MAE 180A: Spacecraft Guidance I, Summer 2009

Homework 2

Due Tuesday, July 14, in class.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Show all work. Required plots should be generated using computer software such as Matlab or Excel. Answers should be written in the blank spaces provided in these homework sheets. Use the back of the page in case you need additional space (not recommended to use more space than provided), for which a clear indication should be written to warn the reader of the presence of text there. Vector quantities are denoted in **bold** letters in what follows.

Student's Name: JAVIER URZAY .... **Student's ID:**........

Question 1 (30 pts). State in words the six fundamental orbital parameters and the geometrical meaning for each of them. For a circular orbit, provide a list with the parameters that are undefined and those that best describe the orbit geometry and spacecraft location.

1) SEMI-MAJOR AXIS "a": DEFINES THE SIZE OF THE CONIC ORBIT.

2) ECCENTRICITY "e": DEFINES THE SHAPE OF THE CONIC ORBIT

3) INCLINATION "i": THE ANGLE BETWEEN THE NORTH-POLE AXIS AND THE ANGULAR MOMENTUM

4) LONGITUDE OF THE ASCENDING NODE "Ω": THE ANGLE BETWEEN THE LINE OF NODES AND THE VERNAL EQUINOX DIRECTION

5) ARGUMENT OF THE PERIAPSIS "ω": THE ANGLE BETWEEN THE LINE OF NODES AND THE PERIAPSIS DIRECTION.

6) TRUE ANOMALY AT EPOCH "ν": THE ANGLE BETWEEN THE PERIAPSIS DIRECTION AND THE POSITION VECTOR.

FOR A CIRCULAR ORBIT, ω AND ν ARE UNDEFINED, SINCE THERE IS NO PERIAPSIS. TO LOCATE THE SPACECRAFT ALONG THE ORBIT, ONE CAN USE THE TRUE LONGITUDE AT EPOCH "l_0", WHICH IS THE SUM OF Ω AND THE ANGLE BETWEEN THE LINE OF NODES AND THE POSITION VECTOR.

$\Rightarrow$ FOR A CIRCULAR ORBIT: a, e, i, Ω, ω (UNDEFINED) AND l_0.

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Question 2 (~~10~~ 20 pts)

Select the true answer (only one) out of the choices from the list provided for each question. A complementary and *brief* mathematical proof of your answer on the available space would be welcome, but it is **not** needed in order to get full credit.

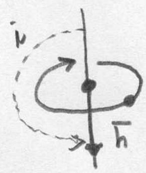
3.1 An equatorial retrograde orbit has an inclination i of

a) 45°

b) 90°

c) 0°

d) 180°



3.2 The topocentric-horizon system of coordinates

a) is an inertial system.

b) is a non-inertial system.

c) does not rotate with the Earth.

d) is irrelevant for spacecraft orbital calculations.

3.3 The latitude of the Cape Canaveral launch site used by NASA is 28°, which represents the

a) geocentric latitude of the site.

b) geocentric longitude of the site.

c) geodetic latitude of the site.

d) topocentric longitude of the site.

3.4 A sidereal day is

a) longer than a mean solar day since the Earth is not a perfect sphere.

b) shorter than a mean solar day.

c) equal to a mean solar day.

d) longer than a mean solar day because of the orbital path of the Earth around the Sun and the rotation of the Earth about its polar axis.

3.5 The longitude of the ascending node of an equatorial orbit is

a) always 0°.

b) 90°.

c) undefined.

d) −90°.

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3.6 The fundamental plane in the geocentric-equatorial system of coordinates is

a) the ecliptic.

b) the galactic equator.

c) the orbit plane.

d) the celestial equator.

3.7 A satellite launched from Cape Canaveral (latitude 28°30′ N, longitude 80°33′ W)

a) can be directly inserted into an equatorial orbit without any additional orbital maneuvers if the launch azimuth angle is conveniently chosen.

b) cannot be directly inserted into an equatorial orbit without additional orbital maneuvers, regardless of the launch azimuth angle.

c) is always painted in pink color to facilitate the visual tracking from the Earth's surface.

d) is always inserted into a $i = 0°$ inclination orbit because of local policies of the base.

3.8 Without any additional maneuvers, a space capsule launched by Eurpean Space Agency from the Baikonur Cosmodrome (latitude 45°54′ N, longitude 63°18′ E) in Kazakhstan can be directly inserted into an orbit of minimum inclination

a) $i = 0°$.

$\cos(i) = \cancel{\sin(\beta_0)}^{1} \cos(L_0) \rightarrow i = L_0$ MIN INCLINATION

b) $i = 63°18'$.

c) $i = 45°54'$.

d) $i = \infty$.

3.9 The vernal equinox direction

a) represents the intersection between the ecliptic plane and the Greenwich meridian.

b) always points towards the Sagittarius constellation.

c) is always normal to the ecliptic plane.

d) represents the intersection between the ecliptic plane and the celestial equator.

3.10 The precession motion of the Earth axis of rotation makes the vernal equinox direction to

a) rotate counter-clockwise if viewed from the north pole.

b) rotate clockwise if viewed from the north pole.

c) stay in the same position forever.

d) the precession motion of the Earth does not affect the orientation of the vernal equinox.

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Problem (50 pts)

A radar site at Cape Canaveral (latitude 28.5° N, longitude 80.5° W) detected a spacecraft passing directly overhead with the following data:

$\rho = 0.5\ \mathrm{DU}_\oplus$, $Az = 45°$, $EI = 90°$, for position relative to the radar site,

and

$\dot{\rho} = 0\ \mathrm{DU}_\oplus/\mathrm{TU}_\oplus$, $\dot{Az} = 0$, $\dot{EI} = 3\ \mathrm{rad}/\mathrm{TU}_\oplus$, for velocity relative to the radar site,

where ρ, Az and EI are the position, and the azimuth and elevation angles relative to the radar site (see figure). Assume that the longitude of the radar site with respect to the vernal equinox direction is $\theta = 45°$ at the time of observation.

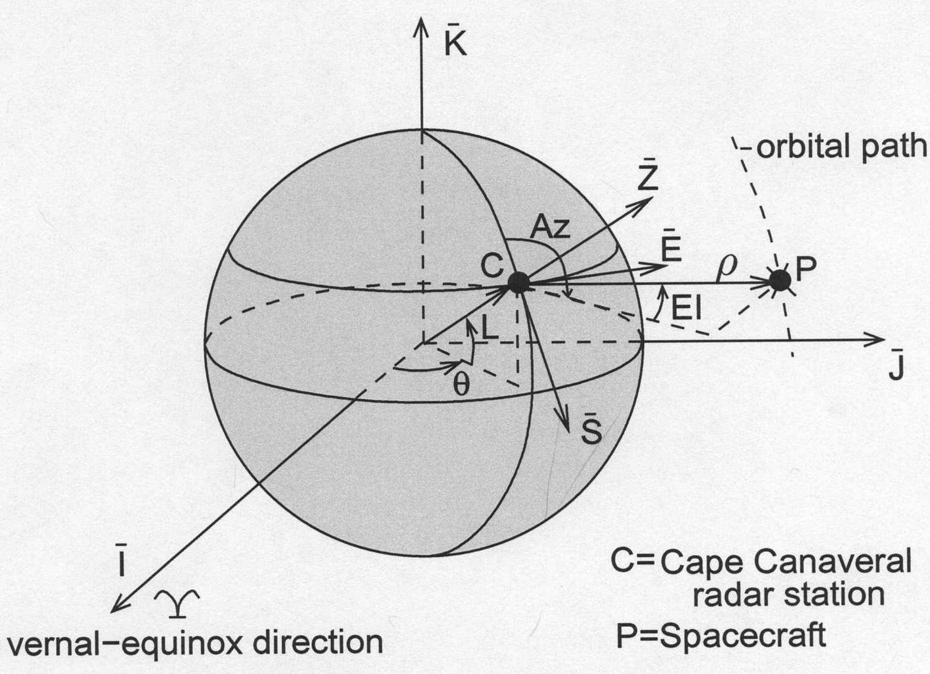

Figure 1: The figure shows the latitude of the radar site L, and its longitude θ *with respect to the vernal equinox direction.* The $\{S, E, Z\}$ system is the topocentric-horizon coordinate system, and the $\{I, J, K\}$ is the geocentric-equatorial coordinate system.

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Part I (25pt)

a) Determine the components of the spacecraft position vector $\boldsymbol{\rho}$ in the topocentric-horizon coordinate system.

$$\vec{\rho} = -\rho \cancel{\cos(EI)}^{\,0} \cos(Az)\,\vec{S} + \rho\cancel{\cos(EI)}^{\,0}\sin(Az)\,\vec{E} + \rho\sin(EI)\,\vec{Z} = \underline{0.5\,\vec{Z}}\ DU_\oplus$$

b) Determine the components of the spacecraft velocity vector $\dot{\boldsymbol{\rho}}$ in the topocentric-horizon coordinate system.

$$\dot{\vec{\rho}} = \dot{\rho}_S\,\vec{S} + \dot{\rho}_E\,\vec{E} + \dot{\rho}_Z\,\vec{Z}$$

WITH

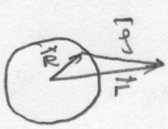
$$\dot{\rho}_S = -\dot{\rho}\cancel{\cos(EI)}^{\,0}\cos(Az) + \rho\,\dot{EI}\sin(EI)\cos(Az) + \rho\,\dot{Az}\cancel{\cos(EI)}^{\,0}\sin(Az) = 1.06$$

$$\dot{\rho}_E = \dot{\rho}\cos(EI)\sin(Az) - \rho\,\dot{EI}\sin(EI)\sin(Az) + \rho\,\dot{Az}\cos(Az)\cos(EI) = -1.06$$

$$\dot{\rho}_Z = \cancel{\dot{\rho}}^{\,0}\sin(EI) + \rho\,\dot{EI}\cancel{\cos(EI)}^{\,0} = 0$$

$$\Rightarrow \dot{\vec{\rho}} = 1.06\,\vec{S} - \underline{1.06\,\vec{E}}\quad DU_\oplus/TU_\oplus$$

c) Obtain the spacecraft position vector $\mathbf{r}$ from the center of the Earth - assuming that the Earth is perfectly spherical - in the topocentric-horizon coordinate system.



$$\vec{r} = \vec{R} + \vec{\rho} = 1.0\,\vec{Z} + \vec{\rho} = \underline{1.5\,\vec{Z}}\quad DU_\oplus$$

d) Transform the $\mathbf{r}$ vector into geocentric-equatorial coordinates.

TRANSFORMATION MATRIX:

$$\tilde{D} = \begin{pmatrix} \sin L\cos\theta & \sin L\sin\theta & -\cos L \\ -\sin\theta & \cos\theta & 0 \\ \cos L\cos\theta & \cos L\sin\theta & \sin L \end{pmatrix} =$$

$$= \begin{pmatrix} 0.34 & 0.34 & -0.88 \\ -0.71 & 0.71 & 0 \\ 0.62 & 0.62 & 0.48 \end{pmatrix}$$

SO THAT

$$\vec{r}_{IJK} = \tilde{D}^T r_{SEZ} = \begin{pmatrix} 0.34 & -0.71 & 0.62 \\ 0.34 & 0.71 & 0.62 \\ -0.88 & 0 & 0.48 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1.5 \end{pmatrix} =$$

$$= \underline{0.93\,\vec{I} + 0.93\,\vec{J} + 0.72\,\vec{K}}\quad DU_\oplus$$

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e) Determine the absolute velocity **v** of the spacecraft in terms of geocentric-equatorial coordinates.

FIRST CALCULATE $\dot{\vec{\rho}}_{IJK} = \tilde{D}^T \dot{\rho}_{SEZ} = \begin{pmatrix} 0.34 & -0.71 & 0.62 \\ 0.34 & 0.71 & 0.62 \\ -0.88 & 0 & 0.48 \end{pmatrix} \begin{pmatrix} 1.06 \\ -1.06 \\ 0 \end{pmatrix} =$

$= 1.11\vec{I} - 0.39\vec{J} - 0.93\vec{K} \quad DU_\oplus/TU_\oplus$

THEN $\vec{V}_{IJK} = \dot{\vec{\rho}}_{IJK} + \vec{\omega}_\oplus \wedge \vec{r}_{IJK} = \dot{\vec{\rho}}_{IJK} + \begin{vmatrix} i & j & k \\ 0 & 0 & 0.058 \\ 0.93 & 0.93 & 0.72 \end{vmatrix} =$

$= \dot{\rho}_{IJK} - 0.054\vec{I} + 0.054\vec{J} = \underline{1.05\vec{I} - 0.34\vec{J} - 0.93\vec{K}} \quad DU_\oplus/TU_\oplus$

f) Calculate the local Greenwich Sidereal Time (GST) in hours and minutes.

$\underset{45^\circ}{\theta} = \theta_g + \underbrace{\text{LATITUDE WITH RESPECT TO GREENWICH MERIDIAN}}_{80.5^\circ W}$

$\Rightarrow \theta_g = 125.5^\circ = 125.5^\circ \cdot 1h/15^\circ = 8.36\,h = \underline{8\,h\ 22\ MIN}$

SOLUTION USING RIGHT VALUES OF $\vec{r}$ AND $\vec{v}$

Part II (25pt). For this second part, if you have **not** been able to calculate the absolute position and velocity vectors of the spacecraft, you can assume that **r** = 0.67**I** + 0.67**J** + 0.55**K** and **v** = 0.61**I** – 0.01**J** – 0.74**K**, which may **not** be the right results for Part I, but still you can obtain full credit in this part if you solve it consistently.

h) Calculate the specific mechanical of the spacecraft.

$\mathcal{E} = \frac{|v|^2}{2} - \frac{\mu}{|r|} = \frac{1.44^2}{2} - \frac{1}{1.5} = \underline{0.37} \quad DU_\oplus^2/TU_\oplus^2$

i) Calculate the specific angular momentum of the spacecraft.

$\vec{h} = \vec{r} \wedge \vec{v} = \underline{-0.62\vec{I} + 1.62\vec{J} - 1.29\vec{K}} \quad DU_\oplus^2/TU_\oplus$

THEN

j) Determine the eccentricity vector and characterize the shape of the orbit.

$\vec{e} = \frac{1}{\mu}\left[\left(|v|^2 - \frac{\mu}{|r|}\right)\vec{r} - (\vec{r}\cdot\vec{v})\vec{v}\right] = 1.41\vec{r} + 0.009\vec{v} =$

$= \underline{1.32\vec{I} + 1.31\vec{J} + 1.01\vec{K}}$

$e = |\vec{e}| = 2.11 \rightarrow$ HYPERBOLIC

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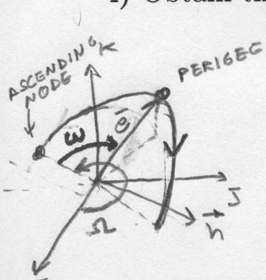
k) Calculate the perigee and apogee altitudes of the orbit.

$$p = \frac{|h|^2}{\mu} = 2.16^2 = 4.67 \ DU_\oplus$$

THEN $r_p = \dfrac{p}{1+e} = 1.50 \ DU_\oplus \rightarrow H_p = r_p - r_\oplus = \underline{\underline{0.5 \ DU_\oplus}} \rightarrow$ NOT A BALLISTIC MISSILE.

THE APOGEE IS UNDEFINED SINCE THE ORBIT IS HYPERBOLIC.

l) Obtain the orbit inclination. Is it a retrograde or direct orbit?.



$$\cos(i) = \frac{\vec{h} \cdot \vec{K}}{|h|} = \frac{-1.29}{2.16} = -0.59 \rightarrow \underline{\underline{i = 126.7^\circ}}$$

RETROGRADE ORBIT $\left(i > \frac{\pi}{2}\right)$

m) Calculate the longitude of the ascending node, the argument of the periapsis, and the true anomaly at epoch.

NODE VECTOR: $\vec{n} = \vec{K} \wedge \vec{h} = -1.62 \vec{I} + 0.62 \vec{J}$

THEN $\cos(\Omega) = \dfrac{\vec{n} \cdot \vec{I}}{|n|} = \dfrac{-1.62}{1.73} = -0.93 \rightarrow \underline{\underline{\Omega = 200.5^\circ}}$ $(n_j < 0)$

$$\cos(\omega) = \frac{\vec{n} \cdot \vec{e}}{|n||e|} = \frac{-2.95}{1.73 \cdot 2.11} = -0.81 \rightarrow \underline{\underline{\omega = 143.9^\circ}} \quad (e_K > 0)$$

$$\cos(\nu) = \frac{\vec{e} \cdot \vec{r}}{|e||r|} = \frac{3.16}{2.11 \cdot 1.5} \approx 1 \rightarrow \underline{\underline{\nu \approx 0^\circ}}$$ $\rightarrow$ THE SPACECRAFT IS AT PERIGEE.

n) Determine the argument of latitude at epoch, the true longitude at epoch and the longitude of the periapsis.

$$u_0 = \omega + \nu = \underline{\underline{143.9^\circ}}$$

$$l_0 = u_0 + \Omega = \underline{\underline{344.4^\circ}}$$

$$\Pi = \Omega + \omega = \underline{\underline{344.4^\circ}}$$

o) What minimum set of parameters, of those calculated above, would you choose in order to determine the orbit orientation, orbit geometry and object location?.

FOR INSTANCE p, e, i, Ω, ω AND ν.

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e) Determine the absolute velocity **v** of the spacecraft in terms of geocentric-equatorial coordinates.

f) Calculate the local Greenwich Sidereal Time (GST) in hours and minutes.

SOLUTION USING THESE VALUES

Part II (25pt). For this second part, if you have **not** been able to calculate the absolute position and velocity vectors of the spacecraft, you can assume that $\mathbf{r} = 0.67\mathbf{I} + 0.67\mathbf{J} + 0.55\mathbf{K}$ and $\mathbf{v} = 0.61\mathbf{I} - 0.01\mathbf{J} - 0.74\mathbf{K}$, which may **not** be the right results for Part I, but still you can obtain full credit in this part if you solve it consistently.

h) Calculate the specific mechanical of the spacecraft.

$$\mathcal{E} = \frac{0.96^2}{2} - \frac{1}{1.09} = -0.45 \quad DU_\oplus^2 / TU_\oplus^2$$

i) Calculate the specific angular momentum of the spacecraft.

$$\vec{h} = \vec{r} \wedge \vec{v} = -0.49\,\vec{I} + 0.83\,\vec{J} - 0.41\,\vec{K}$$

j) Determine the eccentricity vector and characterize the shape of the orbit.

$$\vec{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\vec{r} - (\vec{r}\cdot\vec{v})\,\vec{v}\right] = 0.078\,\vec{I} + 0.047\,\vec{J} + 0.0002\,\vec{K}$$

$e = |\vec{e}| = \underline{\underline{0.0091}}$ ELLIPTIC (ALMOST CIRCULAR)

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e) Determine the absolute velocity $\mathbf{v}$ of the spacecraft in terms of geocentric-equatorial coordinates.

f) Calculate the local Greenwich Sidereal Time (GST) in hours and minutes.

SOLUTION USING THESE VALUES

Part II (25pt). For this second part, if you have **not** been able to calculate the absolute position and velocity vectors of the spacecraft, you can assume that $\mathbf{r} = 0.67\mathbf{I} + 0.67\mathbf{J} + 0.55\mathbf{K}$ and $\mathbf{v} = 0.61\mathbf{I} - 0.01\mathbf{J} - 0.74\mathbf{K}$, which may **not** be the right results for Part I, but still you can obtain full credit in this part if you solve it consistently.

h) Calculate the specific mechanical of the spacecraft.

$$\mathcal{E} = \frac{0.96^2}{2} - \frac{1}{1.09} = -0.45 \quad DU_\oplus^2 / TU_\oplus^2$$

i) Calculate the specific angular momentum of the spacecraft.

$$\vec{h} = \vec{r} \wedge \vec{v} = -0.49\,\vec{I} + 0.83\,\vec{J} - 0.41\,\vec{K}$$

j) Determine the eccentricity vector and characterize the shape of the orbit.

$$\vec{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\vec{r} - (\vec{r}\cdot\vec{v})\,\vec{v}\right] = 0.078\,\vec{I} + 0.047\,\vec{J} + 0.0002\,\vec{K}$$

$e = |\vec{e}| = \underline{\underline{0.0091}}$ ELLIPTIC (ALMOST CIRCULAR)

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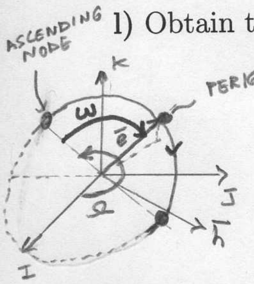
k) Calculate the perigee and apogee altitudes of the orbit.

$$p = \frac{|h|^2}{\mu} = 1.05^2 = \underline{\underline{1.10}}\ DU_\oplus$$

THEN $r_p = \frac{p}{1+e} = 1.09\ DU_\oplus \rightarrow H_p = r_p - r_\oplus = \underline{\underline{0.09}}\ DU_\oplus$

$r_a = \frac{p}{1-e} = 1.11\ DU_\oplus \rightarrow H_a = r_a - r_\oplus = \underline{\underline{0.11\ DU_\oplus}}$

l) Obtain the orbit inclination. Is it a retrograde or direct orbit?.



$$\cos(i) = \frac{\vec{h}\cdot\vec{K}}{|h|} = \frac{-0.41}{1.05} = -0.39 \rightarrow i = \underline{\underline{112.9^\circ}}$$

RETROGRADE ORBIT ($i > \frac{\pi}{2}$)

m) Calculate the longitude of the ascending node, the argument of the periapsis, and the true anomaly at epoch.

NODE VECTOR: $\vec{n} = \vec{K} \wedge \vec{h} = -0.83\vec{I} - 0.49\vec{J}$

$\cos(\Omega) = \frac{\vec{n}\cdot\vec{I}}{|\vec{n}|} = \frac{-0.83}{0.96} = -0.86 \rightarrow \Omega = \underline{\underline{210.1^\circ}}$ $(n_j < 0)$

$\cos(\omega) = \frac{\vec{n}\cdot\vec{e}}{|n||e|} = \frac{-0.0087}{0.96 \cdot 0.0091} = -0.99 \rightarrow \omega = \underline{\underline{174.8^\circ}}$ $(e_k > 0)$

$\cos(\nu) = \frac{\vec{e}\cdot\vec{r}}{|e||r|} = \frac{0.0084}{0.0091 \cdot 1.09} = -0.84 \rightarrow \nu = \underline{\underline{327.8^\circ}}$ $(\vec{r}\cdot\vec{v} < 0)$

n) Determine the argument of latitude at epoch, the true longitude at epoch and the longitude of the periapsis.

$$u_0 = \omega + \nu = 502.6^\circ = \underline{\underline{142.6^\circ}}$$

$$l_0 = u_0 + \Omega = \underline{\underline{352.7^\circ}}$$

$$\Pi = \Omega + \omega = 384.9^\circ = \underline{\underline{24.9^\circ}}$$

o) What minimum set of parameters, of those calculated above, would you choose in order to determine the orbit orientation, orbit geometry and object location?.

FOR INSTANCE p, e, i, Ω, ω, AND ν