ME 471: Turbulent Combustion, Spring 2012
Stanford University
Final Exam
Tuesday, June 12

Part I: Questions
(closed books, closed notes, no calculator allowed)
30 minutes

Guidelines: Please turn in a *neat* and *clean* exam solution. Answers should be written in the blank spaces provided in these exam sheets. Vector quantities are denoted in **bold** letters in what follows.

Student's Name: JAVIER URZAY **Student's ID:**

Question 1 (25 pts)

Explain the concept of steady flamelet models in non-premixed turbulent combustion, and how are they typically implemented in RANS of turbulent diffusion flames.

SEE PAGE 26 AND FIGS 5 + 6 OF THE INSTRUCTOR NOTES

+25

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Question 2 (25 pts)

Select true (T) or false (F) for each of the statements in the list provided for each question, or fill in the gaps when indicated by the symbol (........). A complementary and *brief* mathematical proof / sketch of your answer on the back of the page would be welcome, but it is **not** needed in order to get full credit.

2.1 In the mixing of two reactants in a turbulent flow,

a) the scalar dissipation rate is always a sink in the equation for the mean (T/~~F~~)

b) the scalar dissipation rate is minimum in zones where only one of the reactants exists (~~T~~/F)

e) for order-unity Schmidt numbers, the molecular mixing of two reactants typically occurs in time scales which are much shorter than the integral time scale (~~T~~/F)

f) For chemical reactions to occur in combustion, fuel and oxidizer must be mixed at the (...MOLECULAR....) level.

2.2 For single-step chemistry flames in the limit of large activation energies,

a) the strain rate at extinction of a laminar diffusion flame scales as $\sqrt{D/t_F}$, where D is the fuel mass diffusivity and t_F is the flame-transit time or characteristic residence time within the preheat region (T/~~F~~)

b) at diffusion-flame extinction in counterflow mixing layers, the diffusion time through the mixing layer is of the same order as the flame transit time through the preheat region of a stoichiometric laminar planar premixed flame (~~T~~/F)

c) the scalar dissipation rate at stoichiometry χ_{st} represents the (..INVERSE OF THE DIFFUSION TIME..........) through the stoichiometric surface.

d) the characteristic reaction-zone thickness in diffusion flames is of order (...$\frac{\Delta^{-1/3}}{\beta}\delta_M$...............), where (.......Δ.......) is the (.DAMKÖHLER.......) number, (.......β..............) is the (...ZEL'DOVICH.....) number, and (......δ_M.............) is the (.MIXING-LAYER...) thickness.

2.3 In turbulent non-premixed flames,

a) the mixture fraction is a conserved scalar because (...IT IS NEITHER DESTROYED OR GENERATED BY CHEMICAL REACTIONS..).

b) the flamelet-progress-variable-approach (FPVA), a mostly unique mapping of all tabulated chemical states is provided in terms of mixture fraction and progress variable (~~T~~/F)

c) the Burke-Schumann approximation is accurate for calculating extinction of turbulent diffusion flames (T/~~F~~)

d) the problem $\rho\chi(Z)d^2\phi/dZ^2 = \dot{w}_\phi$, for any scalar ϕ, and with χ the scalar dissipation rate and Z the mixture fraction, represents a convection-diffusion problem (T/~~F~~)

e) the probability density function for the mixture fraction is typically modeled by using a (.....BETA..........) function.

+ 25/19 EACH

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Part II: Problem

(open books, open notes, calculator allowed)

1 hour 30 minutes

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Problem (50 pts)

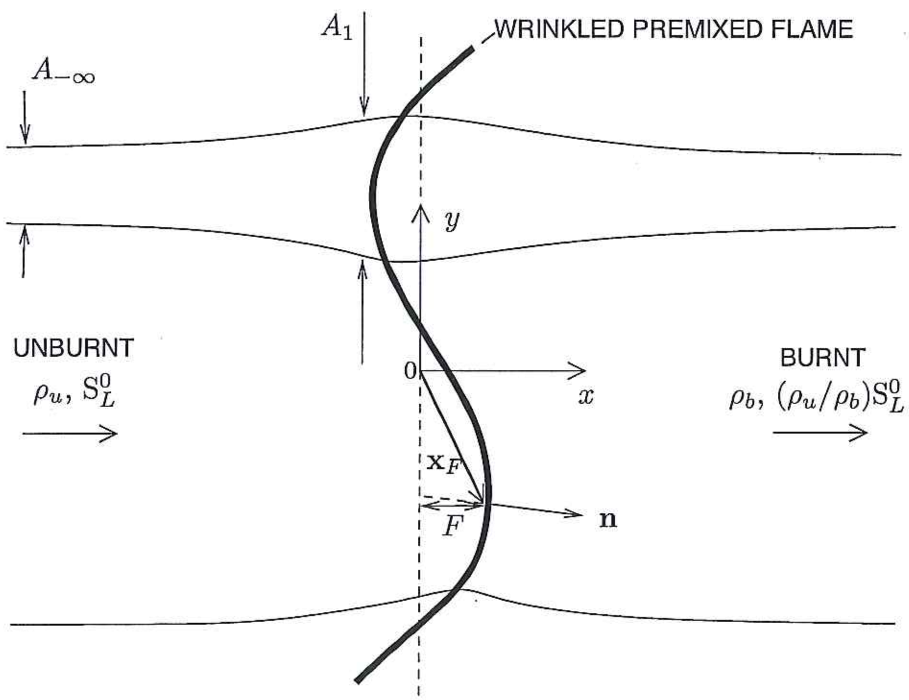

Figure 1: Schematics of the problem

In the hydrodynamic or "Darrieus-Landau" instability of premixed flames, a stream tube with cross-sectional area $A_{-\infty}$ and upstream flow velocity $u_{-\infty} = S_L^0$ widens due to flow divergence ahead of the flame, with S_L^0 the planar propagation velocity. This divergence effect is generated by the thermal expansion at the front that induces a flow component normal to the flame contour. The streamlines become deflected as they cross the front. At large distances from the front, the streamlines are parallel again, but the downstream velocity is $u_{\infty} = (\rho_u/\rho_b)u_{-\infty} > u_{-\infty}$. At the cross section A_1, where the density is still equal to unburnt density ρ_u, the flow velocity is $u_1 = (A_{-\infty}/A_1)u_{-\infty} < u_{-\infty}$ because of mass conservation. Since the unperturbed flame propagates with $u_{-\infty}$, the burning velocity is larger than u_1 and the flame will propagate upstream, thereby enhancing the initial perturbation.

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In the thin-flame limit, in which the flame is treated as a gasdynamic discontinuity, the parameter $\epsilon = k\delta_L^0 \ll 1$ is small, in that the characteristic wavelength of the transverse flame corrugations, $\lambda = 2\pi/k$, with k the wavenumber, is much larger than the unperturbed flame thickness δ_L^0. In the limit $\epsilon \to 0$, the infinitesimally thin flame separates the burnt and unburnt regions, in which the density is constant ($\rho = \rho_u$ in the unburnt zone and $\rho = \rho_b$ in the burnt zone) and in which molecular transport is negligible.

In this problem, the Darrieus-Landau instability is studied in the presence of curvature-correction effects (a.k.a. diffusive-thermal effects) on the propagation velocity, which here is assumed to be given by the linear relationship $S_L = S_L^0(1-\mathcal{L}\kappa)$, where S_L^0 is the planar propagation velocity on the unburnt side, $\mathcal{L}$ is a Markstein length and κ is the local curvature. In order to obtain a dispersion equation that relates the growth rate σ of the instability with the wavenumber k, the thermal-expansion ratio $\alpha = (\rho_u - \rho_b)/\rho_u$ and the Markstein length $\mathcal{L}$, follow the next steps:

1.1 Write down the continuity and momentum conservation equations for the $x-$ and $y-$ components of the flow velocity $\mathbf{v} = u\mathbf{e_x} + v\mathbf{e_y}$, with $\mathbf{e_i}$ unit vectors. Use dimensionless variables $\mathbf{v}^* = \mathbf{v}/S_L^0$, $\rho^* = \rho/\rho_u$, $P^* = P/\rho_u(S_L^0)^2$, $\mathbf{x}^* = \mathbf{x}/\delta_L^0$ and $t^* = t/t_F$. In this formulation, $\delta_L^0 = S_L^0 t_F$ is the flame thickness, t_F is the flame-transit time, P is the pressure and the remaining symbols are defined in the problem statement.

MOLECULAR TRANSPORT IS NEGLIGIBLE ON THE BURNT AND UNBURNT ZONES
+ $\rho = \rho_u$ ON THE UNBURNT REGION
+ $\rho = \rho_b$ ON THE BURNT REGION

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y}$$

+2.5

1.2 What are the boundary conditions for the velocity at $x \to \pm\infty$?

AT $\underline{x} \to +\infty$, $\underline{v} = \frac{\rho_u S_L^0}{\rho_b}\underline{e_x}$; AT $\underline{x} \to -\infty$, $\underline{v} = S_L^0 \underline{e_x}$ +2.5

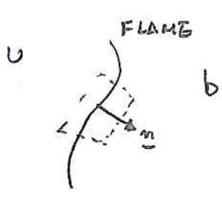
1.3 In the limit $\epsilon \to 0$, the equations written in part 1.1. are only valid on both sides of the flame because of thermal expansion, which induces an infinite density gradient in the first approximation. Jump conditions for the velocity and pressure can be derived in the following manner:

1.3.1 If $\mathbf{x}_F$ is the flame position and $\mathbf{n}$ is the unit normal vector to the front towards the burnt side (see Fig.1), write an equation that states the continuity of the burning rate $\dot{m} = \rho S_L$ across the flame in terms of the relative velocity $(\mathbf{v} - d\mathbf{x}_F/dt)\cdot\mathbf{n}$ on both sides of the flame. (Note: Recall that if $d\mathbf{x}_F/dt$ is the flame speed in the laboratory frame, then the kinematic balance requires that $S_L = (\mathbf{v} - d\mathbf{x}_F/dt)\cdot\mathbf{n}$ on the unburnt region.)

$$\rho_u\left(\underline{v}^{(u)} - \frac{d\underline{x}_F}{dt}\right)\cdot\underline{n} = \rho_b\left(\underline{v}^{(b)} - \frac{d\underline{x}_F}{dt}\right)\cdot\underline{n}$$

+2.5

1.3.2 Additionally, write a momentum balance equation across the flame in terms of the relative velocity $(\mathbf{v} - d\mathbf{x}_F/dt)\cdot\mathbf{n}$ and pressure on both sides of the flame.



$$-\rho_u \underline{v}^{(u)}\left[\underline{v}^{(u)} - \frac{d\underline{x}_F}{dt}\right]\cdot\underline{n} + \rho_b\underline{v}^{(b)}\left[\underline{v}^{(b)} - \frac{d\underline{x}_F}{dt}\right]\cdot\underline{n} = \left(P^{(u)} - P^{(b)}\right)\cdot\underline{n}$$

+2.5

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1.3.3 Use the the thermal-expansion coefficient $\alpha = (\rho_u - \rho_b)/\rho_u$ and the nondimensional variables defined above in order to express the equations obtained in 1.3.1 and 1.3.2 in dimensionless form.

SINCE $\frac{\rho_b}{\rho_u} = \frac{-\rho_u + \rho_u + \rho_b}{\rho_u} = 1 - \alpha$, THE JUMP CONDITIONS BECOME

$$-\alpha \frac{d\underline{x}_F}{dt} = (1-\alpha)\, \underline{v}^{(b)} \cdot \underline{n} - \underline{v}^{(u)} \cdot \underline{n}$$

+2.5

$$\frac{d\underline{x}_F}{dt} \cdot \underline{n} \left(\underline{v}^{(u)} - (1-\alpha)\underline{v}^{(b)} \right) = \underline{v}^{(u)} \underline{v}^{(u)} \cdot \underline{n} - (1-\alpha)\underline{v}^{(b)} \underline{v}^{(b)} \cdot \underline{n} + \left(P^{(u)} - P^{(b)}\right) \underline{n}$$

1.3.4 By employing the ansatz $G = x - F(y,t) = G_0$ at the flame, with G_0 a constant and $F(y,t)$ the displacement function, show that the scalar product $\mathbf{n} \cdot d\mathbf{x}_F/dt$ can be written as

$$\mathbf{n} \cdot \frac{d\mathbf{x}_F}{dt} = \frac{F_t}{\sqrt{1 + F_y^2}}, \tag{1}$$

where $F_t = \partial F/\partial t$ and $F_y = \partial F/\partial y$.

SINCE $\dot{\underline{x}}_F = \dot{x}_F \underline{e}_x + \dot{y}_F \underline{e}_y$, WITH $0 = \dot{x}_F - F_t - F_y \dot{y}_F$ FROM THE ANSATZ, AND ALSO

$$\underline{n} = \frac{\nabla G}{|\nabla G|} = \frac{\underline{e}_x - F_y \underline{e}_y}{(1+F_y^2)^{1/2}}, \quad \text{THEN} \quad \dot{\underline{x}}_F \cdot \underline{n} = \frac{\dot{x}_F - F_y \dot{y}_F}{(1+F_y^2)^{1/2}} = \frac{F_t}{(1+F_y^2)^{1/2}}$$

+2.5

1.3.5 Use equation (1) derived above to show that the jump conditions obtained in part 1.3.3 can be expressed as

$$\alpha F_t = u^{(u)} - (1-\alpha)u^{(b)} - F_y\left[v^{(u)} - (1-\alpha)v^{(b)}\right], \tag{2}$$

$$F_t\left[u^{(u)} - (1-\alpha)u^{(b)}\right] = u^{(u)}\left(u^{(u)} - F_y v^{(u)}\right) - (1-\alpha)u^{(b)}\left[u^{(b)} - F_y v^{(b)}\right] + P^{(u)} - P^{(b)}, \tag{3}$$

$$F_t\left[v^{(u)} - (1-\alpha)v^{(b)}\right] = v^{(u)}\left(u^{(u)} - F_y v^{(u)}\right) - (1-\alpha)v^{(b)}\left[u^{(b)} - F_y v^{(b)}\right] - (P^{(u)} - P^{(b)})F_y, \tag{4}$$

where the superindexes $^{(u)}$ and $^{(b)}$ denote variables evaluated on the unburnt and burnt sides, respectively.

DIRECT PROOF BY SUBSTITUTING (1) IN THE JUMP CONDITIONS AND DECOMPOSING THE MOMENTUM BALANCE INTO X AND Y COMPONENTS.

+2.5

1.4 For $\epsilon \ll 1$, the velocity components, the pressure and the displacement function may be expanded in power series of ϵ as

$$u = U_0 + \epsilon U_1 + O(\epsilon^2), \quad v = \epsilon V_1 + O(\epsilon^2), \quad P = P_0 + \epsilon P_1 + O(\epsilon^2), \quad \text{and} \quad F = \epsilon F_1 + O(\epsilon^2). \tag{5}$$

Similarly, a coordinate transformation

$$x = \xi + F(\eta, \tau), \quad y = \eta, \quad \text{and} \quad t = \tau \tag{6}$$

becomes useful in order to set the flame and the jump conditions at $\xi = 0$.

Use these expansions prove that, to leading order, the jump conditions derived in part 1.3.5 reduce to

$$U_0^{(u)} = 1, \quad U_0^{(b)} = 1/(1-\alpha), \quad \text{and} \quad P_0^{(u)} - P_0^{(b)} = \alpha/(1-\alpha). \tag{7}$$

TO LEADING ORDER, THE JUMP CONDITIONS IN (2)-(4) BECOME

$$U_0^{(u)} - U_0^{(b)}(1-\alpha) = 0 \Rightarrow U_0^{(u)} = 1, \quad U_0^{(b)} = \frac{1}{1-\alpha}$$ 2.5

AND $$0 = U_0^{(u)2} - (1-\alpha)\, U_0^{(b)2} + P_0^{(u)} - P_0^{(b)} \Rightarrow P_0^{(u)} - P_0^{(b)} = \frac{\alpha}{1-\alpha}$$ 2.5

1.5 Use the asymptotic expansions (5) and the coordinate transformation (6) to show that the jump conditions (2)-(4) reduce to

$$\alpha F_{1\tau} = U_1^{(u)} - (1-\alpha)U_1^{(b)}, \tag{8}$$
$$P_1^{(u)} - P_1^{(b)} = 2(U_1^{(b)} - U_1^{(u)}), \tag{9}$$
$$V_1^{(u)} - V_1^{(b)} = \frac{\alpha F_{1\eta}}{1-\alpha}, \tag{10}$$

+2.5

IN THE SECOND APPROXIMATION

(2) BECOMES $\alpha F_{1t} = U_1^{(u)} - (1-\alpha)\, U_1^{(b)}$

(3) BECOMES $0 = 2U_0^{(u)}U_1^{(u)} - 2(1-\alpha)\, U_0^{(b)} U_1^{(b)} + P_1^{(u)} - P_1^{(b)} \rightarrow P_1^{(u)} - P_1^{(b)} = 2(U_1^{(b)} - U_1^{(u)})$

(4) BECOMES: $0 = V_1^{(u)} U_0^{(u)} - (1-\alpha)\, V_1^{(b)} U_0^{(b)} - (P_0^{(u)} - P_0^{(b)})\, F_{1\eta}$

$$\Rightarrow V_1^{(u)} - V_1^{(b)} = \frac{\alpha}{1-\alpha} F_{1\eta}$$

1.6 Use the asymptotic expansions (5) and the coordinate transformation (6) to obtain the second approximation of the asymptotic expansion of the conservation equations written in part 1.1.

$$\left.\begin{aligned} &\frac{\partial U_1}{\partial \xi} + \frac{\partial V_1}{\partial \eta} = 0 \\ &\rho\left(\frac{\partial U_1}{\partial \tau} + U_0 \frac{\partial U_1}{\partial \xi}\right) = -\frac{\partial P_1}{\partial \xi} \\ &\rho\left(\frac{\partial V_1}{\partial \tau} + U_0 \frac{\partial V_1}{\partial \xi}\right) = -\frac{\partial P_1}{\partial \eta} \end{aligned}\right\}$$ 2.5

1.7 If all perturbations vanish far away from the flame, what are the boundary conditions for U_1, V_1 and P_1 at $\xi \to \pm\infty$?

$U_1, V_1, P_1 \to 0$ AT $\xi \to \pm\infty$ 2.5

1.8 To analyze the stability of the solution of the problem obtained in part 1.6, assume a normal mode decomposition for the velocity and pressure disturbances:

$$\begin{pmatrix} U_1 \\ V_1 \\ P_1 \end{pmatrix} = \begin{pmatrix} \hat{U}_1 \\ \hat{V}_1 \\ \hat{P}_1 \end{pmatrix} \exp(\gamma\xi)\exp(\sigma\tau - ik\eta) \tag{11}$$

where $\hat{U}_1$, $\hat{V}_1$ and $\hat{P}_1$ are constants, σ is a growth rate and k is the wavenumber of the transverse perturbations. The parameter γ is obtained from imposing the boundary conditions stated in part 1.7.

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To obtain γ, substitute (11) into the conservation equations written in part 1.6. You may have to solve the determinant of the coefficient matrix and use the boundary conditions stated in part 1.7 and the leading-order jump conditions for U_0 obtained in part 1.4.

$$\left.\begin{array}{l} U_1 = \hat{U}_1 e^{\gamma\xi} e^{\sigma\tau - ik\eta} \\ V_1 = \hat{V}_1 e^{\gamma\xi} e^{\sigma\tau - ik\eta} \\ P_1 = \hat{P}_1 e^{\gamma\xi} e^{\sigma\tau - ik\eta} \end{array}\right\}$$

SUBSTITUTING IN 1.6:

$$\begin{pmatrix} \gamma & -ik & 0 \\ \sigma + U_0\gamma & 0 & \gamma/\rho \\ 0 & \sigma + U_0\gamma & -ik/\rho \end{pmatrix} \begin{pmatrix} \hat{U}_1 \\ \hat{V}_1 \\ \hat{P}_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow \det(A) = 0$$

$$\Rightarrow \frac{1}{\rho}(k^2 - \gamma^2)(\sigma + U_0\gamma) = 0, \quad \text{WITH } U_0^{(u)} = 1 \text{ AND } U_0^{(b)} = 1/_{1-\alpha}$$

+2.5

THEN $\gamma_1 = k\ (\xi < 0)$, $\gamma_2 = -\sigma(1-\alpha)\ (\xi > 0)$, $\gamma_3 = -k\ (\xi > 0)$ ARE THE ONLY VALUES THAT SATISFY THE BCS.

1.9 Using the three values of γ_i $(i = 1, 2, 3)$ obtained in part 1.8, calculate the corresponding eigenvectors $\mathbf{w}_i = (\hat{U}_{1,i}, \hat{V}_{1,i}, \hat{P}_{1,i})$ and the perturbations U_1, V_1 and P_1 on both sides of the flame.

EIGENVECTORS: $\gamma_1 = k\ (\xi < 0)$

$$\begin{pmatrix} k & -ik & 0 \\ \sigma + k & 0 & k \\ 0 & \sigma + k & -ik \end{pmatrix} \begin{pmatrix} w_{11} \\ w_{12} \\ w_{13} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\gamma_2 = -k\ (\xi > 0)$

$$\begin{pmatrix} -k & -ik & 0 \\ \sigma - \frac{k}{1-\alpha} & 0 & -\frac{k}{1-\alpha} \\ 0 & \sigma - \frac{k}{1-\alpha} & -\frac{ik}{1-\alpha} \end{pmatrix} \begin{pmatrix} w_{12} \\ w_{22} \\ w_{23} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\gamma_3 = -\sigma(1-\alpha)\ (\xi > 0)$

$$\begin{pmatrix} -\sigma(1-\alpha) & -ik & 0 \\ 0 & 0 & -\sigma \\ 0 & 0 & -\frac{ik}{(1-\alpha)} \end{pmatrix} \begin{pmatrix} w_{13} \\ w_{23} \\ w_{33} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \underline{W_1} = a \begin{pmatrix} 1 \\ -i \\ -1 - \sigma/k \end{pmatrix} \qquad \underline{W_2} = b \begin{pmatrix} 1 \\ +i \\ -1 + \frac{(1-\alpha)\sigma}{k} \end{pmatrix} \qquad \underline{W_3} = c \begin{pmatrix} 1 \\ i(1-\alpha)\sigma/k \\ 0 \end{pmatrix}$$

+2.5

THEN: $U_1 = a e^{k\xi} e^{\sigma\tau - ik\eta}$, $V_1 = -ia\, e^{k\xi} e^{\sigma\tau - ik\eta}$, $P_1 = -(1 + \sigma/k)\, a e^{k\xi} e^{\sigma\tau - ik\eta}$ FOR $\xi < 0$

$U_1 = \left[b e^{-k\xi} + c e^{-\sigma(1-\alpha)\xi}\right] e^{\sigma\tau - ik\eta}$, $V_1 = \left[+ib e^{-k\xi} + i(1-\alpha)\sigma/k\, c e^{-\sigma(1-\alpha)\xi}\right] e^{\sigma\tau - ik\eta}$, $P_1 = \left[-\left(1 - (1-\alpha)\frac{\sigma}{k}\right) b e^{-\sigma(1-\alpha)\xi}\right] e^{\sigma\tau - ik\eta}$ FOR ξ

1.10 Since, as stated above, in this analysis the propagation velocity S_L is a function of the curvature κ, $S_L = S_L^0(1 - \mathcal{L}\kappa)$, or equivalently,

$$\left(\mathbf{v}^{(u)} - \frac{d\mathbf{x}_F}{dt}\right) \cdot \mathbf{n} = (1 - \alpha)\left(\mathbf{v}^{(b)} - \frac{d\mathbf{x}_F}{dt}\right) \cdot \mathbf{n} = 1 - \mathrm{Ma}\kappa \qquad (12)$$

in dimensionless notation, answer the following questions:

1.10.1 What is the definition and physical meaning of the Markstein number Ma in this formulation?

SINCE DISTANCES ARE NONDIMENSIONALIZED WITH δ_L^0, THEN $\delta_L^0 k = k^*$ IS THE DIMENSIONLESS CURVATURE AND $Ma = \frac{\mathcal{L}}{\delta_L^0}$ IS THE RATIO OF THE MARKSTEIN LENGTH TO THE FLAME THICKNESS

+2.5

1.10.2 Prove that the flame curvature $\kappa = -\nabla \cdot \mathbf{n}$ can be written as

$$\kappa = \frac{F_{\eta\eta}}{(1 + F_\eta^2)^{3/2}}. \qquad (13)$$

SINCE $\underline{n} = \frac{e_x - F_y e_y}{(1 + F_y^2)^{1/2}}$ THEN $\kappa = -\nabla \cdot \underline{n} = \frac{F_{yy}(1 + F_y^2) - F_y^2 F_{yy}}{(1 + F_y^2)^{3/2}} = \frac{F_{yy}}{(1 + F_y^2)^{3/2}} = \frac{F_{\eta\eta}}{(1 + F_\eta^2)^{3/2}}$

+2.5

1.10.3 Using the expansions (5), and the results (12) and (13), prove that the velocity perturbations are related to F by the equations

$$U_1^{(u)} = F_{1\tau} - \mathrm{Ma}F_{1\eta\eta}, \quad \text{and} \quad U_1^b = F_{1\tau} - \frac{\mathrm{Ma}F_{1\eta\eta}}{1 - \alpha} \qquad (14)$$

on both sides of the flame.

TO LEADING ORDER (12) BECOMES $U_0^{(u)} = (1-\alpha)U_0^{(b)} = 1$

AND TO SECOND ORDER: $U_1^{(u)} - F_{1t} = (1-\alpha)(U_1^{(b)} - F_{1t}) = -Ma\, F_{1\eta\eta}$

+2.5

$$\Rightarrow \left\{\begin{array}{l} U_1^{(u)} = F_{1t} - Ma\, F_{1\eta\eta} \\ U_1^{(b)} = F_{1t} - \frac{Ma\, F_{1\eta\eta}}{1-\alpha} \end{array}\right.$$

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DDG | DEFINING $F_1 = \hat{F}_1 e^{\sigma t - i \xi_1 k}$

1.11 Finally, obtain the dispersion relation $\sigma = \sigma(k, \alpha, \mathrm{Ma})$ by i) substituting the perturbations obtained in part 1.9 into the jump conditions (8)-(10), ii) using relations (14), and iii) solving the resulting linear system.

FOLLOWING THE PROCEDURE ABOVE, THE SYSTEM OF EQUATIONS

$$\left.\begin{aligned} a &= \sigma \hat{F}_1 + k^2 Ma \hat{F}_1 \\ b+c &= \left(\sigma \hat{F}_1 + k^2 Ma \hat{F}_1\right) / (1-\alpha) \\ a+b+(1-\alpha)\frac{\sigma}{k} c &= \frac{\alpha k \hat{F}_1}{1-\alpha} \\ 0 &= 2c + a\left(\frac{\sigma}{k} - 1\right) + b\left(1 + (1-\alpha)\frac{\sigma}{k}\right) \end{aligned}\right\}$$

WHICH CAN BE SOLVED FOR

$\hat{a} = \frac{a}{\hat{F}_1}$, $\hat{b} = \frac{b}{\hat{F}_1}$, $\hat{c} = \frac{c}{\hat{F}_1}$ AND $\sigma = \sigma(k)$

TO GIVE $$\sigma = \frac{k}{2-\alpha}\left\{\left(1 + k^2 Ma^2 - \frac{2kMa}{1-\alpha} + \frac{\alpha(2-\alpha)}{1-\alpha}\right)^{1/2} - (1 + kMa)\right\}$$

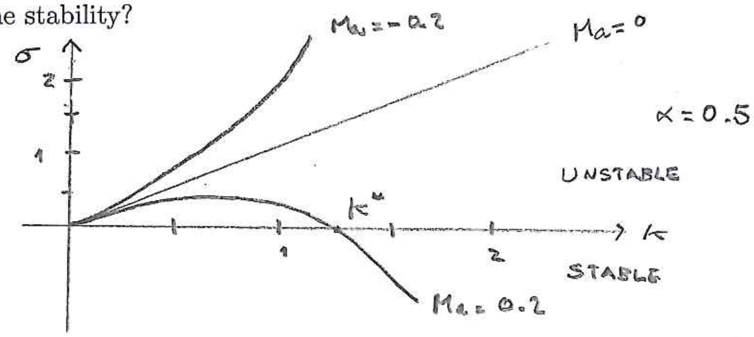
+2.5

1.12 Assuming that in part 1.11 you have obtained the result

$$\sigma = \frac{k}{2-\alpha}\left\{\sqrt{1 + k^2\mathrm{Ma}^2 - \frac{2k\mathrm{Ma}}{1-\alpha} + \frac{\alpha(2-\alpha)}{(1-\alpha)}} - (1 + k\mathrm{Ma})\right\}, \tag{15}$$

for the growth rate σ, sketch σ as a function of k for fixed $\alpha = 0.5$ and for a) $\mathrm{Ma} = 0$, b) $\mathrm{Ma} = 0.2$ and c) $\mathrm{Ma} = -0.2$. According to simplified single-step chemistry analyses at large activation energies, what is the critical Lewis number Le_c for which Ma becomes negative? What is the impact of positive and negative Ma on flame stability?

+5



FOR $Ma \leq 0$ THE FLAME IS ALWAYS UNSTABLE BY HYDRODYNAMIC + DIFFUSIVE-THERMAL INSTABILITIES

IN PARTICULAR, $Ma < 0$ WHEN $Le < Le_c = 1 - \frac{1}{\beta}\left[\frac{2}{1-\alpha}\ln\left(\frac{1}{1-\alpha}\right)\right] \Big/ \int_0^{\frac{\alpha}{1-\alpha}} \frac{\ln(1+x)}{x}\,dx$

WITH $Le_c \sim 0.5$ FOR PRACTICAL APPLICATIONS

FOR $Ma > 0$ THE FLAME IS HYDRODYNAMICALLY UNSTABLE AT LONG WAVELENGTHS BUT DIFFUSIVE-THERMALLY STABLE AT SHORT WAVELENGTHS, WITH k^* A CUTOFF WAVENUMBER GIVEN BY $k^* = \frac{1}{2}\frac{\alpha}{Ma}$.