MAE 180A: Spacecraft Guidance I, Summer 2009

Homework 4

Due ~~Tuesday~~ THURSDAY, July ~~28~~ 30, in class.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Show all work. Required plots should be generated using computer software such as Matlab or Excel. Answers should be written in the blank spaces provided in these homework sheets. Use the back of the page in case you need additional space (not recommended to use more space than provided), for which a clear indication should be written to warn the reader of the presence of text there. Vector quantities are denoted in **bold** letters in what follows.

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Question 1 (35 pts)

Select the true answer (only one) out of the choices from the list provided for each question. A complementary and *brief* mathematical proof of your answer on the available space would be welcome, but it is **not** needed in order to get full credit.

1.1 For a mission to Jupiter by using a Hohmann transfer,

a) the spacecraft has to be launched with a negative hyperbolic excess speed (in the opposite direction to the Earth's orbital motion).

b) the spacecraft has to be launched with a positive hyperbolic excess speed (in the same direction as the Earth's orbital motion).

c) the spacecraft has to be launched at the escape speed.

d) the launch azimuth angle must be 180°.

1.2 A sun-synchronous satellite

a) always rotates about the Sun with the same period of rotation of the Sun about its axis.

b) has most of the time the Sun in its field of view because of the regression of the line of nodes of its orbit.

c) never sees the Sun.

d) is synchronous with the Moon.

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1.3 The Molniya orbit is

a) a nearly-equatorial orbit intended for the International Space Station (ISS).

b) is a geosynchronous orbit.

c) has a period of 1 week.

d) a high-inclination, eccentric orbit of nearly 12 h period intended to give communication capabilities to high-latitude areas such as Russia.

1.4 The ground trace of a circular geostationary (geosynchronous-equatorial) orbit is

a) a stationary spot on the same place of the equator.

b) a distorted 8−shape trace that goes from the northern hemisphere to the southern hemisphere.

c) a square of side equal to the periapsis radius.

d) a 5−shaped trace.

1.5 The regression of the line of nodes and the rotation of the line of apsides are produced by

a) the eccentricity of the Earth and are more important in low-inclination, low-altitude orbits.

b) the eccentricity of the Earth and are more important in nearly-polar orbits.

c) the precession of the Earth's rotation axis and are more important in nearly-polar orbits.

d) the solar radiation.

1.6 In a deceleration gravity-assist maneuver around Neptune,

a) the relative velocity of the spacecraft with respect to Neptune increases.

b) the relative velocity of the spacecraft with respect to Neptune decreases.

c) the velocity of the spacecraft with respect to the Sun remains constant.

d) the velocity of the spacecraft with respect to the Sun decreases.

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Problem (65pts)

An european space probe is sent in a mission to Jupiter for research of the Jovian soil and atmosphere. The probe is composed of an Orbiter and a Jupiter ballistic Lander, and the mass-distribution specifications are

	Structure	Propellant (MMH / N_2O_4)
Orbiter	2717 kg	1875 kg
Lander	500 kg	0 kg

The specific impulse of the orbiter propulsion system is $I_s = 320$ s. Detailed design calculations have shown that an impulse $\Delta V_{CM} = 1$ km/s must be reserved for control maneuvers such as trajectory-control and attitude-control maneuvers during the interplanetary flight. The space probe is placed into the payload fairing of an Ariane-5 rocket before launch at the Korou base (latitude 5° N) in the French Guiana, and it is to be launched from this place with an azimuth angle of $\beta = 90°$ in a direct orbit. During the launch phase, the First Stage (i.e. the Solid Rocket Boosters) and the Second Stage (i.e. the Vulcain Engine) of the Ariane-5 are jettisoned, and the space probe, which is still attached to a restartable Upper Stage, is inserted into a low-Earth orbit of 300 nm altitude. The Upper Stage is responsible for changing planes to the ecliptic and giving the appropriate $C_3 = V_\infty^2$ to the space probe towards Jupiter; once these tasks are completed, the Upper Stage is jettisoned and the probe (lander + orbiter) continues alone on its way to Jupiter along the ecliptic.

Simplifying assumption: The Earth and Jupiter orbits are assumed to be circular and are both contained in the ecliptic plane, which is inclined 23.4° with respect to the Earth's equator.

Useful Data: See pages 360, 361 and 368 of textbook and its appendix A to obtain the necessary astrodynamic constants. Additionally 1 $\mathrm{AU}/\mathrm{TU}_\odot = 3.768\ \mathrm{DU}_\oplus/\mathrm{TU}_\oplus$, 1 $\mathrm{DU}_{♃}/\mathrm{TU}_{♃} = 1.41\ \mathrm{AU}/\mathrm{TU}_\odot$.

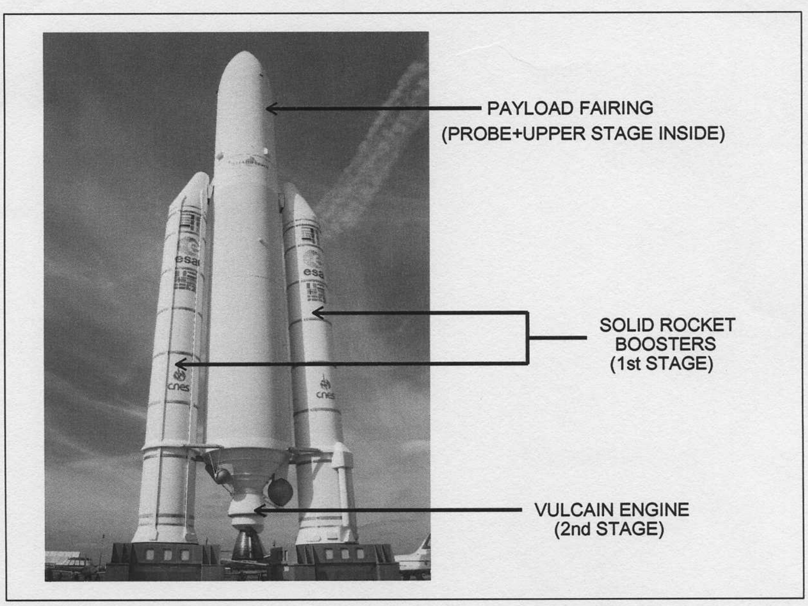

Figure 1: The Ariane-5 rocket in dual launch configuration.

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Part I: Departure and Interplanetary Flight (35 pt)

Consider a Hohmann trajectory from the Earth to Jupiter as depicted in figure 2.

$r_{ES} = 1$ AU

$r_{JS} = 5.2$ AU

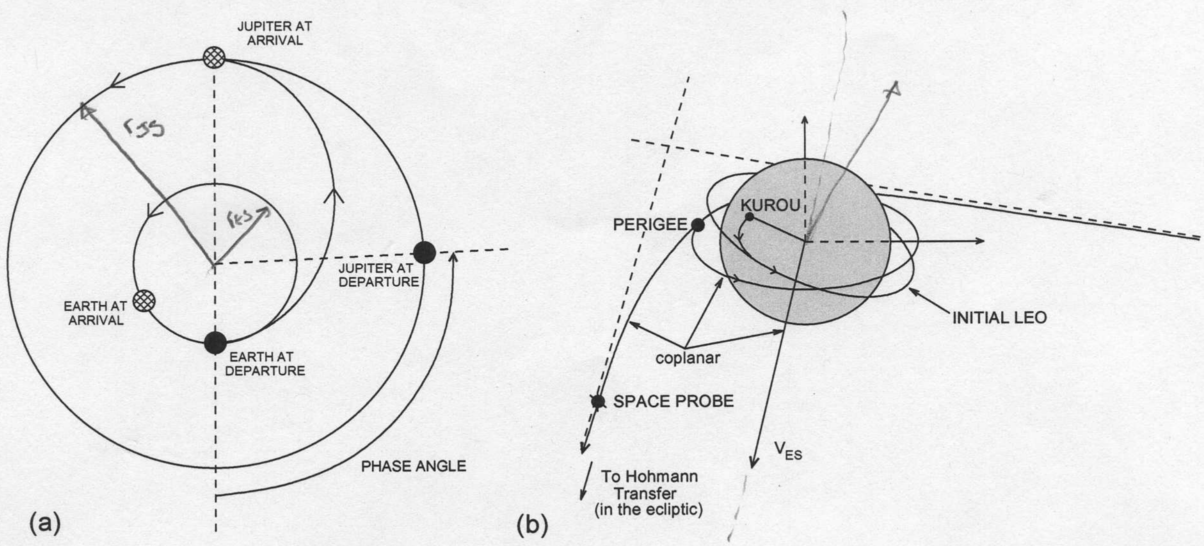

Figure 2: (a) Interplanetary Hohmann trajectory. (b) Departure phase at the Earth.

a) Calculate the circular velocity of the probe + Upper Stage in the circular parking LEO.

$$V_{CS} = \left(\frac{\mu_\oplus}{r_{CS}}\right)^{1/2} = \underline{\underline{0.96}}\ DU_\oplus/TU_\oplus \qquad \text{WITH} \quad r_{CS} = 300\text{ km} + r_\oplus = 1.087\ DU_\oplus$$

b) Calculate the ΔV_1 needed to change planes from the initial circular parking LEO to an identical circular orbit on the ecliptic plane.

$$\Delta V_1 = 2 V_{CS} \sin\left(\frac{\theta}{2}\right) = \underline{\underline{0.47}}\ \frac{DU_\oplus}{TU_\oplus} = \underline{\underline{3.72}}\ \frac{km}{s}; \quad \text{WITH} \quad \theta = \underbrace{5^\circ}_{\text{MIN INCLINATION FOR } \beta = 90^\circ \text{ WITH RESPECT TO EQUATOR}} + \overbrace{23.4^\circ}^{\text{INCLINATION ECLIPTIC-EQUATOR}} = 28.4^\circ$$

c) Calculate the orbital velocity of the Earth around the Sun.

$$V_{ES} = \left(\frac{\mu_\odot}{r_{ES}}\right)^{1/2} = \underline{\underline{1}}\ AU/TU_\odot = \underline{\underline{28.78}}\ km/s = \underline{\underline{3.768}}\ DU_\oplus/TU_\oplus$$

d) Determine the velocity of the probe at the perihelion of the transfer ellipse.

$r_{JS} = 5.2$ AU

$$V_{1t} = \left[2\mu_{\odot} \left(\frac{1}{r_{ES}} - \frac{1}{r_{ES} + r_{JS}} \right) \right]^{1/2} = \underline{\underline{1.295}} \ \frac{AU}{TU_{\odot}} = 4.88 \ \frac{DU_{\oplus}}{TU_{\oplus}}$$

e) Obtain the burnout velocity, the C_3 of the Upper Stage, and the ΔV_2 necessary to attain the injection velocity calculated in part d) for the interplanetary trajectory..

THE C_3 OF THE U.S. IS $C_3 = (V_{1t} - V_{ES})^2 = V_\infty^2 = 1.23 \ \frac{DU_{\oplus}^2}{TU_{\oplus}^2} \approx \underline{\underline{77 \ km^2/s^2}}$

WHERE $V_\infty = V_{1t} - V_{ES} = 1.11 \ DU_{\oplus}/TU_{\oplus} = 0.29 \ AU/TU_{\odot}$

THE BURNOUT VELOCITY IS $V_{bo} = \left[V_\infty^2 + \frac{2\mu_{\oplus}}{r_{LEO}} \right]^{1/2} = \underline{\underline{1.75}} \ DU_{\oplus}/TU_{\oplus} = 0.46 \ AU/TU_{\odot}$

SO THAT $\Delta V_2 = V_{bo} - V_{cs} = \underline{\underline{0.79}} \ \frac{DU_{\oplus}}{TU_{\oplus}} = 0.21 \ \frac{AU}{TU_{\odot}} = 6.24 \ km/s$

f) Determine the velocity of the probe at the aphelion of the Hohmann transfer ellipse. .

$$V_{2t} = \left[2\mu_{\odot} \left(\frac{1}{r_{JS}} - \frac{1}{r_{ES} + r_{JS}} \right) \right]^{1/2} = \underline{\underline{0.25}} \ AU/TU_{\odot}$$

g) Calculate the time of flight transfer in days.

$$TOF = \pi \left[\frac{(r_{ES} + r_{JS})^3}{8\mu_{\odot}} \right]^{1/2} = 17.15 \ TU_{\odot} = \underline{\underline{997}} \ \text{DAYS}$$

h) Obtain the needed phase angle at departure between the Earth and Jupiter for the Hohmann transfer to succeed. If this launch opportunity is missed, how long does it take to recover the same relative position between both planets?

$$\gamma_1 = \nu_{\substack{JUPITER\\ARRIVAL}} - \omega_{JUPITER/SUN} \cdot TOF = \pi - 0.530 \ \frac{rad}{yr} \cdot \frac{1 \ yr}{365 \ days} \cdot \frac{58.1 \ days}{1 \ TU_{\odot}} \cdot TOF =$$

$$= 1.69 \ rad = \underline{\underline{97.1^\circ}}$$

IT TAKES 1 SYNODIC PERIOD TO RECOVER THE SAME POSITION:

$$\tau_s = \frac{2\pi}{\omega_{ES} - \omega_{JS}} = \frac{2\pi}{6.28 - 0.53} = \underline{\underline{1.09}} \ \text{YEARS}$$

Part II: Arrival and Jovian Landing Mission (25 pt)

After performing all the control maneuvers ΔV_{CM}, the space probe arrives at Jupiter with a hyperbolic trajectory as the one depicted in figure 3. The sequence of the Lander jettisoning and Orbiter insertion into a parking orbit is as follows:

- Phase 1) The Jupiter Lander is ejected from the Orbiter at some distance from Jupiter while the probe is still cruising at the hyperbolic excess speed at arrival with a minimum approach distance equal to $2b/3$, where b is the effective collision distance.
- Phase 2) Upon ejection, the Lander follows a ballistic trajectory towards Jupiter along the original hyperbolic trajectory.
- Phase 3) A simultaneous Orbit Deflection Maneuver (ODM) is performed by the Orbiter for avoiding ballistic collision with Jupiter, by which a tangential burn of magnitude ΔV_{ODM} is produced to increase the altitude of the perijove 100000 km above Jupiter's surface.
- Phase 4) The rockets of the Orbiter are activated at the perijove to transfer from the hyperbolic trajectory to a circular parking orbit of the same radius.
- Phase 5) The Jupiter Lander impacts Jupiter's surface.

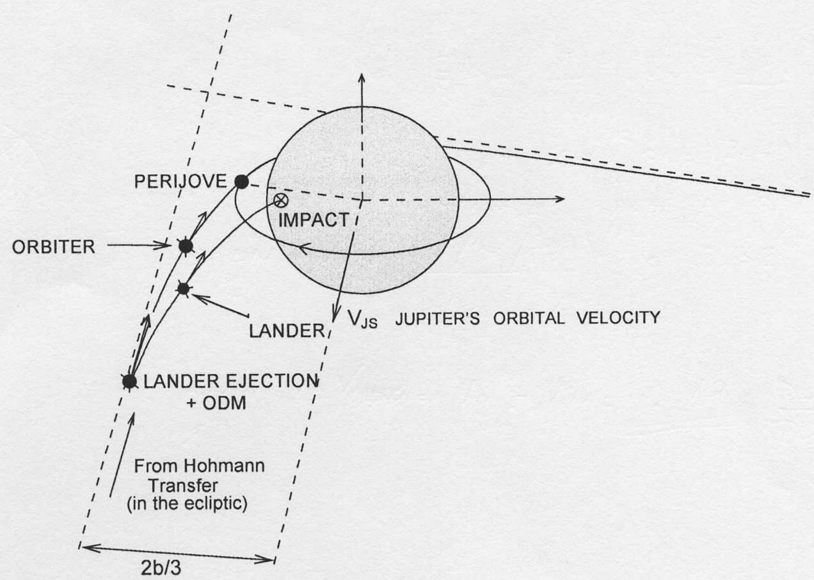

Figure 3: Arrival phase at Jupiter.

Answer the following questions:

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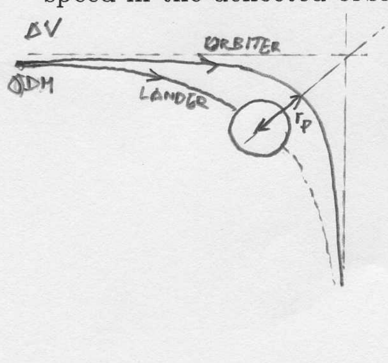
a) Calculate the orbital velocity of Jupiter around the Sun and the hyperbolic excess velocity of the probe before the ODM at arrival.

$$V_{JS} = \left(\frac{\mu_\odot}{r_{JS}}\right)^{1/2} = 0.44 \ AU/TU_\odot \Rightarrow V_\infty = V_{JS} - V_{2b} = \underline{\underline{0.19 \ AU/TU_\odot}} = \underline{\underline{0.13 \ DU_♃/TU_♃}}$$

b) Calculate the effective collision section b.

$$b = \frac{r_♃}{V_\infty}\left(V_\infty^2 + \frac{2\mu_♃}{r_♃}\right)^{1/2} = \underline{\underline{10.9}} \ \frac{DU_♃}{TU_♃}$$

c) Calculate the impulse ΔV_{ODM} in the rocket to perform the above specified Orbital Deflection Maneuver and increase the perijove altitude to 100000 km. (Hint: the minumum approach distance for both deflected and initial orbits must be the same, so that from there you can calculate the hyperbolic speed in the deflected orbit)



$$\frac{2}{3}b = \frac{r_P}{V_\infty'}\left(V_\infty'^2 + \frac{2\mu_♃}{r_♃}\right)^{1/2},$$

WITH $r_P = r_♃ + 100 \cdot 10^3 \ km = 2.40 \ DU_♃$

AND V_∞' HYPERBOLIC SPEED AFER IMPULSE

$$\text{THEN} \quad V_\infty' = \left(\frac{2\mu_♃/r_P}{\frac{(2/3 b)^2}{r_P^2} - 1}\right)^{1/2} = 0.32 \ DU_♃/TU_♃$$

$$\text{AND} \quad \Delta V_{ODM} = V_\infty' - V_\infty = \underline{\underline{0.19}} \ \frac{DU_♃}{TU_♃} = \underline{\underline{8.01}} \ km/s$$

d) Calculate the impulse ΔV_3 necessary to transfer to the above specified circular parking orbit of 500000 km altitude.

AT THE PERIJOVE OF THE HYPERBOLA:

$$V_P = \left(V_\infty'^2 + 2\frac{\mu_♃}{r_P}\right)^{1/2} = 0.74 \ DU_♃/TU_♃$$

FOR THE CIRCULAR PARKING ORBIT: $V_{CS} = \left(\frac{\mu_♃}{r_P}\right)^{1/2} = 0.51 \ DU_♃/TU_♃$

$$\text{THEN} \quad \Delta V_3 = V_P - V_{CS} = \underline{\underline{0.23}} \ \frac{DU_♃}{TU_♃} = \underline{\underline{0.32}} \ \frac{AU}{TU_\odot} = \underline{\underline{9.53}} \ km/s$$

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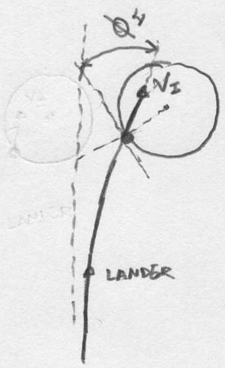
e) Calculate the velocity and flight-path angle (i.e. the reentry angle) of the Jupiter Lander. For this part assume an infinitely thin Jovian atmosphere. Give a couple of possible causes of why the Jupiter Lander may not be able to fulfill its mission goals of safe landing and research of the Jovian soil.



AT IMPACT: $r = r_{\text{♃}} = \dfrac{|a_\ell|(e_\ell^2-1)}{1+e_\ell \cdot \cos\nu_I}$

$\cos\nu_I = -0.167 \rightarrow \nu_I = 99.6^\circ$

SO THAT $\cos\phi_I = \dfrac{1+e_\ell\cos\nu_I}{(1+2e_\ell\cos\nu_I+e^2)^{1/2}} = 0.64$

THEN $\underline{\underline{\phi_I \cong -51^\circ}}$

AND $V_I\, r_{\text{♃}} \cos\phi_I = V_\infty \dfrac{2b}{3} \rightarrow V_I = \underline{\underline{1.47}}\ \dfrac{DU_{\text{♃}}}{TU_{\text{♃}}} = \underline{\underline{61.9\ km/s}}$

Part III: Space Probe Design Conclusions (5 pt)

Obtain the total impulse $\Delta V_T = \Delta V_1 + \Delta V_2 + \Delta V_3 + \cancel{\Delta V_4} + \Delta V_{CM} + \Delta V_{ODM}$, and the mass of propellant needed to produce such impulse. Is there enough propellant on board the probe to perform the mission?.

TOTAL IMPULSE: $\Delta V_T = 3.72 + 6.24 + 9.53 + 8.01 + 1 = \underline{\underline{28.4}}\ km/s$

TOTAL INITIAL MASS: $M_t = 2717 + 1875 + 1500 = 5092\ kg$

1) THE PROPELLANT CONSUMPTION BEFORE ODM IS ONLY PRODUCED BY ΔV_{CM} (ΔV_1 AND ΔV_2 ARE PERFORMED BY THE UPPER STAGE)

SO THAT $M_p = M_i \left(1 - e^{-\frac{\Delta V_{CM}}{I_g g}}\right) = 0.27 \cdot 5092 = 1390\ kg$

2) THE PROPELLANT CONSUMPTION AFTER ODM IS PRODUCED BY ΔV_3 AND ΔV_{ODM} WITH AN INITIAL MASS $M_t = 2713 + (1875 - 1390) = 3198\ kg$

$\Rightarrow M_p = M_t \left(1 - e^{-\frac{(\Delta V_3 + \Delta V_{ODM})}{I_g g}}\right) = 0.996 \cdot 3198\ kg = 3185\ kg$

SO THAT THE PROBE NEEDS $3185 + 1390 = \underline{\underline{4575.2\ kg}}$ OF PROPELLANT AT LAUNCH → THERE IS NOT ENOUGH PROPELLANT!