ME 471: Turbulent Combustion, Spring 2012

Stanford University

Homework 2: Review of turbulent flows (II) and Premixed Turbulent Combustion (I)

Due Tuesday, May 8 in class.

Guidelines: Please turn in a *neat* and *clean* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Show all work. Required plots should be generated using computer software such as Matlab or similar program. Answers should be written in the blank spaces provided in these homework sheets. Use the back of the page in case you need additional space (not recommended to use more space than provided), for which a clear indication should be written to warn the reader of the presence of text there. Vector quantities are denoted in **bold** letters in what follows.

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Solve either Question 1 or Question 2, and solve Problem 1.

Question 1 (50 pts)

a) Obtain a relation between the Favre average $\widetilde{\phi}$ and the Reynolds average $\overline{\phi}$ for a scalar field $\phi = \phi(\mathbf{x}, t)$. Why is Favre averaging used in formulations of turbulent combustion?

FAVRE: $\phi = \widetilde{\phi} + \phi''$ SUCH THAT $\overline{\phi'' \rho} = 0$

REYNOLDS: $\phi = \overline{\phi} + \phi'$

THEN $\overline{\rho\phi} = \overline{\rho(\widetilde{\phi} + \phi'')} = \overline{\rho}\widetilde{\phi}$, AND ALSO

$\overline{\rho\phi} = \overline{\rho(\overline{\phi} + \phi')} = \overline{\rho}\,\overline{\phi} + \overline{\rho'\phi'} \Rightarrow \boxed{\overline{\rho}\widetilde{\phi} = \overline{\rho}\,\overline{\phi} + \overline{\rho'\phi'}}$ ✓ +10

⇒ THERE ISN'T A SIMPLE RELATION BETWEEN FAVRE AVERAGES AND REYNOLDS AVERAGES. THE CORRELATION $\overline{\rho'\phi'}$ IS REQUIRED TO TRANSFORM ONE INTO THE OTHER. FAVRE AVERAGES ARE USED BECAUSE THEY CAN ABSORB THE $\overline{\rho'\phi'}$ TERM THAT OTHERWISE WOULD APPEAR IN THE REYNOLDS AVERAGED CONTINUITY EQ. +10

b) Derive a transport equation for the scalar variance $\widetilde{\phi''^2}$. To do this, follow these steps: First derive a transport equation for $\widetilde{\phi^2}$, and then obtain another transport equation for $\left(\widetilde{\phi}\right)^2$. Then use the relation $\widetilde{\phi''^2} = \widetilde{\phi^2} - \left(\widetilde{\phi}\right)^2$ to obtain the transport equation for the scalar variance (use back of the page for more space). Also, give a brief description of each of the different terms in your equation.

SEE PAGE 13 OF INSTRUCTOR'S NOTES FOR DERIVATION +30

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Question 2 (50 pts)

Select true (T) or false (F) for each of the statements in the list provided for each question. A complementary and *brief* mathematical proof / sketch of your answer on the back of the page would be welcome, but it is **not** needed in order to get full credit.

(X INDICATE CORRECT CHOICE)

2.1 In turbulent flows at high Reynolds numbers,

+ 50/14 EACH

a) the maximum kinetic energy is located within the Kolmogorov range (T/F)

b) the viscous dissipation generated by the motion of the large eddies is much larger than the turbulent dissipation (T/F)

c) the large scales have turnover times which are much faster than the turnover time of the Kolmogorov eddies (T/F)

d) large-eddy simulations resolve all scales in the flow up to a maximum wave number beyond which physical subgrid-scale models may (or may not) be employed (T/F).

2.2 In a turbulent flow carrying a scalar ϕ with a single integral length scale $\ell = \ell_\phi$, characteristic large-eddy turnover velocity u', molecular diffusivity D, and Schmidt number $\mathrm{Sc} = \nu/D$,

a) molecular diffusion of the scalar takes place at the integral scale ℓ; this is why the turbulent diffusivity D_t is usually defined by relations of the type $u'\ell/D_t = O(1)$ (T/F)

b) there is no molecular diffusion in turbulent flows because in the asymptotic limit of infinite Reynolds numbers $u'\ell/D \to \infty$ the Euler equations are recovered; this is why computing turbulent reacting flows becomes so much more straightforward than computing laminar reacting flows (T/F)

c) molecular diffusion of the scalar ϕ takes place at scales of the same order as the Batchelor scale, $\ell_B = \mathrm{Sc}^{-1/2}\ell_k$, where ℓ_k is the Kolmogorov scale (T/F)

d) the Reynolds number based on the turbulent diffusivity D_t, (i.e. $u'\ell/D_t$) is typically much larger than the Reynolds number based on the molecular diffusivity D, (i.e. $u'\ell/D$) (T/F).

2.3 In the mixing of two reactants in a turbulent flow,

a) the scalar dissipation rate is always a sink in the scalar variance equation (T/F)

b) the scalar dissipation rate indicates zones where molecular mixing is taking place (T/F)

c) the scalar dissipation rate is maximum in zones where only one of the reactants exists (T/F)

d) the scalar dissipation of interest for turbulent combustion is defined as the Reynolds-average of the turbulent diffusivity multiplied by the square of the gradient of the scalar fluctuations (neglecting density variations) (T/F)

e) for order-unity Schmidt numbers, the molecular mixing of two reactants typically occurs in time scales which are much faster than the integral time scale (T/F)

f) macroscopic mixing or stirring is sufficient for having chemical reaction between both reactants (T/F).

Student's Name:

Problem 1 (50 pts)

A cylindrical premixed burner in a power plant is designed to burn a stoichiometric mixture of gaseous methane (CH_4) and air in a chamber pressurized at 20 bar. A sketch of the burner is shown below. Just upstream of the combustor intake, the CH_4−air feed stream has been passed through a grid to promote turbulence, in such a way that a streamwise turbulent intensity 15% and an approximate integral length scale 3 cm are obtained. a) Give an estimate of the resolution requirements required for the direct numerical simulation of this burner in terms of the total number of grid points N, the ~~minimum~~ MAX grid spacing Δx (make the assumptions that you most like regarding grid topology), the ~~minimum~~ MAX time step Δt and the total number of time steps N_t (assume an explicit fully-compressible code). b) Based on the premixed turbulent-combustion diagram explained in class, characterize the combustion regime in which this burner is likely to be operating. (Note: In case you need additional data, you can use external references but make sure you cite them below)

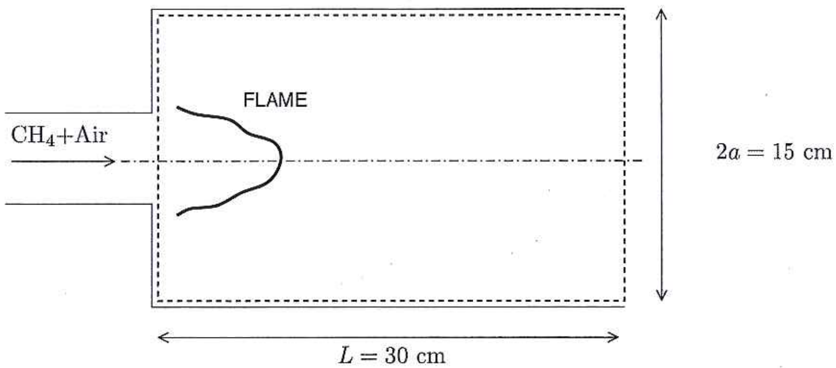

Figure 1: Sketch of the burner. In this figure, $2a$ denotes the diameter of the burner and L its axial length. The dashed line represents the boundaries of the computational domain.

THE INFLOW MEAN VELOCITY $\bar{U}$ NEEDS TO BE OF THE SAME ORDER AS THE PREMIXED FLAME VELOCITY S_L^0 FOR STABILIZING THE FLAME (UP TO A MULTIPLICATIVE FACTOR OF ORDER UNITY THAT INDICATES FLOW INHOMOGENEITIES AND AN INCLINATION ANGLE). FOR ESTIMATION PURPOSES, $\bar{U} \sim S_L^0 \sim 15\ cm/s$ (SEE LIÑÁN AND WILLIAMS, 1993 PAGE 54). THEN $0.15 = u'/\bar{U} \rightarrow u' = 2.2\ cm/s$. SINCE $\ell = 0.03\ m$, THEN $Re_\ell = \frac{u'\ell}{\nu} \sim 900$ WITH $\nu \sim \nu_{1ATM}^{AIR}/20 \sim 7.5\cdot 10^{-7} \frac{m^2}{s}$. SIMILARLY, $\delta_L^0 \sim D_T/S_L^0 \sim \nu/S_L^0 \sim 10\ \mu m$. (NOTE SMALL THICKNESS DUE TO ELEVATED PRESSURE)

IN THIS PROBLEM, THE SIZE OF THE KOLMOGOROV EDDIES IS OF ORDER $\ell_K \sim \ell Re_\ell^{-3/4} \sim 0.18\ mm$

THE EXTENT OF THE COMPUTATIONAL DOMAIN IS ALREADY LARGE ENOUGH TO CAPTURE THE INFLOW INTEGRAL LENGTH SCALE. HOWEVER, SINCE $\ell_K > \delta_L^0$, THE MAXIMUM Δx IS SET BY THE FLAME THICKNESS,

$\delta_L^0 \sim \Delta x N_f \sim 10 \Delta x \rightarrow \Delta x \sim \delta_L^0/10 \sim 1\ \mu m$. +10 THE TOTAL NUMBER OF GRID POINTS IS

$N = N_r N_\theta N_x \sim \frac{L}{\Delta x} \cdot \frac{a}{\Delta x} \cdot N_\theta \sim 10^{15}$ POINTS ($N_\theta \sim N_r$) +10 (WOULD BE EVEN LARGER IF $N_f \sim 100 \Rightarrow$ TOO LARGE COMBUSTOR FOR DNS!)

THE MAXIMUM CHEMICAL TIME STEP REQUIRED MAY BE ESTIMATED AS $\Delta t_c \sim \delta^2 \frac{\delta_L^{02}}{\nu} \sim 10^{-6}\ s$, WITH $\delta \sim 0.1$ THE DIMENSIONLESS THICKNESS OF THE FUEL-CONSUMPTION LAYER. THE MAXIMUM TIME STEP BASED ON ACOUSTIC CONSIDERATIONS IS OF ORDER $\Delta t_s \sim \Delta x / c \sim 10^{-8}\ s$, WITH $c \sim 340\ m/s$ THE SPEED OF SOUND WAVES. +10

THEN, $\Delta t = \min(\Delta t_s, \Delta t_c) = 10^{-8}\ s$ +10. THE MINIMUM NUMBER OF TIME STEPS REQUIRED IS $N_t \sim \frac{L}{\bar{U}\Delta t} \sim 10^8$ FOR A 1 FLOW-THROUGH TIME. SINCE $u'/S_L < 1$ AND $\delta_L^0 < \ell_K$, THE BURNER IS OPERATING IN THE WRINKLED FLAMELETS. +10