. Compute the Mach number in the post-shock flow Ma_2 as well as the relative variation in the stagnation pressure $(P_{01} - P_{02})/P_{01}$ and specific entropy $(s_1 - s_2)/c_v$.
2. A supersonic flow parallel to a wall at $\mathrm{Ma}_1 = 3.0$ and $P_1 = 3$ bar encounters a corner that deflects the stream outwards at an angle θ creating an expansion fan. Compute the angle θ required to decrease the static pressure of the gas to $P_2 = 0.1$ bar. What is the associated Mach number Ma_2?
3. Sketch i) the supersonic flow at $\mathrm{Ma}_1 = 4.5$ around a symmetric wedge of semi-angle $\delta = 15°$ at pressure $P_1 = 1$ bar, ii) the same flow when the semi-angle is increased to $\delta = 70°$.
4. Compute the Chapman-Jouget deflagration and detonation velocities, as well as the Mach number of the reactants and products in the wave frame, for a stoichiometric H_2-air mixture at temperature $T_1 = 298$ K and pressure $P_1 = 1$ bar. Assume that the heat released by the combustion reaction is $q = 3423$ KJ/Kg and that the gas constant is $R_g = 383$ J/KgK.

Problem 1 (30 pts)

Consider the flat plate depicted below as a model of a supersonic interceptor aircraft flying in the stratosphere at an altitude $h = 20$ km. The aircraft flies at $\mathrm{Ma}_1 = 3.0$ with an angle of attack $\alpha = 30°$.



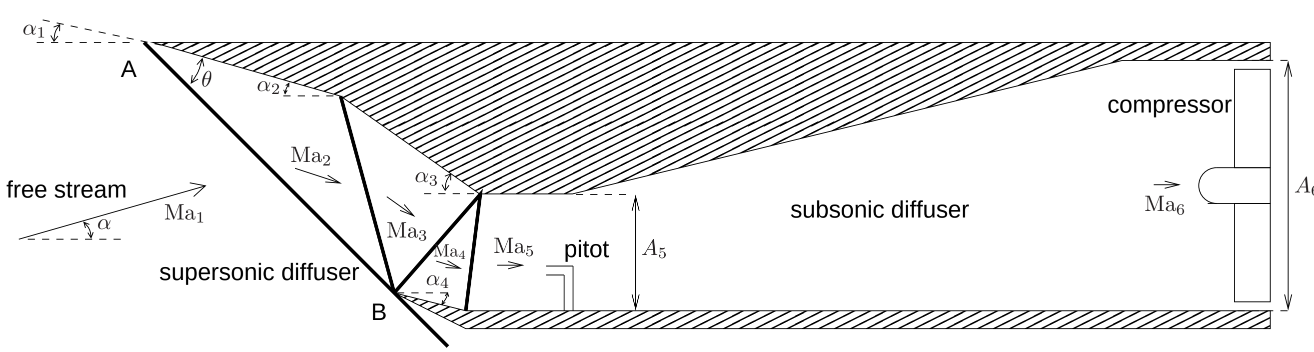
a) Determine the wave-drag and compression-lift coefficients given by $C_D = 2D/(\rho_1 U_1^2 S)$ and $C_L = 2D/(\rho_1 U_1^2 S)$, respectively. In this formulation, S is the length of the plate, while D and L are the drag and lift forces per unit spanwise length.

b) Because of the non-zero angle of attack, the solution in the wake of the aircraft involves a tangential discontinuity A that in reality leads to a shear layer with constant static pressure across. Compute the streamline angle of deflection α_∞ with respect to the incident horizontal free stream.

c) Far from the aircraft, the leading shock B becomes weaker and resembles a weak shock or compression Mach wave whose streamline deflection angle is just 0.1% of the aircraft angle of attack α. This wave creates a sonic boom that is perceived in populated areas on the ground. Compute the angle of inclination of the Mach wave on the ground and the associated sound pressure level (in decibels) as it passes through the populated areas.

Problem 3 (30 pts)

The Concorde was powered by four Rolls-Royce/Snecma Olympus-593 turbojet engines capable of producing 38,050 lbs of thrust. Each one included compressor, combustor, turbine and afterburner stages. At supersonic flight speeds, the air had to be decelerated to subsonic velocities before reaching the compressor. However, slowing down a supersonic stream necessarily involves shock waves and loss of stagnation pressure. For that reason, the intake design for Concorde's engines was especially critical and involved complex ramp and nozzle assemblies, including a variable-geometry intake control system that shifted from a straight duct at subsonic speeds to increasing the ramp angles at supersonic speeds, thus transforming the intake front into a supersonic diffuser. Downstream from the shock train, the flow became subsonic and it was further decelerated through a subsonic diffuser for increasing as much as possible the static pressure upstream from the compressor. A model description of the intake at design conditions is depicted in the figure below, where $\mathrm{Ma}_1 = 2.2$, $\alpha_1 = 10°$, $P_1 = 0.3$ bar and $A_6/A_5 = 1.4$. A pitot-tube measurement right downstream from the shock train gives $\mathrm{Ma}_5 = 0.7$ and $P_5 = 1.4$ bar. Consider only the weak solution for oblique shocks.



a) Calculate the angle of attack α such that the angle formed by the leading shock and the upper cowl lip is $\theta = 30°$, as shown in the figure.

b) Calculate the Mach number Ma_6 upstream from the compressor.

c) Determine the pressure-recovery ratio P_6/P_{01}.