ME 355: Compressible Flows, Spring 2016

Stanford University

Final Exam

Friday, June 3

Guidelines: Please turn in *neat* and *clean* exam solutions that give all the formulae that you have used as well as details that are required for the grader to understand your solution. Attach these sheets to your solutions. Assume $\gamma = 1.4$ and $c_p = 1.0$ KJ/KgK for all problems.

Student's Name: JAVIER URZAY **Student's ID:**

PART 1: Closed books, closed notes, calculators allowed
Time: 40 mins

Questions (30 pts)

1. Using the conservation equations in a control volume that includes an oblique shock front, state the continuity, momentum and energy jump constraints across the shock wave, indicating which variables (or group of variables) remain invariant and which ones are discontinuous. SEE PAGE 19 NOTES
2. Explain why a curved shock wave generates vorticity in an initially irrotational flow. SEE PAGE 64 NOTES
3. Explain the structure of a ZND detonation indicating its main regions in terms of pressure, temperature and velocity variations across the front, and sketch the approximate thermodynamic trajectory of a fluid particle in a pressure / specific-volume diagram. SEE PAGE 82 NOTES

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PART II: Open books, open notes, calculators allowed
Time: 120 mins

Problem 1 (40 pts)

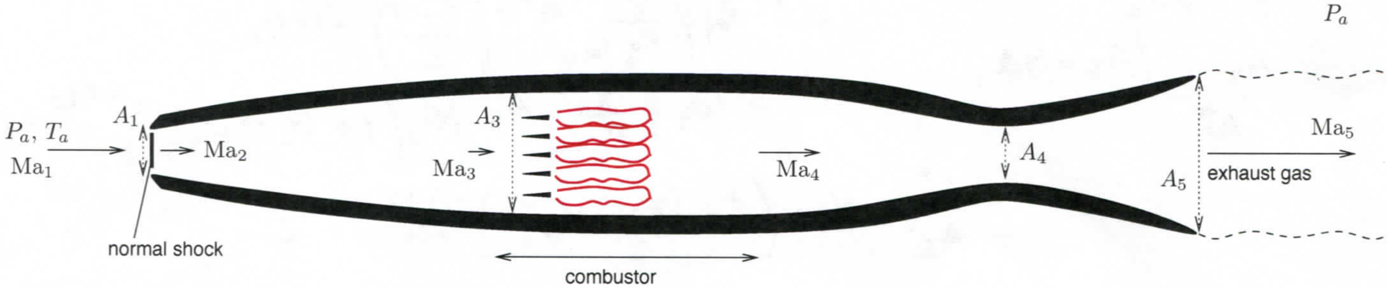
A simplified model of a RAMJET engine is depicted in the figure below. RAMJETS differ from SCRAMJETS in that combustion in the former occurs subsonically rather than supersonically. Supersonic air at $U_1 = 2$ km/s, $T_a = 270$ K and $P_a = 0.1$ bar is ingested through a nozzle of cross section $A_1 = 20$ cm^2 that creates a normal shock. The flow passes through a diffuser of area ratio $A_3/A_1 = 2$ before entering the combustion chamber. The heat release from combustion leads to a 20% increase in the stagnation temperature. The combustion products flow through a converging-diverging nozzle of area ratio $A_4/A_5 = 0.2$, with $A_4/A_3 = 0.5$.

a) The Mach number, pressure and temperature of the air entering the combustor.

b) The Mach number, pressure and temperature of the gas leaving the combustor.

c) The Mach number, pressure and temperature of the exhaust gases.

d) The specific impulse of the engine, $I_s = F/(\dot{m}g)$, where F is the engine thrust, $\dot{m}$ is the mass flow rate, and g is the gravitational acceleration.



a) NORMAL SHOCK (TABLE II)

$6.1 = Ma_1 = \frac{U_1}{\sqrt{\gamma R_g T_a}}$ $\quad Ma_2$

$P_a, T_a \quad P_2\, T_2$

$\Rightarrow Ma_2 = \boxed{0.4}$

$P_2/P_a = 43.2, \; T_2/T_a = 8.2 \rightarrow \boxed{P_2 = 4.3 \text{ bar} \quad T_2 = 2214 \text{ K}}$

TO ISENTROPIC RELATION TO OBTAIN: $T_{o_2} = T_2\left(1 + \left(\frac{\gamma-1}{2}\right) Ma_2^2\right) = 2284.5 \text{ K} = T_{o3}$

2 → 3 ISENTROPIC DIFFUSER: $A_2 Ma_2 \left(1 + \frac{(\gamma-1)}{2} Ma_2^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}} = A_3 Ma_3 \left(1 + \frac{(\gamma-1)}{2} Ma_3^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$

FLOW WITH HEAT ADDITION: $A_2 = A_1$

$$\Rightarrow \frac{A_3}{A_2} = \frac{Ma_2\left(1+\left(\frac{\gamma-1}{2}\right)Ma_2^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}}{Ma_3\left(1+\left(\frac{\gamma-1}{2}\right)Ma_3^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}} = 2 \Rightarrow \text{OBTAIN SOLUTION } Ma_3 \ (Ma_3<1)$$

$$\frac{1}{0.628} = 1.592$$

$$\frac{A_3}{A^*} = \frac{A_3}{A_2}\frac{A_2}{A^*} = 2 \cdot 1.592 = 3.18 \Rightarrow \text{DOES NOT COME IN MY TABLE!}$$

(ITERATE TO SOLVE FOR Ma_3 $\Rightarrow$ $\boxed{Ma_3 = 0.19}$

$$\Rightarrow \frac{P_3}{P_2} = \frac{\left(1+\left(\frac{\gamma-1}{2}\right)Ma_2^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1+\left(\frac{\gamma-1}{2}\right)Ma_3^2\right)^{\frac{\gamma}{\gamma-1}}} = 1.09 \Rightarrow \boxed{P_3 = 4.67 \text{ bar}}$$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = 1.02 \Rightarrow \boxed{T_3 = 2268.5}$$

b) $T_{04} = 1.2 \cdot T_{03} = 1.2\, T_{02} =$

$$\Rightarrow q = c_p(T_{04}-T_{03}) = c_p T_{03}\left(\frac{T_{04}}{T_{03}}-1\right) = \frac{c_p T_{03}}{5}$$

$$\Rightarrow \mathcal{F}^2(Ma_4) = \mathcal{F}^2(Ma_3)\left(1+\frac{q}{c_pT_{03}}\right) = 1.2\,\underbrace{\mathcal{F}^2(Ma_3)}_{0.0329} = 0.0395$$

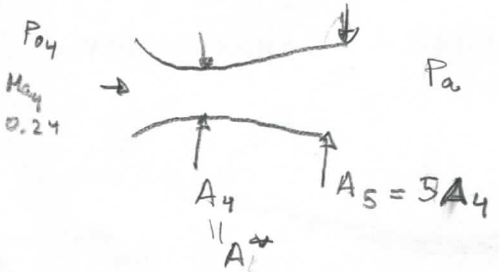
0.0329 FROM

$$\mathcal{F}(Ma) = \frac{Ma}{1+\gamma Ma^2}\left[1+\left(\frac{\gamma-1}{2}\right)Ma^2\right]^{1/2}$$

$$\Downarrow \quad \boxed{Ma_4 \approx 0.24}$$

THEN $T_{04} = 2741$ K $\Rightarrow$ $T_4 = \dfrac{T_{04}}{1+\frac{\gamma-1}{2}Ma_4^2} = \boxed{2710 \text{ K}}$; $\dfrac{P_4}{P_3} = \dfrac{1+\gamma Ma_3^2}{1+\gamma Ma_4^2} = 0.972 \Rightarrow \boxed{P_4 = 4.54 \text{ bar}}$

c) IN ORDER TO KNOW THE CONDITIONS AT THE EXIT WE NEED TO COMPUTE THE CHARACTERISTIC PRESSURES OF THE CD NOZZLE:



IF THE FLOW IS CHOKED,

$$\dot{m}^* = P_{04}\sqrt{\frac{\gamma}{R_g T_{04}}}A^*\left(\frac{1+\gamma}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} =$$

$$= P_{04}\sqrt{\frac{\gamma}{R_g T_{04}}}A_5\, Ma_5\left(1+\frac{(\gamma-1)}{2}Ma_5^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

$$\Rightarrow \frac{A^*}{A_5} = Ma_5\left(1+\frac{(\gamma-1)}{2}Ma_5^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}} = \frac{1}{5}$$

$\rightarrow Ma_5^{SUP} = 3.17$ TABLES

$\rightarrow Ma_5^{SUB} = 0.21$ ITERATION

$\Rightarrow$ PRESSURE FOR SONIC THROAT (STILL SUBSONIC FLOW AT EXIT):

$$\boxed{P_C = P_{04}\left(1+\left(\frac{\gamma-1}{2}\right)\underbrace{Ma_5^2}_{SUB}\right)^{-\frac{\gamma}{\gamma-1}} = \left(\frac{1+\frac{\gamma-1}{2}Ma_4^2}{1+\frac{\gamma-1}{2}Ma_5^2}\right)^{\frac{\gamma}{\gamma-1}}P_4 = 1.01\,P_4 = 4.5 \text{ bar}}$$

$\Rightarrow$ PRESSURE FOR SONIC THROAT AND SUPERSONIC FLOW AT EXIT

$$\boxed{P_{OE} = \frac{\left(1+\frac{\gamma-1}{2}Ma_4^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1+\frac{\gamma-1}{2}\underbrace{Ma_5^2}_{SUP}\right)^{\frac{\gamma}{\gamma-1}}}P_4 = 0.022\,P_4 = 0.1 \text{ bar}}$$

SINCE $P_{0E} \simeq P_a$, THE CD NOZZLE IS OPTIMALLY EXPANDED,

WITH $Ma_5 = \boxed{3.17}$

$$P_5 = P_{0E} \doteq P_a = \boxed{0.1 \text{ bar}}$$

$$T_5 = \frac{T_{04}}{1 + \frac{\gamma-1}{2} Ma_5^2} = \boxed{910.7 \text{ K}}$$

$$U_5 = Ma_5 \sqrt{\gamma R_g T_5} = 1.91 \text{ km/s}$$

d) MASS FLOW RATE :

$$\dot{m} = \dot{m}^* = P_{04} \sqrt{\frac{\gamma}{R_g T_{04}}} A^* \left(\frac{1+\gamma}{2}\right)^{\frac{-\gamma+1}{2(\gamma-1)}}$$

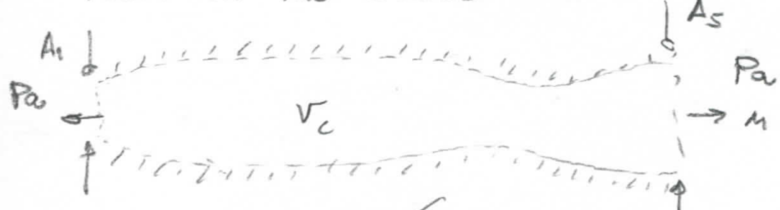
WITH $P_{04} = P_4 \left(1 + \frac{\gamma-1}{2} Ma_4^2\right)^{\frac{\gamma}{\gamma-1}} = 4.72$ bar

$$A^* = \frac{A^*}{A_3} \cdot \frac{A_3}{A_2} A_1 = 0.5 \cdot 2 A_1 = A_1 = 20 \text{ cm}^2$$

NOTE ($A^* = A_4$)

SUBSTITUTING ABOVE: $\dot{m} = \underline{0.73}$ kg/s

FORCE ON THE ENGINE



$$A_5 = \frac{A_5}{A_4} \cdot \frac{A_4}{A_3} \frac{A_3}{A_1} A_1 = 5 \cdot 0.5 \cdot 2 A_1 = 5 A_1$$

$$\cancel{\frac{d}{dt}\int_{V_c} \rho \vec{w}\, dv} + \int_{S_c} \rho \vec{U} \vec{U} \cdot d\vec{s} = -\int_{S_c} P \cdot d\vec{s}$$

$$-\rho_1 U_1^2 A_1 \vec{e}_x + \rho_5 U_5^2 A_5 \vec{e}_x = -\int_{S_{LATERAL}} P\, d\vec{s} = -\vec{F}$$

$$\vec{F} = -\dot{m}(U_5 - U_1)\vec{e}_x = \underline{65.7 \text{ N}} \rightarrow \text{IN THE } +x \text{ DIRECTION!}$$

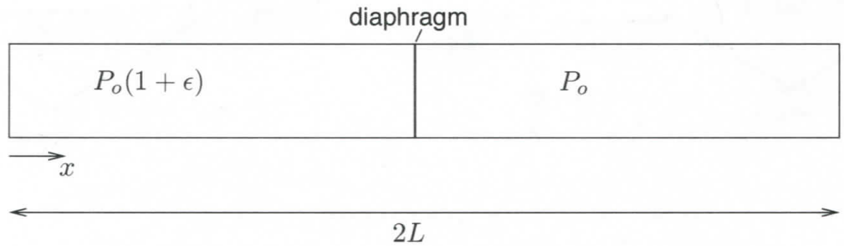
THE NORMAL SHOCK AT THE ENTRANCE PRODUCES MORE DRAG THAN THE THRUST THE ENGINE CAN PRODUCE

$$I_s = \frac{F_s}{\dot{m} g} = 8.59 \text{ s}$$

USELESS NUMBER SINCE THERE IS NO THRUST FORWARD!

Problem 2 (30 pts)

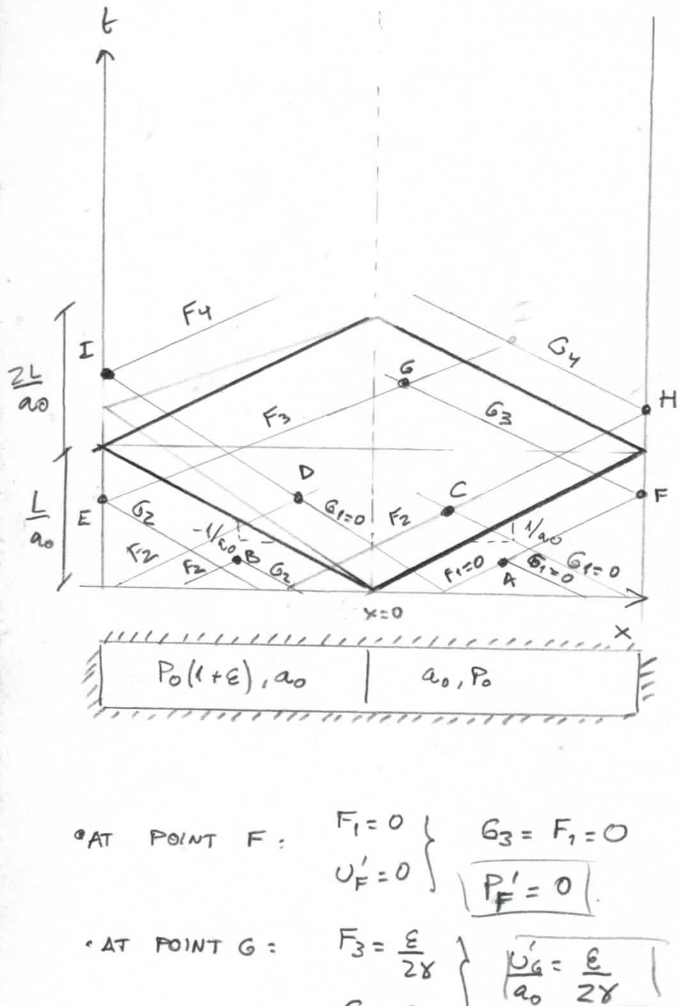
A closed duct of length $2L$ is initially divided into two equal cavities filled with the same gas at the same temperature T_o and separated by a diaphragm. Initially, the pressure of the gas in the right cavity is P_o, while the pressure of the gas in the left cavity is $P_o(1+\epsilon)$, with $\epsilon \ll 1$. At $t=0$ the diaphragm is removed and acoustic waves propagate into both cavities. Draw the velocity and pressure profiles in the duct as a function of x for a) $t = L/(2a_0)$, b) $t = L/a_0$, and c) $t = 2L/a_0$, where a_0 is the speed of sound. Justify your sketches by appropriately computing the flow variables using the acoustics theory.



SINCE $\varepsilon \ll 1 \Rightarrow$ USE LINEAR THEORY. :

$$F = \frac{1}{2}\left(\frac{u'}{a_0} + \frac{p'}{\rho_0 a_0^2}\right) \qquad \frac{p'}{\rho_0 a_0^2} = F + G$$

$$G = \frac{1}{2}\left(\frac{p'}{\rho_0 a_0^2} - \frac{u'}{a_0}\right) \qquad \frac{u'}{a_0} = F - G$$



• AT POINT A: $F_1 = G_1 = 0 \Rightarrow u'_A = p'_A = 0$

• AT POINT B: $F_2 = \frac{1}{2}\frac{\varepsilon P_o}{\rho_0 a_0^2} = \frac{\varepsilon}{2\gamma}$, $G_2 = \frac{1}{2}\frac{\varepsilon P_o}{\rho_0 a_0^2} = \frac{\varepsilon}{2\gamma}$ $\Rightarrow$ $\boxed{u'_B = 0}$, $\boxed{p'_B = \varepsilon P_o}$

THE GAS BELOW THE TWO CHARACTERISTICS EMERGING FROM $x=0, t=0$ HAS NOT NOTICED THE RUPTURE OF THE DIAPHRAGM

• AT POINT C: $F_2 = \frac{\varepsilon}{2\gamma}$, $G_1 = 0$ $\Rightarrow$ $\boxed{\frac{u'_c}{a_0} = F_2 - G_1 = \frac{\varepsilon}{2\gamma}}$, $\frac{p'_c}{\rho_0 a_0^2} = F_2 + G_1 = \frac{\varepsilon}{2\gamma} \Rightarrow \boxed{p'_c = \frac{\varepsilon P_o}{2}}$

• AT POINT D: SAME AS C.

• AT POINT E: $G_2 = \frac{\varepsilon}{2\gamma}$, $\boxed{u'_E = 0}$ $\Rightarrow$ $F_3 = G_2 = \frac{\varepsilon}{2\gamma}$, $\frac{p'_E}{\rho_0 a_0^2} = F_3 + G_3 = \frac{\varepsilon}{\gamma} \Rightarrow \boxed{p'_E = \varepsilon P_o}$

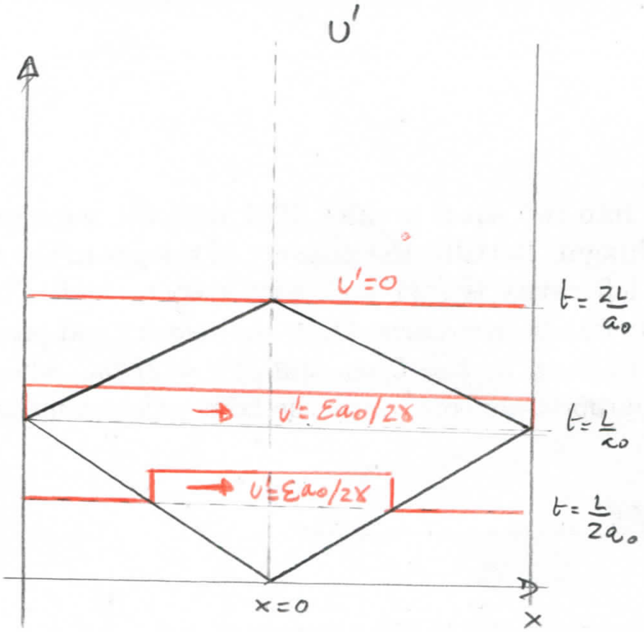
• AT POINT F: $F_1 = 0$, $u'_F = 0$ $\Rightarrow$ $G_3 = F_1 = 0$, $\boxed{p'_F = 0}$

• AT POINT H: $F_2 = \frac{\varepsilon}{2\gamma}$, $\boxed{u'_H = 0}$ $\Rightarrow$ $G_4 = F_2 = \varepsilon/2\gamma$, $\frac{p'_H}{\rho_0 a_0^2} = F_2 + G_4 = \frac{\varepsilon}{\gamma} \rightarrow \boxed{p'_H = \varepsilon P_o}$

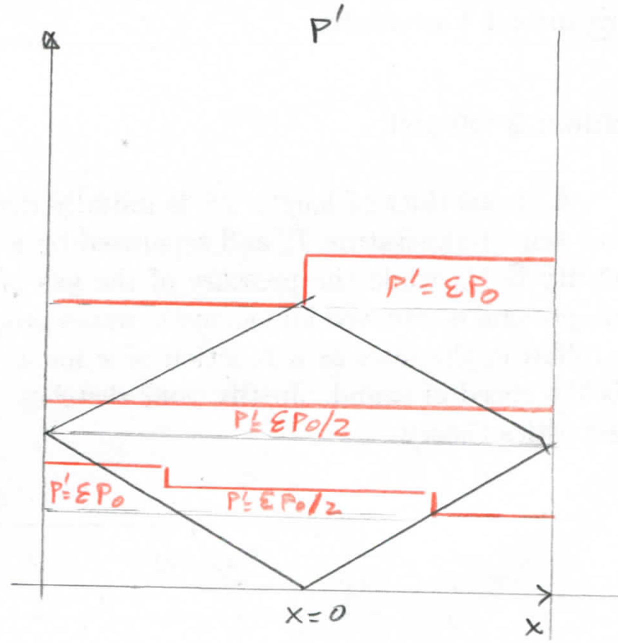
• AT POINT G: $F_3 = \frac{\varepsilon}{2\gamma}$, $G_3 = 0$ $\Rightarrow$ $\boxed{\frac{u'_G}{a_0} = \frac{\varepsilon}{2\gamma}}$, $\frac{p'_G}{\rho_0 a_0^2} = \frac{\varepsilon}{2\gamma} \rightarrow \boxed{p'_G = \frac{\varepsilon P_o}{2}}$

• AT POINT I: $G_1 = 0$, $\boxed{u'_I = 0}$ $\Rightarrow$ $F_4 = G_1 = 0$, $\boxed{p'_I = 0}$

DRAWINGS $\longrightarrow$



U'
U' = 0
U' = εa0/2γ
U' = εa0/2γ
t = 2L/a0
t = L/a0
t = L/2a0
x = 0
x



P'
P' = εP0
P' = εP0/2
P' = εP0
P' = εP0/2
x = 0
x