MAE 180A: Spacecraft Guidance I, Summer 2009

Midterm Exam

Thursday, July 16.

Part I: Questions. (40pt)

Guidelines: Please turn in a *neat* and *clean* exam solution. Answers should be written in the blank spaces provided in these exam sheets. Vector quantities are denoted in **bold** letters in what follows. This part of the exam is **closed book, closed notes, no use of calculator allowed.**

Student's Name:.....JAVIER URZAY..................... **Student's ID:**...........................

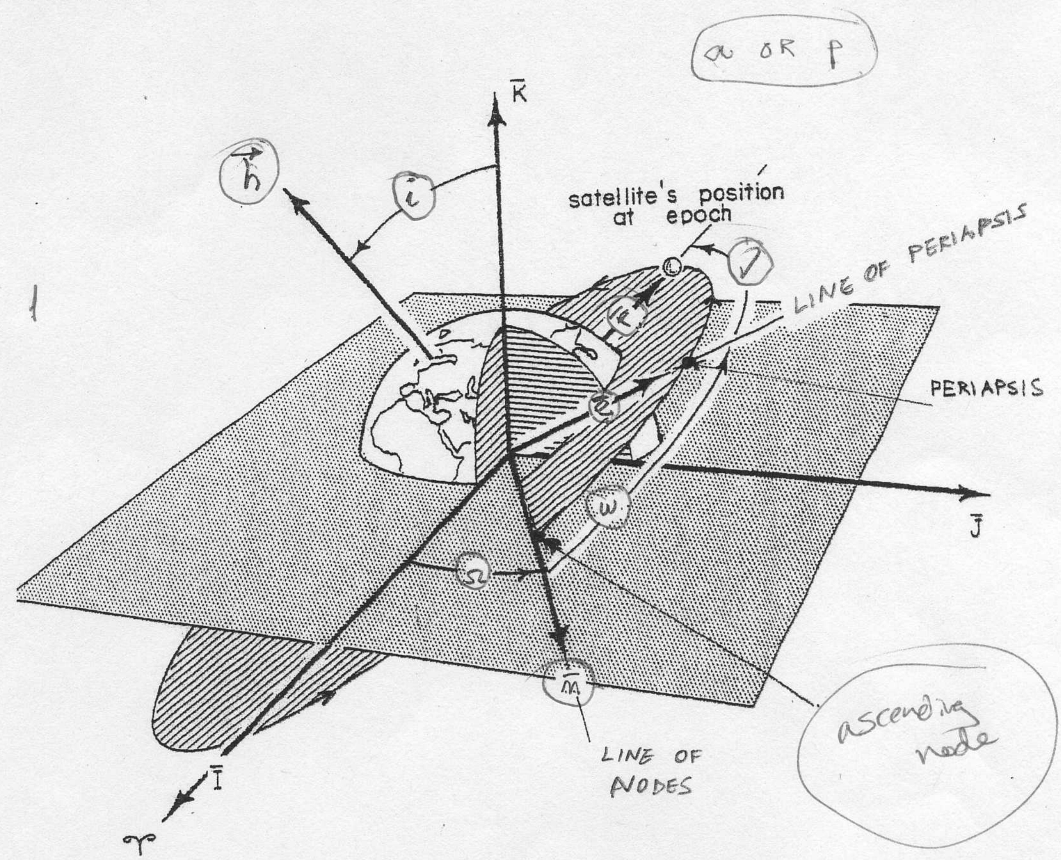
Question 1 (20 pts).

+5

a) Indicate on the following figure the six main orbital parameters and the fundamental vectors that define those parameters. Also indicate on the figure where is the ascending node and locate the line of nodes.

0 ≈ 0.5

1



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+10 b) Provide a separate list with the names of the orbital parameters and vectors that you indicated on part a) and the geometrical meaning for each of them.

+1 - SEMI-MAJOR AXIS "a": DEFINES THE SIZE OF THE CONIC ORBIT

+1 - ECCENTRICITY "e": DEFINES THE SHAPE OF THE CONIC ORBIT

+1 - INCLINATION "i": THE ANGLE BETWEEN THE NORTH-POLE AXIS AND THE ANGULAR MOMENTUM.

+1 - LONGITUDE OF THE ASCENDING NODE "Ω": THE ANGLE BETWEEN THE LINE OF NODES AND THE VERNAL EQUINOX DIRECTION.

+1 - ARGUMENT OF THE PERIAPSIS "ω": THE ANGLE BETWEEN THE LINE OF NODES AND THE PERIAPSIS DIRECTION.

+1 - TRUE ANOMALY AT EPOCH "ν": THE ANGLE BETWEEN THE PERIAPSIS DIRECTION AND THE POSITION VECTOR.

AND THE VECTORS THAT DEFINE THESE PARAMETERS ARE:

+1 - THE NODE VECTOR "$\bar{n}$": FROM THE CENTER OF THE EARTH TO THE ASCENDING NODE

+1 - THE POSITION VECTOR "$\bar{r}$": FROM THE CENTER OF THE EARTH TO THE SPACECRAFT.

+1 - THE ECCENTRICITY VECTOR "$\bar{e}$": FROM THE CENTER OF THE EARTH TO THE PERIAPSIS.

+1 - THE ANGULAR MOMENTUM VECTOR: FROM THE CENTER OF THE EARTH ON A PERPENDICULAR LINE TO THE ORBITAL PLANE.

+2 c) What is the name of the direction pointing on the Aries constellation (Υ) direction?

THE VERNAL-EQUINOX DIRECTION.

+2 d) What is the name of the system of coordinates shown in the picture? Is this coordinate system inertial or non-inertial?.

THE GEOCENTRIC-EQUATORIAL SYSTEM. (INERTIAL)

+1 e) What is the name of the fundamental (dotted) plane on this picture?

THE CELESTIAL EQUATOR.

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Question 2 (20 pts) 2 pts EACH

Select the true answer (only one) out of the choices from the list provided for each question. A complementary and *brief* mathematical proof of your answer on the available space would be welcome, but it is **not** needed in order to get full credit.

3.1 An equatorial direct orbit has an inclination i of

a) 45°

b) 90°

c) 0°

d) 180°

3.2 The heliocentric-ecliptic system of coordinates

a) is centered at the Earth and the x-axis points towards the Saggitarius constellation.

b) is centered at the Earth and the x-axis points towards the Aries constellation.

c) is perpendicular to the celestial equator.

d) is centered at the Sun.

3.3 The latitude of the Cape Canaveral launch site used by NASA is 28°, which represents the

a) geocentric latitude of the site.

b) geocentric longitude of the site.

c) geodetic latitude of the site.

d) topocentric longitude of the site.

3.4 A mean solar day is

a) longer than a sidereal day since the Earth is not a perfect sphere.

b) longer than a sidereal day because of the orbital path of the Earth around the Sun and the rotation of the Earth about its polar axis.

c) equal to a sidereal day.

d) shorter than a sidereal day.

3.5 The argument of the periapsis of a circular orbit is

a) always 0°.

b) 90°.

c) undefined.

d) −90°.

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3.6 The fundamental plane in the perifocal system of coordinates is

a) the ecliptic.

b) the galactic equator.

c) the orbit plane.

d) the celestial equator.

3.7 If the launch site is very close to the equator then

a) the spacecraft can be generally inserted into an equatorial orbit very easily.

b) it is impossible to insert a spacecraft into an equatorial orbit.

c) the orbit is always polar.

d) the orbit is always retrograde.

3.8 The specific mechanical energy of a satellite orbiting around the Earth

a) is not constant and varies sinusoidally with time.

b) is constant but only for US-manufactured satellites.

c) is constant if the satellite is not subject to any dissipative forces or external forces other than gravitational interactions.

d) is always constant.

3.9 The orbital period of a planet in a circular orbit of radius r_1 around the Sun is P_1. The orbital period of another planet in a circular heliocentric orbit of radius $r_1/3$ is

a) longer.

b) shorter.

c) equal.

d) undefined.

3.10 The angular momentum in an orbital motion is

a) constant and parallel to the orbit plane.

b) constant and normal to the orbit plane.

c) not constant, and it is parallel to the orbit plane.

d) not constant, and it is normal to the orbit plane.

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Part II: Problem. (60pt)

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Problem (60 pts)

The PAVE-PAWS (Phased Array Warning System) is a United States' Space Command radar system capable of detecting intercontinental ballistic missiles (ICBMs). An unidentified object is detected by this system orbiting at a position $\mathbf{r} = 1.5\mathbf{K}\ DU_\oplus$ with velocity $\mathbf{v} = 0.4\mathbf{I} - 0.2\mathbf{K}\ DU_\oplus/TU_\oplus$, both referred to the geocentric-equatorial system of coordinates. *Give all your results in both canonical units and dimensional units.*

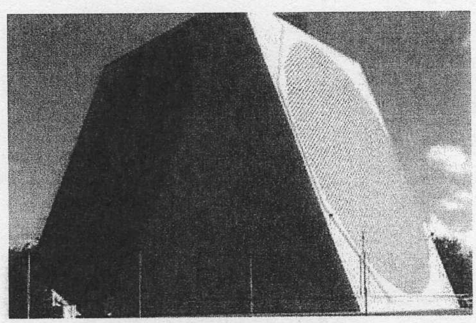

Figure 1: The PAVE-PAWS radar at Clear AFS (Alaska).

a) Calculate the specific mechanical energy of the object.

+6 $|\vec r| = 1.5\ DU_\oplus$, $|\vec v| = 0.45\ DU_\oplus/TU_\oplus$

THEN $\mathcal{E} = \frac{|\vec v|^2}{2} - \frac{\mu_\oplus}{|\vec r|} = \underline{-0.57}\ DU_\oplus^2/TU_\oplus^2 = \underline{-35.4}\ km^2/s^2$

b) Calculate the specific angular momentum of the orbit.

+6 $\vec h = \vec r \wedge \vec v = \begin{vmatrix} i & j & k \\ 0 & 0 & 1.5 \\ 0.4 & 0 & -0.2 \end{vmatrix} = \underline{0.6\vec J}\ DU_\oplus^2/TU_\oplus = \underline{30253.3}\ km^2/s$

t3 RIGHT FORM, WRONG RESULT IN a) – d)

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+1 RIGHT FORM, WRONG RESULT e) – g)

–1 FOR ADDITIONAL ORBITAL ELE IN h)

+6 c) Determine the eccentricity and characterize the shape of the orbit.

SINCE $p = \frac{|\bar{h}|^2}{\mu_\oplus} = 0.36 \, DU_\oplus$ AND $a = -\frac{\mu_\oplus}{2\mathcal{E}} = 0.88 \, DU_\oplus$

THEN $e = \left(1 - \frac{p}{a}\right)^{1/2} = \underline{0.77} \Rightarrow$ ELLIPTIC

+6 d) Calculate the perigee and apogee altitudes of the orbit. Is the object a ballistic missile?.

$r_p = \frac{p}{1+e} = 0.20 \, DU_\oplus \rightarrow H_p = r_p - r_\oplus = \underline{-0.8} \, DU_\oplus < 0,$

$r_a = \frac{p}{1-e} = 1.56 \, DU_\oplus \rightarrow H_a = r_a - r_\oplus = \underline{0.56} \, DU_\oplus = \underline{3571.7 km}$

SINCE $r_p < r_\oplus \rightarrow$ BALLISTIC MISSILE.

+6 e) Obtain the orbit inclination. Is it a retrograde or direct orbit?.

$\cos(i) = \frac{\bar{h} \cdot \bar{K}}{|\bar{h}|} = 0 \rightarrow \underline{i = 90^\circ}$, DIRECT ORBIT

f) Calculate the longitude of the ascending node, the argument of the periapsis, and the true anomaly at epoch.

+9

* NODE VECTOR $\vec{n} = \bar{K} \wedge \bar{h} = -0.6 \vec{I}$

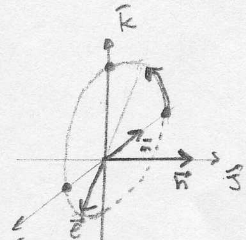
* ECCENTRICITY VECTOR $\vec{e} = \frac{1}{\mu_\oplus}\left[\left(|\bar{v}|^2 - \frac{\mu_\oplus}{|\bar{r}|}\right)\vec{r} - (\bar{r} \cdot \bar{v})\bar{v}\right] = 0.12\vec{I} - 0.76\vec{K}$

THEN $\cos(\Omega) = \frac{\bar{n} \cdot \bar{I}}{|\bar{n}|} = -1 \rightarrow \underline{\Omega = 180^\circ}$

$\cos(\omega) = \frac{\vec{n} \cdot \vec{e}}{|\bar{n}||\vec{e}|} = \frac{-0.072}{0.6 \cdot 0.77} = -0.16 \rightarrow \underline{\omega = 261^\circ} \; (e_K < 0)$

$\cos(\nu) = \frac{\vec{e} \cdot \vec{r}}{|\vec{e}||\bar{r}|} = \frac{-1.14}{0.77 \cdot 1.5} = -0.987 \rightarrow \underline{\nu = 189.2^\circ} \; (\bar{r} \cdot \bar{v} < 0)$

+6 g) Determine the argument of latitude at epoch, the true longitude at epoch and the longitude of the periapsis.



$U_0 = \omega + \nu = \underline{450.2^\circ} = \underline{90.2^\circ}$

$l_0 = \Omega + U_0 = \underline{270.2^\circ}$

$\Pi = \Omega + \omega = \underline{441^\circ} = \underline{81^\circ}$

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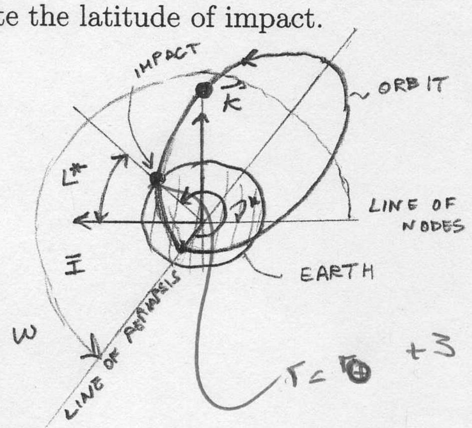
+6

h) What minimum set of parameters, of those calculated above, would you choose to determine the orbit orientation, orbit geometry and object location?.

$p, e, \Omega, \omega, i, \nu$

i) Calculate the latitude of impact.

+9



+3

L^* = LATITUDE OF IMPACT

ν^* = TRUE ANOMALY OF IMPACT

FROM THE TRAJECTORY EQUATION:

$$r_\oplus = \frac{p}{1 + e\cos\nu^*} \Rightarrow 1 = \frac{0.36}{1 + 0.77\cos\nu^*}$$

+3

$$\cos\nu^* = -0.83 \rightarrow \nu^* = 213.8^\circ$$

(NOTE: $\bar{r}\cdot\bar{v} < 0$ UPON IMPACT

+3

THEN $L^* = 360^\circ - \nu^* - (\omega - 180^\circ) = \underline{65.2^\circ}$