MAE 180A: Spacecraft Guidance I, Summer 2009

Homework 1

Due Tuesday, July 6, in class.

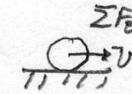
Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Show all work. Required plots should be generated using computer software such as Matlab or Excel. Answers should be written in the blank spaces provided in these homework sheets. Use the back of the page in case you need additional space (not recommended to use more space than provided), for which a clear indication should be written to warn the reader of the presence of text there. Vector quantities are denoted in **bold** letters in what follows.

Student's Name: JAVIER URZAY .... **Student's ID:**............................

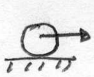
Question 1 (10 pts)

a) State briefly in words the physical meaning of the three Newton's laws of motion, and put a physical or mechanical example for each of them.

1ST LAW) EVERY BODY CONTINUES IN STATE OF REST OR IN UNIFORM MOTION IN A STRAIGHT LINE UNLESS IT IS COMPELLED TO CHANGE THAT STATE BY FORCES IMPRESSED UPON IT. EG. $\Sigma F_{ext} = 0 \rightarrow U = 0$ { $\Sigma F_{EXT} = 0 \rightarrow U =$ con

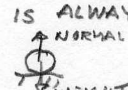


2ND LAW) THE RATE OF CHANGE OF MOMENTUM IS PROPORTIONAL TO THE FORCE IMPRESSED AND IS IN THE SAME DIRECTION AS THAT FORCE.



$\Sigma \vec{F}_{EXT} \neq 0 \Rightarrow \Sigma \vec{F}_{EXT} = m\vec{a} = \frac{d\vec{p}}{dt}$

3RD LAW) TO EVERY FORCE THERE IS ALWAYS OPPOSED AND EQUAL REACTION.



b) State in words the definitions of an inertial reference system and a non-inertial reference system. Newton formulated his laws with respect to a reference system fixed relative to the *stars*; is the system used by Newton a truly inertial reference system? Is the Earth an inertial or a non-inertial reference system?

* INERTIAL REFERENCE SYSTEM IS A REFERENCE FRAME THAT MOVES AT CONSTANT TRANSLATIONAL VELOCITY AND DOES NOT ROTATE.

* NON-INERTIAL REFERENCE SYSTEM IS A REFERENCE FRAME THAT ACCELERATES OR ROTATES.

THE INERTIAL REFERENCE SYSTEMS ARE "IDEAL" FRAMES, IN THAT THE EARTH, SUN AND THE STARS MOVE, ROTATE, AND ARE SUBJECT TO ACCELERATIONS. HOWEVER, IF SUCH NON-INERTI[AL] EFFECTS ARE SMALL, THE REFERENCE SYSTEM CAN BE TREATED AS AN INERTIAL ONE. THE EARTH IS USUALLY APPROXIMATED AS AN INERTIAL FRAME, EVEN THOUGH, STRICTLY SPEAKING IT IS A NON-INERTIAL SYSTEM.

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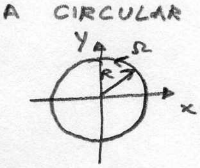
Question 2 (10 pts).
The motion of a planet around a star can be described by its position vector along the orbit referred to the inertial space, $\mathbf{r} = R\cos(\Omega t)\mathbf{i} + R\sin(\Omega t)\mathbf{j}$, with R and Ω constants, $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x and y coordinates respectively, and t is the time coordinate.

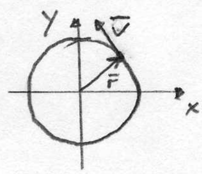
a) Show that the motion of the planet follows a direct circular orbit of radius R around the star at an angular velocity Ω.

$\vec{r} = R\cos(\Omega t)\vec{i} + R\sin(\Omega t)\vec{j} = x\vec{i} + y\vec{j}$, THEN $|\vec{r}|^2 = x^2 + y^2 = R^2(\cos^2\Omega t + \sin^2\Omega t)$

AND $\underline{R^2 = x^2 + y^2}$, WHICH REPRESENTS A CIRCULAR ORBIT



b) Obtain the velocity vector of the planet, and show that it is perpendicular to $\mathbf{r}$ at all times.

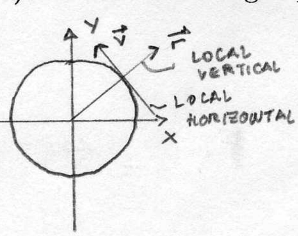


$\vec{v} = \dot{\vec{r}} = \dfrac{d\vec{r}}{dt} = -R\Omega\sin(\Omega t)\vec{i} + R\Omega\cos(\Omega t)\vec{j}$

WHICH IS ALWAYS PERPENDICULAR TO $\vec{r}$ SINCE

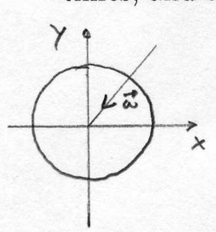
$\vec{v}\cdot\vec{r} = -R\Omega\sin(\Omega t)\cos(\Omega t) + R\Omega\cos(\Omega t)\sin(\Omega t) = \underline{0}$

c) Calculate the flight-path angle and zenith angle of the planet on its orbit around the star.



* SINCE $\vec{v}\cdot\vec{r} = 0$ THE VELOCITY VECTOR IS PARALLEL TO THE LOCAL HORIZONTAL $\Rightarrow$ FLIGHT-PATH ANGLE $\underline{\phi = 0}$.
* THEN, THE ZENITH ANGLE IS $\gamma = 90^\circ - \phi = \underline{90^\circ}$

d) Obtain the acceleration vector of the planet, and show that its direction is towards the origin at all times, and that the modulus of the acceleration is always proportional to the distance from the origin.



$\vec{a} = \ddot{\vec{r}} = \dfrac{d^2\vec{r}}{dt^2} = -R\Omega^2\cos(\Omega t)\vec{i} - R\Omega^2\sin(\Omega t)\vec{j} = \underline{-\Omega^2\vec{r}}$ → TOWARDS THE ORIGIN,

THEN $\underline{|\vec{a}| = \Omega^2|\vec{r}|}$ → THE ACCEL IS PROPORTIONAL TO $|\vec{r}|$

e) Calculate the specific angular momentum vector of the planet, and show that it is CONSTANT AND perpendicular to the orbital plane at all times.

$|\vec{h}| = |\vec{r}\wedge\vec{v}| = |\vec{r}||\vec{v}|\cdot\sin\gamma = R\cdot(\Omega R)\sin(90^\circ) = \underline{\Omega R^2}$ AND DIRECTED OUTWARDS FROM THE PAPER.

(ALSO $\vec{h} = \vec{r}\wedge\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & 0 \\ v_x & v_y & 0 \end{vmatrix} = +(r_x v_y - v_x r_y)\vec{k} = \underline{\Omega R^2\vec{k}}$)

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Question 3 (10 pts)

Select the true answer (only one) out of the choices from the list provided for each question. A complementary and *brief* mathematical proof of your answer on the available space would be welcome, but it is not needed to get full credit.

3.1 The orbital period of a planet in a circular orbit of radius r_1 around the Sun is P_1. The orbital period of another planet in a circular heliocentric orbit of radius $3r_1$ is

a) $P_1/3$

b) $-3P_1$

c) $3^3 P_1$

~~d)~~ $3^{3/2} P_1$

$$P_1 = 2\pi\sqrt{\frac{r_1^3}{\mu_\odot}}, \quad P_2 = 2\pi\sqrt{\frac{r_2^3}{\mu_\odot}},$$

SINCE $r_2 = 3r_1 \Rightarrow$ $P_2 = \left(2\pi\sqrt{\frac{r_1^3}{\mu_\odot}}\right)\cdot 3^{3/2} = \underline{3^{3/2}P_1}$

3.2 The angular momentum in a central motion is

a) constant and parallel to the orbit plane.

~~b)~~ constant and normal to the orbit plane.

c) not constant, and it is parallel to the orbit plane.

d) not constant, and it is normal to the orbit plane.

$\vec{h} = \vec{r}\wedge\vec{v}$, $\vec{h}\perp\vec{r}$ AND $\vec{h}\perp\vec{v}$,

AND $|\vec{h}| = r^2\dot{\theta} = 2\times$ AEREOLAR VELOCITY = CO

3.3 The specific mechanical energy of a satellite orbiting around the Earth

a) is not constant and varies sinusoidally with time.

b) is constant but only for US-manufactured satellites.

~~c)~~ is constant if the satellite is not subject to any dissipative forces or external forces other than gravitational interactions.

d) is always constant.

3.4 The burnout velocity of a space probe intended to leave the Earth is $1.0 \cdot v_{ESC}$, where v_{ESC} is the escape speed at the burnout point. Then the hyperbolic excess speed of the space probe is

a) $2.0 \cdot v_{ESC}$

b) $0.5 \cdot v_{ESC}$

~~c)~~ zero.

d) infinity.

$$\underbrace{V_\infty}_{\text{HYPERBOLIC EXCESS VEL}} = \left[\left(\underbrace{V_{BO}}_{\text{BURNOUT VELOCITY}}\right)^2 - \left(\underbrace{V_{ESC}}_{\text{ESCAPE VELOCITY}}\right)^2\right]^{1/2} = \left(V_{ESC}^2 - V_{ESC}^2\right)^{1/2} = \underline{0}$$

↳ VELOCITY OF THE PROBE FAR FROM THE PLANET

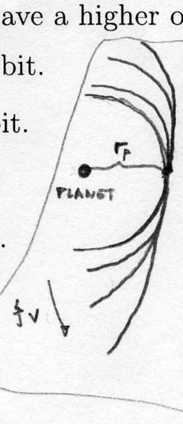
3.5 If a hyperbolic, parabolic, elliptic and circular orbits intersect at one point P in the space, a satellite at point P would have a higher orbital speed if it is orbiting on the

~~a)~~ hyperbolic orbit.

b) parabolic orbit.

c) elliptic orbit.

d) circular orbit.



ON A HYPERBOLIC ORBIT: $V = \left[2\left(\frac{\mu}{2|a|} + \frac{\mu}{r_P}\right)\right]^{1/2}$ $(a<0)$

ON A PARABOLIC ORBIT: $V = \left[2\left(\frac{\mu}{r_P}\right)\right]^{1/2}$ $(a\to\infty)$

ON AN ELLIPTIC ORBIT: $V = \left[2\left(-\frac{\mu}{2a} + \frac{\mu}{r_P}\right)\right]^{1/2}$ $(a>0)$

ON A CIRCULAR ORBIT: $V = \left[2\left(-\frac{\mu}{2r_P} + \frac{\mu}{r_P}\right)\right]^{1/2} = \left(\frac{\mu}{r_P}\right)^{1/2}$

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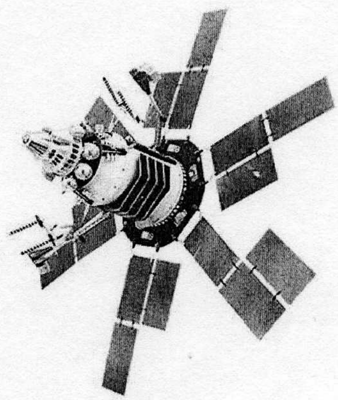
Problem 1 (20 pts)

The Molniya 3-3 satellite was launched in 1975 by the Soviet Union for communication, surveillance and military purposes, and it was inserted into a high-inclination Earth orbit of perigee altitude 2,646.5 km and apogee altitude 37,715.4 km. *Give all your results in both canonical units and dimensional units.* Find:



$r_a = 37\,715.4 \text{ km} + r_\oplus = 5.91 + 1 = \underline{\underline{6.91 \text{ DU}_\oplus}}$

$r_p = 2\,646.5 \text{ km} + r_\oplus = 0.41 + 1 = \underline{\underline{1.41 \text{ DU}_\oplus}}$

Figure 1: The Molniya 3-3 satellite.

a) the orbit eccentricity,

$$e = \frac{r_a - r_p}{r_a + r_p} = \underline{\underline{0.66}}$$

b) the specific angular momentum of the satellite,

SINCE SEMI-LATUS RECTUM: $p = (1+e)\, r_p = 2.34 \text{ DU}_\oplus = 14938.2 \text{ km}$

THEN $|\bar{h}| = \sqrt{\mu_\oplus p} = \underline{\underline{1.53}} \text{ DU}_\oplus^2/\text{TU}_\oplus = \underline{\underline{77130.1}} \text{ km}^2/\text{s}$

c) the perigee velocity,

SINCE $|h|$ IS CONSTANT AND THE FLIGHT-PATH ANGLE IS ZERO AT PERIGEE AND APOGEE: $|\bar{h}| = r_p V_p \Rightarrow V_p = \frac{|\bar{h}|}{r_p} = \underline{\underline{1.08}} \frac{\text{DU}_\oplus}{\text{TU}_\oplus} = \underline{\underline{8.57}} \text{ km}/\text{s}$

d) the apogee velocity,

SIMILARLY, $|\bar{h}| = r_a V_a \Rightarrow V_a = \frac{|\bar{h}|}{r_a} = \underline{\underline{0.22}} \frac{\text{DU}_\oplus}{\text{TU}_\oplus} = \underline{\underline{1.72}} \text{ km}/\text{s}$

e) the specific mechanical energy of the satellite,

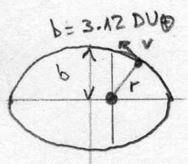
$$\mathcal{E} = -\frac{\mu_\oplus}{2a} = -\frac{\mu_\oplus}{(r_a + r_p)} = \underline{\underline{-0.12}}\ \frac{DU_\oplus^2}{TU_\oplus^2} = \underline{\underline{-7.51}}\ \frac{km^2}{s^2}$$

f) the orbit period,

SEMI-MAJOR AXIS: $a = \frac{r_a + r_p}{2} = 4.16\ DU_\oplus = 26533.1\ km$

THEN $\mathbb{P} = 2\pi \left(\frac{a^3}{\mu_\oplus}\right)^{1/2} = \underline{\underline{53.3}}\ TU_\oplus = 43011.6\ s = \underline{\underline{11.94}}\ h$

g) the satellite velocity when its altitude is 10, 000 km,

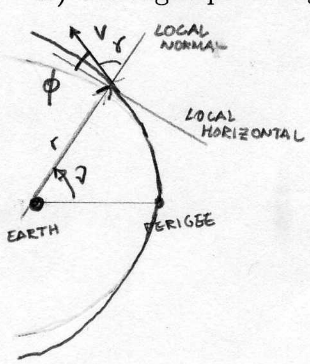


$r = 10000\ km + r_\oplus = 2.56\ DU_\oplus$

SINCE THE ENERGY IS CONSERVED: $\mathcal{E} = -0.12 = \frac{v^2}{2} - \frac{\mu_\oplus}{r}$

THEN $v = \left[2\left(\mathcal{E} + \frac{\mu_\oplus}{r}\right)\right]^{1/2} = \underline{\underline{0.73}}\ \frac{DU_\oplus}{TU_\oplus} = \underline{\underline{5.81}}\ km/s$

h) the flight-path angle at altitude 10, 000 km,



CONIC EQUATION: $r = \frac{p}{1 + e\cos\nu}$, THEN $\cos\nu = \frac{1}{e}\left(\frac{p}{r} - 1\right) = -0.13$

AND $\nu = 97.4^\circ$ IS THE TRUE ANOMALY

THEN $\cos\phi = \frac{1 + e\cos\nu}{(1 + 2e\cos\nu + e^2)^{1/2}} = 0.81 \Rightarrow \underline{\underline{\phi = 35.5^\circ}}$

i) and the zenith angle at altitude 10, 000 km.

$$\gamma = 90^\circ - \phi = \underline{\underline{54.4^\circ}}$$

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Problem 2 (25 pts)
The Ballistic Missile Early Warning System (BMEWS) detects an unidentified object with the following parameters:

- altitude: 0.5 $DU_\oplus$,
- speed: 0.8 $DU_\oplus/TU_\oplus$,
- flight-path angle: 40°.

Is it possible that this object is a space probe intended to escape the Earth, an Earth satellite, or a ballistic missile? Justify your answer.

* DISTANCE FROM THE EARTH'S CENTER: $r = 0.5\, DU_\oplus + 1.0\, DU_\oplus = 1.5\, DU_\oplus$

* MECHANICAL ENERGY: $\mathcal{E} = \frac{|\vec{v}|^2}{2} - \frac{\mu_\oplus}{r} = \frac{0.8^2}{2} - \frac{1}{1.5} = -0.34\, \frac{DU_\oplus^2}{TU_\oplus^2}$

SINCE $\mathcal{E} < 0$, THE ORBIT IS NOT HYPERBOLIC NOR PARABOLIC

$\rightarrow$ IT IS NOT A PROBE INTENDED TO ESCAPE THE EARTH.

* SEMIMAJOR AXIS: $a = -\frac{\mu_\oplus}{2\mathcal{E}} = +\frac{1}{2 \cdot 0.34} = 1.44\, DU_\oplus$

* ANGULAR MOMENTUM: $|\vec{h}| = |r||v| \cos\phi = 1.5 \cdot 0.8 \cdot \cos(40^\circ) = 0.92\, \frac{DU_\oplus^2}{TU_\oplus}$

THEN $p = \frac{h^2}{\mu_\oplus} = 0.92^2 = 0.84\, DU_\oplus$

* ECCENTRICITY: $e = \left(1 - \frac{p}{a}\right)^{1/2} = 0.64 \Rightarrow$ ELLIPTICAL ORBIT

SATELLITE (NOT COLLIDES)

MISSILE (COLLIDES WITH EARTH SURFACE)

SINCE $r_p = \frac{p}{1+e} = 0.51\, DU_\oplus < r_\oplus$, THEN THE OBJECT IS A

BALLISTIC MISSILE

Problem 3 (25 pts)

A spacecraft is launched from the Vanderberg Air Force base (VAF) to a circular equatorial parking orbit of altitude 300 nmi above Earth. The spacecraft remains circulating in this orbit until the mission center gives permission for a escape maneuver, during which the spacecraft ignites its main engines tangentially to the circular orbit during a short time, by which it is transferred to a hyperbolic orbit about the equator as the one depicted in the figure. The burnout and orbit transfer point corresponds to the perigee of the hyperbolic orbit. Thus, the spacecraft leaves the Earth at a velocity $\mathbf{v_d} = v_\infty \mathbf{I}$ parallel to the vernal-equinox direction Υ (i.e. the x axis), with $v_\infty = 10$ km/s the spacecraft velocity at a radial distance $r_d = 10 r_\oplus$, where $r_\oplus$ is the radius of the Earth. In this formulation, **I**, **J** and **K** are unit vectors in the x, y and z directions respectively. Notice that v_∞ is close to the hyperbolic excess velocity since the spacecraft is far from the Earth, $r_d \gg r_\oplus$. A radar station R is located at VAF base, which latitude is $L = 28.5°$. At a particular tracking time, the longitude of VAF is $\theta = 30°$ in a westerly direction with respect to the vernal equinox direction Υ. Note that the system of coordinates $\{x, y, z\}$ (i.e. the geocentric-equatorial coordinate system) does not rotate with the Earth, but the radar site does rotate with the Earth.

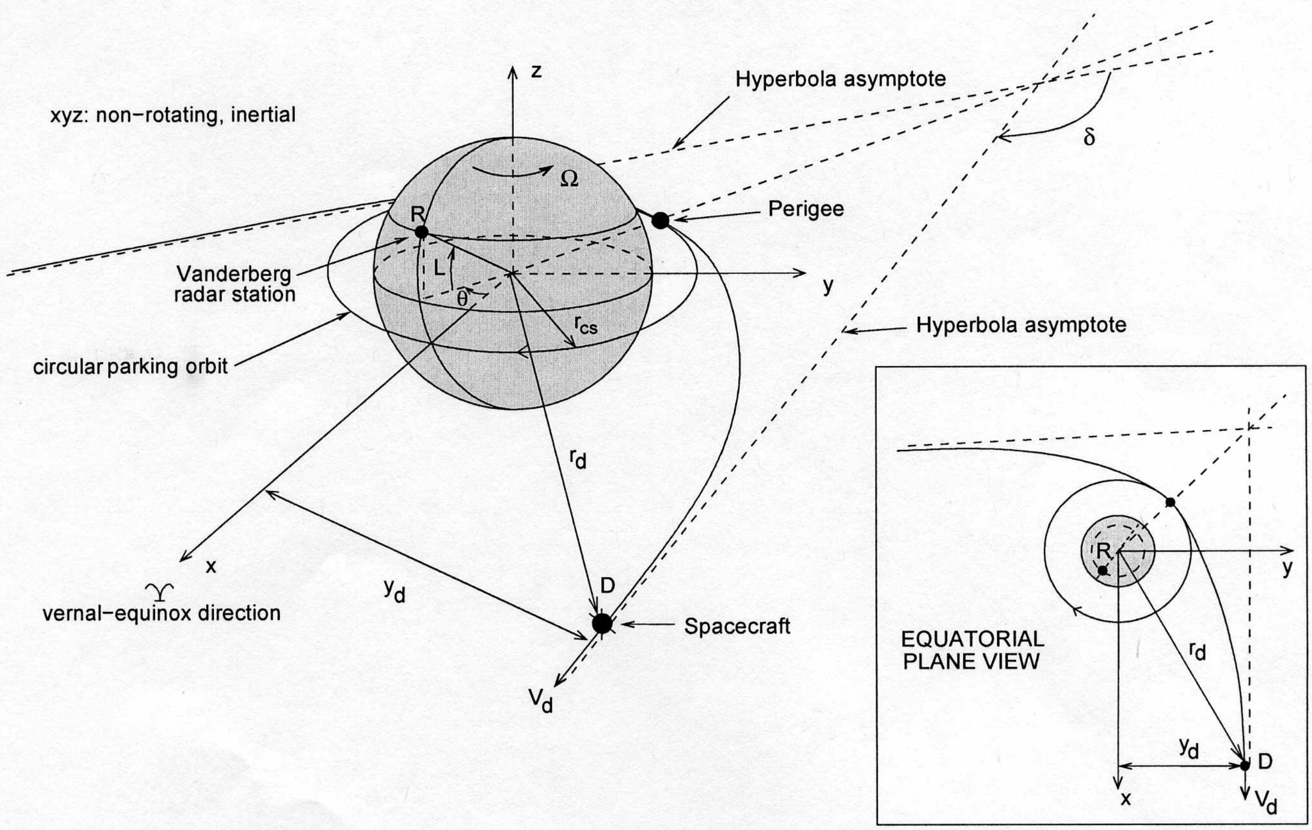

Figure 2: Departure mission sketch.

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a) Calculate the orbital velocity v_{CS} of the spacecraft on the circular parking orbit.

$r_{CS} = 300 \text{ nmi} + r_\oplus = 0.09 + 1.0 = 1.09 \text{ DU}_\oplus$

THEN $V_{CS} = \left(\frac{\mu_\oplus}{r_{CS}}\right)^{1/2} = \left(\frac{1}{1.09}\right)^{1/2} = 0.96 \text{ DU}_\oplus/\text{TU}_\oplus = 7.58 \text{ km/s}$

b) Calculate the specific mechanical energy of the spacecraft on the hyperbolic orbit.

$$\mathcal{E} = \frac{V_d^2}{2} - \frac{\mu_\oplus}{r_d} = \frac{1.26^2}{2} - \overset{\text{NEGLIGIBLE}}{\cancel{\frac{1}{10}}} = 0.79 \frac{\text{DU}_\oplus^2}{\text{TU}_\oplus^2} = 49.6 \frac{\text{km}^2}{\text{s}^2}$$

$V_d = V_\infty = 10 \text{ km/s} = 1.26 \frac{\text{DU}_\oplus}{\text{TU}_\oplus}$

c) Obtain the velocity of the spacecraft at perigee v_p when inserted into the hyperbolic orbit.

REVERSE ORDER

CONSERVATION OF MECHANICAL ENERGY:

$$\mathcal{E} = \frac{V_P^2}{2} - \frac{\mu_\oplus}{r_P} = \frac{V_d^2}{2} - \cancel{\frac{\mu_\oplus}{r_d}} \rightarrow V_P = \left(V_d^2 + \frac{2\mu_\oplus}{r_P}\right)^{1/2} = 1.84 \frac{\text{DU}_\oplus}{\text{TU}_\oplus} = 14.6 \text{ km/s}$$

d) Calculate the perigee radius. $r_P = r_{CS} = 1.09 \text{ DU}_\oplus$

e) Calculate the specific angular momentum of the spacecraft at point D about the center of the Earth O, and compute the distance y_d from point D to the vernal equinox direction Υ.

$|\bar{h}| = V_d \cdot y_d = V_P r_P$ SINCE THE ANGULAR MOMENTUM IS CONSTANT

THEN $|\bar{h}| = V_P r_P = 2.06 \frac{\text{DU}_\oplus^2}{\text{TU}_\oplus} = 101501.8 \frac{\text{km}^2}{\text{s}} \rightarrow y_d = \frac{|h|}{V_d} = 1.59 \text{ DU}_\oplus = 10149.3 \text{ km}$

e) Compute the velocity increment $\Delta v = v_p - v_{CS}$ (this increment is proportional to the energy that the propulsion system must spend to change orbits).

$$\Delta V = V_P - V_{CS} = 0.88 \frac{\text{DU}_\oplus}{\text{TU}_\oplus} = 6.95 \text{ km/s}$$

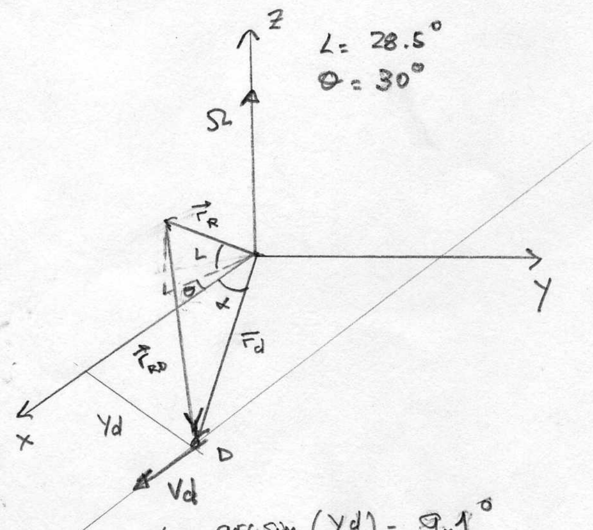
f) Calculate the turning angle δ.

$$\mathcal{E} = -\frac{\mu_\oplus}{2a} = \frac{V_D^2}{2} = 0.79 \frac{\text{DU}_\oplus^2}{\text{TU}_\oplus^2} \rightarrow a = -0.63 \text{ DU}_\oplus$$

THE ECCENTRICITY IS $e = -\frac{r_P}{a} + 1 = 2.73$, SO THAT $\delta = 2 \arcsin\left(\frac{1}{e}\right) = 43.1^\circ$

Student's Name:

g) Obtain the $\{I, J, K\}$ components of the relative position vector of the spacecraft at D with respect to the radar site R.



$L = 28.5^\circ$
$\theta = 30^\circ$

$\alpha = \arcsin\left(\frac{Yd}{rd}\right) = 9.1^\circ$

$\vec{r}_R = r_\oplus \cos L \cos\theta \vec{I} - r_\oplus \cos L \sin\theta \vec{J} + r_\oplus \sin L \vec{K} = 0.76\vec{I} - 0.43\vec{J} + 0.48\vec{K}$ $DU_\oplus$

$\vec{r}_d = 10\, r_\oplus \cos\alpha \vec{I} + 10\, r_\oplus \sin\alpha \vec{J} = 9.87\vec{I} + 1.58\vec{J}$

THEN $\vec{r}_{RD} = \vec{r}_d - \vec{r}_R = 9.11\vec{I} + 2.01\vec{J} - 0.48\vec{K}$ DU

h) Obtain the $\{I, J, K\}$ components of the relative velocity vector of the spacecraft at D with respect to the radar site R,

$\vec{V}_d = \vec{V}_{REL} + \vec{\Omega}_\oplus \wedge \vec{r}_d$,

$\Omega_\oplus = 0.058 \frac{rad}{TU_\oplus}$

THEN $\vec{V}_{REL} = \vec{V}_d - \vec{\Omega} \wedge \vec{r}_d = V_\infty \vec{I} - \begin{vmatrix} I & J & K \\ 0 & 0 & \Omega \\ 9.87 & 1.55 & 0 \end{vmatrix} =$

$= 1.41\vec{I} - (-1.58\Omega\vec{I} + 9.87\Omega\vec{J}) = 1.50\vec{I} - 0.57\vec{J}$ $DU_\oplus / TU_\oplus$