ME 355: Compressible Flows, Spring 2016

Stanford University

Homework 1: Isentropic flow

Due Tuesday, April 19, in class.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions.

Student's Name:.. **Student's ID:**.............................

Questions (40 pts)

1. Air flows isentropically through a constant-area duct at $\text{Ma} = 0.4$. The stagnation pressure at point 1 is $P_0 = 1.3$ bar. What is the stagnation pressure at a downstream point 2?

2. In addition to the conditions described in the previous question, it is also known that the stagnation density at point 1 is $\rho_0 = 1.0$ kg/m^3. What is the static enthalpy at the downstream point 2?

3. Sketch the streamwise variations of pressure, speed of sound and flow velocity for a) a subsonic flow through a divergent nozzle, b) a supersonic flow through a divergent nozzle, and c) a subsonic flow through a convergent nozzle.

4. A gas with $\gamma = 1.4$ flows supersonically through a diverging duct. At section 1, the cross-sectional area is A_1 and the Mach number is $\text{Ma}_1 = 1.6$. At section 2, the cross-sectional area is $A_2 = 5A_1$. Calculate the Mach number Ma_2 at section 2.

Problem 1 (30 pts)

Consider the subsonic diffuser shown in the figure where the inlet Mach number $\text{Ma}_1 < 1$ and the area ratio $A_2/A_1 > 1$ are known parameters.

a) Derive an expression to compute the static-pressure ratio P_2/P_1 as a function of Ma_1, A_2/A_1 and γ.

b) Calculate the static-pressure ratio P_2/P_1 for $\text{Ma}_1 = 0.4$ and $A_2/A_1 = 4$.

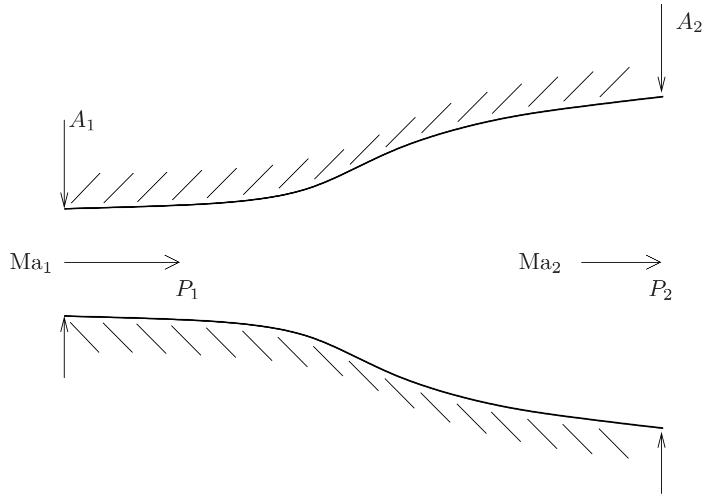
c) Prove that the expression

$$\frac{P_0}{P_1} = \left[1 + \frac{(\gamma - 1)}{2}\text{Ma}_1^2\right]^{\gamma/(\gamma-1)}$$

leads to the incompressible Bernoulli equation

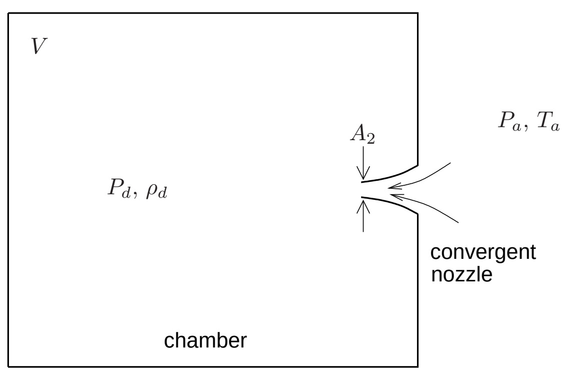
$$P_1 + \rho U_1^2/2 = P_0$$

in the incompressible limit $\text{Ma}^2 \ll 1$, where P_0 is the stagnation pressure, U_1 is the flow velocity, and ρ is the density.



Problem 2 (30 pts)

A small hole is perforated on the wall of a chamber of volume V that is initially void of gas. The hole resembles a convergent nozzle of minimum cross-sectional area $A_2 \ll V^{2/3}$. Air from the atmosphere flows through the hole filling up the chamber, as shown in the figure below. The atmosphere is at pressure P_a and temperature T_a.



a) Show that mass and total internal-energy conservation constraints require that the time evolution of the density ρ_d and pressure P_d of the gas in the chamber are given by

$$\frac{d\rho_d}{dt} = \frac{\dot{m}}{V} \tag{1}$$

and

$$\frac{dP_d}{dt} = \frac{\dot{m}}{V} a_a^2 \tag{2}$$

where $\dot{m}$ is the mass flow rate and a_a is the speed of sound in the atmospheric air.

b) Obtain an expression for the time $t^\star$ during which the hole remains choked as a function of γ, V, A_2, T_a and the gas constant R_g.

c) Plot the time evolution of the density ρ_d/ρ_a and pressure P_d/P_a of the gas in the chamber during the interval $0 < t/t^\star < 5$ for $\gamma = 1.4$.