MAE 180A: Spacecraft Guidance I, Summer 2009

Homework 3

Due Tuesday, July 21, in class.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Show all work. Required plots should be generated using computer software such as Matlab or Excel. Answers should be written in the blank spaces provided in these homework sheets. Use the back of the page in case you need additional space (not recommended to use more space than provided), for which a clear indication should be written to warn the reader of the presence of text there. Vector quantities are denoted in **bold** letters in what follows.

Student's Name: JAVIER URZAY .... **Student's ID:**............................

Question 1 (30 pts)

Select the true answer (only one) out of the choices from the list provided for each question. A complementary and *brief* mathematical proof of your answer on the available space would be welcome, but it is **not** needed in order to get full credit.

3.1 The minimum-energy transfer trajectory of two impulses between two circular coplanar orbits of radius r_1 and r_2 is

a) a circular orbit of radius $2r_1$.

b) a circular orbit of radius $r_2/2$.

c) an elliptic trajectory called the Hohmann transfer, which is tangent to the inner orbit at perigee and tangent to the outer orbit at apogee.

d) an elliptic trajectory of perigee radius $r_p = 2r_1$ and apogee radius $r_a = 3r_2$.

3.2 An elliptic orbit of perigee radius $r_p = 3$ DU and apogee radius $r_a = 6$ DU is used to transfer a spacecraft between two circular coplanar orbits of radius $r_1 = 3$ DU and $r_2 = 8$ DU. Then

a) the spacecraft will arrive safely at the outer circular orbit.

b) it would take too much fuel to perform this maneuver.

c) this transfer orbit cannot be used since the spacecraft will not arrive at the outer circular orbit.

d) the transfer orbit is parallel to the ecliptic plane. $r_a < r_2$

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3.3 A satellite is launched from the Baikonur Cosmodrome to a 45.8°-inclination low-Earth orbit (LEO), and it is about to be transferred to an equatorial orbit of larger radius. Then

a) it is more economical to perform the plane change to the equatorial plane when the satellite is still in the LEO before executing any coplanar maneuver.

~~b)~~ it is more economical to perform the plane change to the equatorial orbit at the intersection of the equatorial plane and the transfer ellipse (i.e. at the descending node of the transfer ellipse).

$\Delta v \propto V$

c) the launch azimuth was 0°.

d) the perigee radius is equal to the radius of the Earth.

3.4 For a given Δv and initial mass of the spacecraft, the larger the specific impulse of the propulsion system,

a) the larger the propellant consumption.

$$M_p = M_0 \left(1 - e^{-\frac{\Delta v}{I_s g}}\right)$$

b) the larger the eccentricity of the orbit attained.

~~c)~~ the smaller the propellant consumption.

d) the smaller the eccentricity of the orbit attained.

3.5 For two identical elliptic orbits, the time of flight from periapsis to a true anomaly $\nu = 150°$ is smaller

~~a)~~ for the orbit around the central body with the largest mass.

b) for the orbit around the central body with the smallest mass.

$$t - T \propto \left(\frac{a^3}{\mu}\right)^{3/2}$$

c) for the orbit with the largest semi-major axis.

BUT $a_1 = a_2$

d) for the orbit with the smallest semi-major axis.

3.6 In a Hohmann transfer from an inner circular orbit to an outer circular orbit

a) the spacecraft first undergoes an acceleration to become injected in the transfer ellipse, and then a deceleration for insertion into the outer circular orbit.

b) the spacecraft first undergoes an deceleration to become injected in the transfer ellipse, and then another deceleration for insertion into the outer circular orbit.

~~c)~~ the spacecraft first undergoes an acceleration to become injected in the transfer ellipse, and then another acceleration for insertion into the outer circular orbit.

d) the spacecraft first undergoes an deceleration to become injected in the transfer ellipse, and then an acceleration for insertion into the outer circular orbit.

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Problem (70 pts)

An Earth satellite is to be transferred from a 300-km-altitude circular parking orbit to a 35,786-km-altitude circular orbit (also called synchronous orbit). The satellite has a total initial mass of 5700 kg, and it is equipped with a small engine that uses a hypergolic mixture of monomethylhydrazine / nitrogen tetroxide as propellant to perform orbital maneuvers, for which the specific impulse is 320 s. Both orbits are coplanar. For this purpose, three different mission designs are proposed:

Part I (20 pt)

The first design consists on a Hohmann transfer from the inner orbit to the outer orbit, as depicted in figure 1.

$r_1 = r_\oplus + 300\ km = 1.047\ DU_\oplus$

$r_2 = r_\oplus + 35786\ km = 6.611\ DU_\oplus$

$r_1 = r_p$

$r_2 = r_a$

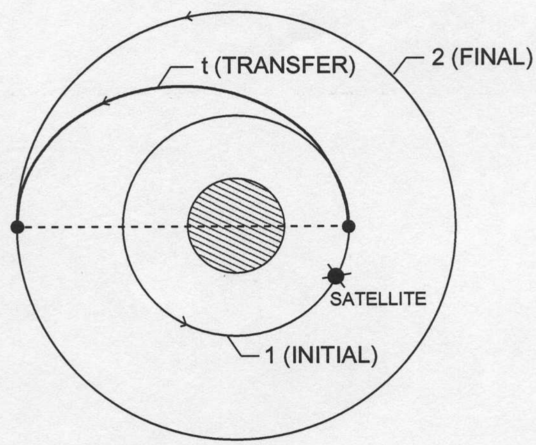

Figure 1:

a) Calculate the orbital velocities on each of the circular orbits.

$$V_1 = \left(\frac{\mu_\oplus}{r_1}\right)^{1/2} = \underline{\underline{0.98}}\ DU_\oplus/TU_\oplus$$

$$V_2 = \left(\frac{\mu_\oplus}{r_2}\right)^{1/2} = \underline{\underline{0.39}}\ DU_\oplus/TU_\oplus$$

b) Calculate the velocity at the perigee of the Hohmann ellipse.

$$V_{1t} = \left[2\mu_\oplus\left(\frac{1}{r_1} - \frac{1}{r_1+r_2}\right)\right]^{1/2} = \underline{\underline{1.28}}\ DU_\oplus/TU_\oplus$$

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c) Calculate the velocity at the apogee of the Hohmann ellipse.

$$V_{2t} = \left[2\mu_{\oplus} \left(\frac{1}{r_2} - \frac{1}{r_1 + r_2} \right) \right]^{1/2} = \underline{\underline{0.70}} \; DU_{\oplus}/TU_{\oplus}$$

d) Obtain the total Δv necessary for the transfer.

$$\Delta v = (V_{1t} - V_1) + (V_2 - V_{2t}) = 0.3 + 0.19 = \underline{\underline{0.49}} \; DU_{\oplus}/TU_{\oplus} =$$

$$= 3.87 \; km/s$$

e) Calculate the time of flight during the transfer.

$$TOF = \frac{P}{2} = \pi \left[\frac{(r_1 + r_2)^3}{8\mu_{\oplus}} \right] = \underline{\underline{23.5}} \; TU_{\oplus}$$

f) Calculate the total mass of propellant spent in the transfer maneuver.

SINCE $\frac{\Delta v}{I_s \cdot g} = \frac{3.87 \cdot 10^3}{320 \cdot 9.8} = 1.23$

THEN $M_p = M_i \left(1 - e^{-\frac{\Delta v}{I_s g}} \right) = 0.71 \cdot 5700 = \underline{\underline{4040}} \; kg$

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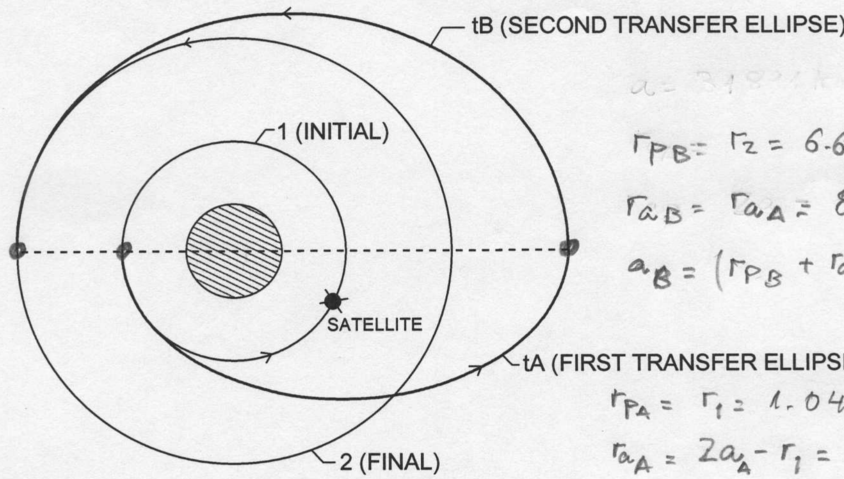
Part II (20 pt)
The second design consists on a bielliptical transfer as depicted in figure 2. The first impulse converts the initial orbit to the first transfer ellipse, which has a semi-major axis of 31,891 km. The second impulse establishes the second transfer ellipse.



$r_{PB} = r_2 = 6.611\ DU_\oplus$

$r_{aB} = r_{aA} = 8.95\ DU_\oplus$

$a_B = (r_{PB} + r_{aB})/2 = 7.78\ DU_\oplus$

$a_A = 31891\ km = 5.0\ DU$

$r_{PA} = r_1 = 1.047\ DU_\oplus$

$r_{aA} = 2a_A - r_1 = 8.95\ DU_\oplus$

Figure 2:

a) Calculate the velocity at the perigee of the first transfer ellipse.

$$V_{PA} = \left[2\mu_\oplus\left(\frac{1}{r_1} - \frac{1}{r_{aA}+r_1}\right)\right]^{1/2} = \underline{\underline{1.31}}\ DU_\oplus/TU_\oplus$$

b) Calculate the velocity at the apogee of the first transfer ellipse.

$$V_{aA} = \left[2\mu_\oplus\left(\frac{1}{r_{aA}} - \frac{1}{r_{aA}+r_1}\right)\right]^{1/2} = \underline{\underline{0.15}}\ DU_\oplus/TU_\oplus$$

c) Calculate the velocity at the apogee of the second transfer ellipse.

$$V_{aB} = \left[2\mu_\oplus\left(\frac{1}{r_{aB}} - \frac{1}{r_{PB}+r_{aB}}\right)\right]^{1/2} = \underline{\underline{0.31}}\ DU_\oplus/TU_\oplus$$

d) Calculate the velocity at the perigee of the second transfer ellipse.

$$V_{PB} = \left[2\mu_\oplus \left(\frac{1}{r_{PB}} - \frac{1}{r_{PB} + r_{aB}} \right) \right]^{1/2} = \underline{\underline{0.42}}\ DU_\oplus/TU_\oplus$$

e) Obtain the total Δv necessary for the transfer.

$$\Delta v = \left(V_{PA} - V_1 \right) + \left(V_{aB} - V_{aA} \right) + \left(V_{PB} - V_2 \right) =$$

$$= 0.33 + 0.16 + 0.03 = \underline{\underline{0.52}}\ DU_\oplus/TU_\oplus$$

$$= 4.11\ km/s$$

f) Calculate the total time of flight during the transfer.

$$TOF = \frac{T_A}{2} + \frac{T_B}{2} = \pi \left[\frac{(r_{PA} + r_{aA})^3}{8\mu_\oplus} \right]^{1/2} + \pi \left[\frac{(r_{PB} + r_{aB})^3}{8\mu_\oplus} \right]^{1/2} =$$

$$= 35.1 + 21.7 = \underline{\underline{56.8}}\ TU_\oplus$$

g) Calculate the total mass of propellant spent in the transfer maneuver.

SINCE $\frac{\Delta v}{I_s g} = 1.31$

THEN $M_P = M_i \left[1 - e^{-\frac{\Delta v}{I_s g}} \right] = 0.73 \cdot 5700 = \underline{\underline{4162\ kg}}$

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Part III (20 pt)

The third design consists on a first tangential burn that converts the initial orbit into a transfer ellipse of eccentricity $e = 0.88$, and a second burn that injects the satellite in the final orbit as depicted in figure 3.

$r_p = r_1 = 1.047\ DU_\oplus$

$r_a = \frac{(1+e)\, r_p}{(1-e)} = 16.40\, DU_\oplus$

$a = \frac{r_p + r_a}{2} = 8.72\ DU_\oplus$

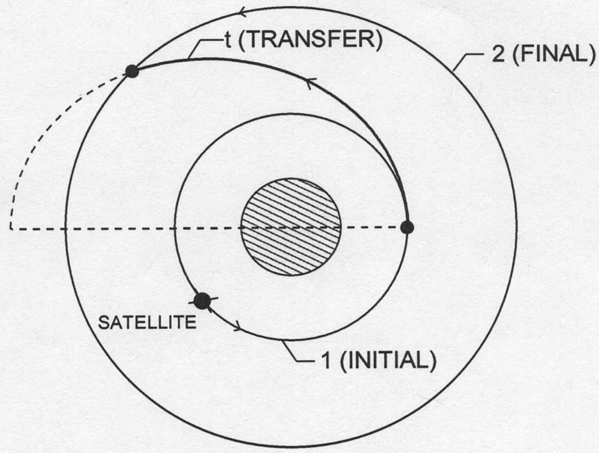

Figure 3:

a) Calculate the velocity at the perigee of the transfer ellipse.

$$V_{1t} = \left[2\mu_\oplus\left(\frac{1}{r_1} - \frac{1}{r_p + r_a}\right)\right]^{1/2} = 1.34\ DU_\oplus/TU_\oplus$$

b) Calculate the velocity at the outer-orbit insertion point in the transfer ellipse.

$$V_{2t} = \left[2\mu_\oplus\left(\frac{1}{r_2} - \frac{1}{r_p + r_a}\right)\right]^{1/2} = 0.43\ DU_\oplus/TU_\oplus$$

c) Calculate the flight path angle at the outer-orbit insertion point in the transfer ellipse.

SINCE $r_2 = \frac{a(1-e^2)}{1+e\cos\nu} \Rightarrow \cos\nu = \frac{1}{e}\left[\frac{a(1-e^2)}{r_2} - 1\right] = -0.79$

$\Rightarrow \nu = 142.9^\circ$

AND $\cos\phi_1 = \frac{1+e\cos\nu}{(1+2e\cos\nu+e^2)^{1/2}} = 0.48 \Rightarrow \phi = \underline{60.7^\circ}$

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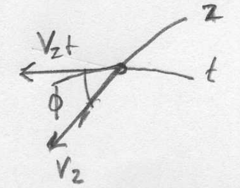
d) Obtain the total Δv necessary for the transfer.

FROM 1 → TRANSFER ELLIPSE: $\Delta V_1 = V_{1t} - V_1 = 0.36 \; DU_\oplus / TU_\oplus$

FROM TRANSFER ELLIPSE → 2: $\Delta V_2 = \left(V_2^2 + V_{2t}^2 - 2 V_2 V_{2t} \cos\phi \right)^{1/2} = 0.41 \; DU_\oplus/$



THEN $\Delta v = \Delta v_1 + \Delta v_2 = \underline{\underline{0.77}} \; DU_\oplus = 6.13 \; km/s$

f) Calculate the total time of flight during the transfer.

THE TRANSFER FLIGHT OCCURS FROM PERIAPSIS TO A TRUE ANOMALY $\vartheta = 142.9°$, FOR WHICH THE ECCENTRIC ANOMALY IS $\cos E = \dfrac{e + \cos\vartheta}{1 + e\cos\vartheta} = 0.27 \rightarrow E = 73.9 = 1.29 \; rad$

THEN $TOF = t - T = \left(\dfrac{a^3}{\mu_\oplus} \right)^{1/2} (E - e \sin E) = 25.7 \cdot 0.44 = \underline{\underline{11.3}} \; TU_\oplus$

g) Calculate the total mass of propellant spent in the transfer maneuver.

SINCE $\dfrac{\Delta v}{I_s g} = 1.95$

THEN $M_p = M_i \left(1 - e^{-\frac{\Delta v}{I_s g}} \right) = 0.86 \cdot 5700 = \underline{\underline{4893}} \; kg$

Part IV (10 pt)
Compare the three proposed mission designs in terms of energy budget, required complexity of the propulsion system, time of flight, and mass of propellant spent in the transfer.

(+) BETTER ⟶ WORSE (−)

IN TERMS OF ENERGY BUDGET	HOHMANN	BIELLIPTICAL	TRANS ELLIPSE
" OF PROPULSION COMPLEXITY	TRANS ELLIPSE	BIELLIPTICAL	HOHMANN
" OF TIME OF FLIGHT	BIELLIPTICAL	HOHMANN	TRANS. ELLIPSE
" OF MASS OF PROPELLANT	TRANS ELLIPSE	BIELLIPTIC	HOHMANN