

SUPPLEMENT TO “CONTRACT PRICING IN CONSUMER  
CREDIT MARKETS”

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In this appendix, we describe the details associated with estimation of the model described in Sections 4 and 5. The first part of the Appendix describes the demand side (Section 4) and the second part describes the supply side (pricing) (Section 5).

APPENDIX B: DETAILS OF THE ESTIMATION

B.1. *Demand Estimation*

HERE WE PRESENT the likelihood function used to estimate the parameters of the demand model. The model can be thought of as a system of four “standard” equations: (i) a tobit down payment equation (with censoring at the minimum down), (ii) a probit purchase equation, (iii) a tobit repayment equation (with censoring at full payment or at the end of our sample), and (iv) a linear price negotiation equation. The model’s four equations are given by

$$(B.1) \quad D_i^* = x_i' \beta_x + \beta_p p_i + \varepsilon_{Di},$$

$$(B.2) \quad U_i^* = x_i' \alpha_x + \alpha_p p_i + \alpha_d I(D_i^* \leq d_i) d_i + \varepsilon_{Di},$$

$$(B.3) \quad \ln(s_i^*) = x_i' \gamma_x + (p_i - D_i) \gamma_L + \varepsilon_{Si},$$

$$(B.4) \quad p_i = x_i' \lambda_x + \lambda_l l_i + \varepsilon_{Pi}.$$

The system’s endogenous variables are negotiated price  $p_i$ , desired down payment  $D_i^*$ , observed down payment  $D_i$  (which is a nonlinear function of  $D_i^*$ ), latent purchase utility net of outside option  $U_i^*$  (here we normalize the outside option term  $\bar{U}_i$  to zero to simplify notation), and the (logarithm of the) fraction of payments made  $s_i^* = S_i^*/T_i$ , where  $S_i^*$  is the number of months of successful payment and  $T_i$  is the loan length in months. The system’s exogenous variables are list price  $l_i$ , minimum down payment  $d_i$ , and a vector of offer, car, applicant, location, and time characteristics  $x_i$ , which is common to all equations. The indicator function in the second (purchase) equation is equal to 1 if an applicant’s optimal down payment is constrained by the minimum down requirement, and is equal to 0 otherwise. In each equation,  $i$  indexes a customer.

The variables  $D_i^*$ ,  $U_i^*$ , and  $s_i^*$  are not observed for all applicants. In particular,  $D_i^*$  is not observed if a borrower does not purchase or makes exactly the required minimum down payment, and  $s_i^*$  is not observed if the applicant does not purchase or if the loan repayment is censored (either due to full payment or due to the end of our sample). The latent utility from purchase  $U_i^*$  is never observed. We discuss the relationship between these latent variables and their

observable counterparts  $D_i$ , a purchase indicator  $Q_i$ , and the observed fraction of payments made  $s_i = S_i/T_i$  in more detail below.

The system's unobservables  $(\varepsilon_Q, \varepsilon_D, \varepsilon_S, \varepsilon_p)$  are assumed to be distributed jointly normal with distribution function  $f(\varepsilon_Q, \varepsilon_D, \varepsilon_S, \varepsilon_p) \sim N(0, \Sigma)$ , where  $\Sigma$  is the variance–covariance matrix given by

$$(B.5) \quad \Sigma = \begin{pmatrix} \sigma_Q^2 & \rho_{QD}\sigma_Q\sigma_D & \rho_{QS}\sigma_Q\sigma_S & \rho_{Qp}\sigma_Q\sigma_p \\ \rho_{QD}\sigma_Q\sigma_D & \sigma_D^2 & \rho_{DS}\sigma_D\sigma_S & \rho_{Dp}\sigma_D\sigma_p \\ \rho_{QS}\sigma_Q\sigma_S & \rho_{DS}\sigma_D\sigma_S & \sigma_S^2 & \rho_{Sp}\sigma_S\sigma_p \\ \rho_{Qp}\sigma_Q\sigma_p & \rho_{Dp}\sigma_D\sigma_p & \rho_{Sp}\sigma_S\sigma_p & \sigma_p^2 \end{pmatrix}.$$

This joint density allows us to derive the likelihood function  $\mathcal{L}(D_i, Q_i, s_i, p_i | l_i, d_i, x_i)$ . We begin by rewriting the joint density as a product of an unconditional density and three conditional densities:

$$(B.6) \quad f(\varepsilon_Q, \varepsilon_D, \varepsilon_S, \varepsilon_p) = f(\varepsilon_S | \varepsilon_Q, \varepsilon_D, \varepsilon_p) f(\varepsilon_Q | \varepsilon_D, \varepsilon_p) f(\varepsilon_D | \varepsilon_p) f(\varepsilon_p).$$

Since the Jacobian of the transformation of  $(\varepsilon_Q, \varepsilon_D, \varepsilon_S, \varepsilon_p)'$  to  $(D_i^*, U_i^*, \ln(s_i^*), p_i)'$  is 1, we can write the joint density of  $(D_i^*, U_i^*, \ln(s_i^*), p_i)'$  as

$$(B.7) \quad \begin{aligned} f(D_i^*, U_i^*, \ln(s_i^*), p_i | l_i, d_i, x_i) &= f(\ln(s_i^*) - x_i' \gamma_x - (p_i - D_i) \gamma_L | p_i^*, U_i^*, D_i^*, l_i, d_i, x_i) \\ &\quad \times f(U_i^* - x_i' \alpha_x - \alpha_p p_i - \alpha_d I(D_i^* \leq d_i) d_i | p_i, D_i^*, l_i, x_i) \\ &\quad \times f(D_i^* - x_i' \beta_x - \beta_p p_i | p_i, l_i, x_i) \\ &\quad \times f(p_i - x_i' \lambda_x - \lambda_l l_i | l_i, x_i). \end{aligned}$$

If  $D_i^*$ ,  $U_i^*$ , and  $\ln(s_i^*)$  were observed for all applicants, this expression would provide the likelihood function for the data. However, since  $D_i^*$  and  $s_i^*$  are not always observed and  $U_i^*$  is never observed, we must rewrite this expression in terms of the observable endogenous variables  $D_i$ ,  $Q_i$ , and  $s_i$ . We proceed in five steps. First, we derive the likelihood of observing a given negotiated price. Since we assume price is observed for all applicants (as mentioned in Section 3.3, we impute cars and prices for nonbuyers by matching them to cars and prices at the same time, dealership, and income category), this step is straightforward. The probability of observing a negotiated price  $p_i$  is given simply by

$$(B.8) \quad p_{p_i} = \phi \left[ \frac{p_i - x_i' \lambda_x - \lambda_l l_i}{\sigma_p} \right],$$

where  $\phi$  denotes the standard normal probability density function (p.d.f.). We do not account for censoring at list price, since this would require integration over  $\varepsilon_{p_i}$  for some (but only few) observations, thus complicating the derivation of the likelihood function with limited benefit.

Second, conditional on a negotiated price, we derive the likelihood of observing a borrower's chosen down payment. We define the observed down payment  $D_i$  as

$$(B.9) \quad D_i = \begin{cases} D_i^* = x'_i \beta_x + \beta_p p_i + \varepsilon_{Di}, & \text{if } D_i^* \geq d_i, \\ d_i, & \text{if } D_i^* < d_i. \end{cases}$$

The first case applies to applicants who purchase a car and make a down payment above the minimum; the second case applies to applicants who purchase a car and make the minimum down payment. In the former case,  $D_i^*$  is observed, and the likelihood of observing a given  $D_i^*$  above the minimum is

$$(B.10) \quad p_{D_i=D_i^*|\varepsilon_{pi}} = \Pr(D_i^* = x'_i \beta_x + \beta_p p_i + \varepsilon_{Di} | \varepsilon_{pi}) \\ = f_{\varepsilon_{Di}|\varepsilon_p}(D_i^* - x'_i \beta_x - \beta_p p_i),$$

where  $f$  is a (conditional) normal distribution function, with mean and variance given by  $\mu_{\varepsilon_{Di}|\varepsilon_p} = \frac{\rho_{Dp}\sigma_D}{\sigma_p} \varepsilon_p$  and  $\sigma_{\varepsilon_{Di}|\varepsilon_p}^2 = \sigma_D^2(1 - \rho_{Dp}^2)$ , respectively. Note that due to the correlation between  $\varepsilon_{Di}$  and  $\varepsilon_{pi}$ , the moments of this distribution depend on  $\varepsilon_{pi}$ .

In the latter case,  $D_i^*$  is unobserved, and the likelihood of observing a minimum down payment is defined by a cumulative distribution function. The likelihood of observing a minimum down payments is

$$(B.11) \quad p_{D_i=d_i|\varepsilon_{pi}} = \Pr(D_i^* < x'_i \beta_x + \beta_p p_i + \varepsilon_{Di} | \varepsilon_{pi}) = F_{\varepsilon_{Di}|\varepsilon_p}(d_i - x'_i \beta_x - \beta_p p_i),$$

where  $F$  is the (conditional) normal cumulative distribution function (c.d.f.) with mean and variance given above. Note that for borrowers who do not purchase, we do not observe a down payment, and accordingly these borrowers' down payment choices do not enter the likelihood function.

Third, we derive the likelihood of observing an applicant's decision to purchase a car. As in a standard discrete choice probit model, we define the observed variable  $Q_i$  to be equal to 1 if the borrower purchases and equal to 0 otherwise. That is,

$$(B.12) \quad Q_i = \begin{cases} 1, & \text{if } U_i^* = x'_i \alpha_x + \alpha_p p_i + \alpha_d I(D_i^* \leq d_i) d_i + \varepsilon_{Qi} \geq 0, \\ 0, & \text{otherwise,} \end{cases}$$

where  $F_{\varepsilon_{Qi}|\varepsilon_p, \varepsilon_D}$  is the (conditional) cumulative normal distribution, with mean  $\mu_{\varepsilon_{Qi}|\varepsilon_p, \varepsilon_D}$  and variance  $\sigma_{\varepsilon_{Qi}|\varepsilon_p, \varepsilon_D}^2$ . These moments can be computed straightforwardly from the covariance matrix  $\Sigma$  and the properties of the multivariate normal distribution. As stated above, the indicator function is equal to 1 if an applicant's optimal down payment is constrained by the minimum down requirement, and is equal to 0 otherwise, meaning we can think of the coefficient

$\alpha_d$  as the (average) shadow price of the down payment constraint, conditional on it being binding.

Calculation of this likelihood is complicated by the fact that  $\varepsilon_D$  is not observed for borrowers who put down exactly the minimum. Thus, for these borrowers, we cannot directly calculate the moments of the conditional distribution function  $F_{\varepsilon_Q|\varepsilon_p, \varepsilon_D}$  and, instead, must integrate over all  $\varepsilon_{Di}$  that result in an observed minimum down payment. This yields the expression for the likelihood of sale:

$$(B.13) \quad P_{Q_i=1|D_i=d_i, \varepsilon_{pi}} = \int_{-\infty}^{d_i - x'_i \beta_x - \beta_p p_i} F_{\varepsilon_Q|\varepsilon_p, \varepsilon_D}(x'_i \alpha_x + \alpha_p p_i + \alpha_d d_i) f(\varepsilon_p, \varepsilon_D) d\varepsilon_D.$$

For applicants who do not purchase, the likelihood of observing a nonpurchase is also complicated by the fact that  $\varepsilon_i$  is not observed for applicants who do not purchase. This is important due to the interaction term in the equation for  $U_i^*$ , which is equal to 1 if the applicant would (if they purchased) put down the minimum and is equal to 0 if the applicant would put down more than the minimum. Since down payment is not observed for applicants who do not purchase, we write the likelihood of observing a nonpurchase in terms of their probability of making the minimum down payment:

$$(B.14) \quad P_{Q_i=0|\varepsilon_{pi}} = 1 - \left\{ P_{D_i=d_i|\varepsilon_{pi}} \int_{-\infty}^{d_i - x'_i \beta_x - \beta_p p_i} F_{\varepsilon_Q|\varepsilon_p, \varepsilon_D}(x'_i \alpha_x + \alpha_p p_i + \alpha_d d_i) \times f(\varepsilon_{pi}, \varepsilon_D) d\varepsilon_D + (1 - P_{D_i=d_i|\varepsilon_{pi}}) \int_{d_i - x'_i \beta_x - \beta_p p_i}^{\infty} F_{\varepsilon_Q|\varepsilon_p, \varepsilon_D}(x'_i \alpha_x + \alpha_p p_i) \times f(\varepsilon_{pi}, \varepsilon_D) d\varepsilon_D \right\}.$$

The term in brackets is the probability of sale for nonbuyers. The first term in the sum is equal to the probability of a nonbuyer putting the minimum down times the probability of purchase conditional on putting the minimum down; the second term is equal to the probability of a nonbuyer putting more than the minimum down times the probability of purchase conditional on putting more than the minimum down. In both cases, the probability of purchase requires integration over the unobservable  $\varepsilon_i$ . All integrals in the likelihood function are computed by simulation. For the above integrals, we simulate values of  $\varepsilon_{Di}$  from the joint distribution  $f(\varepsilon_p, \varepsilon_D)$  in the regions given by the limits of the integrals, use these simulated values to compute likelihoods for each purchase outcome, and average these likelihoods across simulations.

Fourth, we derive the likelihood of observing the loan repayment outcomes: no payments, default after at least one payment, and payments censored due to full payment or the end of our sample, conditional on a negotiated price, a purchase decision, and a financing decision. We begin with equation (B.3) and define the censoring point  $c_i \in (0, 1]$  as the fraction of the loan observed before the end of our data. The observed fraction of payments made,  $s_i$ , is then

$$(B.15) \quad s_i = \begin{cases} s_i^* = \exp(x'_i \gamma_x + (p_i - D_i) \gamma_L + \varepsilon_{Si}), & \text{if } s_i^* < c_i, \\ c_i, & \text{if } c_i \leq s_i^*, \end{cases}$$

where  $\varepsilon_{Si}$  is correlated with  $\varepsilon_{Qi}$ ,  $\varepsilon_{Di}$ , and  $\varepsilon_{pi}$ . The first case applies to buyers with observed default (including default before making a single payment); the second case applies to buyers with censored repayment. For loans that have been repaid in full,  $c_i = 1$ . To account for the correlation between unobservables, we calculate the probability of each repayment outcome conditional on  $\varepsilon_{Qi}$ ,  $\varepsilon_{Di}$ , and  $\varepsilon_{pi}$ . For a given  $\varepsilon_{Qi}$ ,  $\varepsilon_{Di}$ , and  $\varepsilon_{pi}$ , the likelihood of observing censored payments is

$$(B.16) \quad \begin{aligned} p_{s_i=c_i|\varepsilon_{Qi}, \varepsilon_{Di}, \varepsilon_{pi}} &= \Pr(s_i = c_i | \varepsilon_{Qi}, \varepsilon_{Di}, \varepsilon_{pi}) = \Pr(s_i^* \geq c_i | \varepsilon_{Qi}, \varepsilon_{Di}, \varepsilon_{pi}) \\ &= \Pr(\exp(x'_i \gamma_x + (p_i - D_i) \gamma_L + \varepsilon_{Si}) \geq c_i | \varepsilon_{Qi}, \varepsilon_{Di}, \varepsilon_{pi}) \\ &= \Pr(\varepsilon_{Si} < -\ln(c_i) + x'_i \gamma_x + (p_i - D_i) \gamma_L | \varepsilon_{Qi}, \varepsilon_{Di}, \varepsilon_{pi}) \\ &= F_{\varepsilon_S | \varepsilon_Q, \varepsilon_D, \varepsilon_P}(-\ln(c_i) + x'_i \gamma_x + (p_i - D_i) \gamma_L), \end{aligned}$$

where  $F_{\varepsilon_S | \varepsilon_Q, \varepsilon_D, \varepsilon_P}$  is the (conditional) cumulative normal distribution, with mean  $\mu_{\varepsilon_S | \varepsilon_Q, \varepsilon_D, \varepsilon_P}$  and variance  $\sigma_{\varepsilon_S | \varepsilon_Q, \varepsilon_D, \varepsilon_P}^2$  computed based on the covariance matrix  $\Sigma$  and the properties of the multivariate normal distribution.

Similarly, for a given  $\varepsilon_{Qi}$ ,  $\varepsilon_{Di}$ , and  $\varepsilon_{pi}$ , the likelihood of observing no payments is

$$(B.17) \quad \begin{aligned} p_{s_i=0|\varepsilon_{Qi}, \varepsilon_{Di}, \varepsilon_{pi}} &= \Pr(s_i = 0 | \varepsilon_{Qi}, \varepsilon_{Di}, \varepsilon_{pi}) = \Pr(s_i^* < g | \varepsilon_{Qi}, \varepsilon_{Di}, \varepsilon_{pi}) \\ &= \Pr(\exp(x'_i \gamma_x + (p_i - D_i) \gamma_L + \varepsilon_{Si}) < g | \varepsilon_{Qi}, \varepsilon_{Di}, \varepsilon_{pi}) \\ &= \Pr(\varepsilon_{Si} < \ln(g) - x'_i \gamma_x - (p_i - D_i) \gamma_L | \varepsilon_{Qi}, \varepsilon_{Di}, \varepsilon_{pi}) \\ &= F_{\varepsilon_S | \varepsilon_Q, \varepsilon_D, \varepsilon_P}(\ln(g) - x'_i \gamma_x - (p_i - D_i) \gamma_L), \end{aligned}$$

where  $g$  represents the fraction of the loan paid in each installment. The probability of zero payments thus equals the probability of default before the first payment is made. In the estimation, we assume loan payments are made in 20 installments, meaning  $g = 0.05$ .

The likelihood of observing payments through  $s_i^*$  prior to the censoring point is

$$(B.18) \quad p_{s_i=s_i^*|\varepsilon_{Q_i}, \varepsilon_{D_i}, \varepsilon_{p_i}} = F_{\varepsilon_S|\varepsilon_Q, \varepsilon_D, \varepsilon_P}(\ln(s_i^* + g) - x'_i \gamma_x - (p_i - D_i) \gamma_L)$$

$$(B.19) \quad - F_{\varepsilon_S|\varepsilon_Q, \varepsilon_D, \varepsilon_P}(\ln(s_i^*) - x'_i \gamma_x - (p_i - D_i) \gamma_L).$$

We compute the likelihood using a difference in c.d.f.'s rather than a p.d.f. to account for the discrete nature of payment timing.

Note that the above derivations assumed that  $\varepsilon_{Q_i}$ ,  $\varepsilon_{D_i}$ , and  $\varepsilon_{p_i}$  were given. In practice, however,  $\varepsilon_{Q_i}$  is unobserved and  $\varepsilon_{D_i}$  is unobserved whenever a minimum down payment is made. This means that computation of the likelihood requires integration over  $\varepsilon_{Q_i}$  or both  $\varepsilon_{Q_i}$  and  $\varepsilon_{D_i}$ . In the former case, the likelihood of observing a repayment outcome  $s_i$  is given by

$$(B.20) \quad p_{s_i} = \int_{-(x'_i \alpha_x + \alpha_i p_i + \alpha_d d_i)}^{\infty} p_{s_i} f(\varepsilon_{Q_i}, \varepsilon_{D_i}, \varepsilon_{p_i}) d\varepsilon_{Q_i}.$$

In the latter case, the likelihood is given by

$$(B.21) \quad p_{s_i} = \int_{-\infty}^{d_i - x'_i \beta_x - \beta_p p_i} \int_{-(x'_i \alpha_x + \alpha_i p_i + \alpha_d d_i)}^{\infty} p_{s_i} f(\varepsilon_{Q_i}, \varepsilon_{D_i}, \varepsilon_{p_i}) \varepsilon_{Q_i} \varepsilon_{D_i}.$$

The latter expression differs from the first through integration over  $\varepsilon_{D_i}$ . As before, the integrals are computed by simulation. That is, we simulate values of  $\varepsilon_{Q_i}$  and  $\varepsilon_{D_i}$  from the joint distribution  $f(\varepsilon_{Q_i}, \varepsilon_{D_i}, \varepsilon_{p_i})$  in the regions given by the limits of the integrals, use these simulated values to compute likelihoods for each repayment outcome, and average these likelihoods across simulations.

The final step is to combine the set of negotiated price probabilities, down payment probabilities, purchase probabilities, and loan repayment probabilities into a full likelihood function,  $\mathcal{L}(p_i, D_i, Q_i, s_i | d_i, l_i, x_i)$ . Before writing this likelihood function, we define the seven possible outcomes observed in the data.

- $I_0$ : no sale
- $I_1$ : sale, down payment above minimum, no payments
- $I_2$ : sale, down payment above minimum, censored payments
- $I_3$ : sale, down payment above minimum, observed default after at least one payment
- $I_4$ : sale, minimum down payment, no payments
- $I_5$ : sale, minimum down payment, censored payments
- $I_6$ : sale, minimum down payment, observed default after at least one payment.

Using the notation  $i \in I$  to indicate that applicant  $i$  chose outcome  $I$ , we can write the full log-likelihood function for the data as

$$\begin{aligned}
 \text{(B.22)} \quad \log \mathcal{L} = & \sum_i \log(p_{p_i}) + \sum_{i \in I_0} \log(p_{Q_i=0|p_i}) + \sum_{i \notin I_0} \log(p_{Q_i=1|p_i}) \\
 & + \sum_{i \in I_1} \{\log(p_{D_i=D_i^*|p_i}) + \log(p_{s_i=0|p_i, D_i=D_i^*})\} \\
 & + \sum_{i \in I_4} \{\log(p_{D_i=d_i|p_i}) + \log(p_{s_i=0|p_i, D_i=d_i})\} \\
 & + \sum_{i \in I_2} \{\log(p_{D_i=D_i^*|p_i}) + \log(p_{s_i=c_i|p_i, D_i=D_i^*})\} \\
 & + \sum_{i \in I_5} \{\log(p_{D_i=d_i|p_i}) + \log(p_{s_i=c_i|p_i, D_i=d_i})\} \\
 & + \sum_{i \in I_3} \{\log(p_{D_i=D_i^*|p_i}) + \log(p_{s_i=s_i^*|p_i, D_i=D_i^*})\} \\
 & + \sum_{i \in I_6} \{\log(p_{D_i=d_i|p_i}) + \log(p_{s_i=s_i^*|p_i, D_i=d_i})\}.
 \end{aligned}$$

Our estimates of the parameters  $\lambda_x$ ,  $\lambda_l$ ,  $\beta_x$ ,  $\beta_p$ ,  $\alpha_x$ ,  $\alpha_p$ ,  $\alpha_d$ ,  $\gamma_x$ ,  $\gamma_L$ , and  $\Sigma$  maximize this log-likelihood function.

## B.2. Supply Estimation

In this section, we derive the moment conditions used to estimate the supply-side parameter  $\psi$  (see Section 5). We first derive detailed expressions for profit per sale  $\pi_i$  and expected profit per applicant  $\Pi_i$ , and describe how these quantities are computed in practice. We then derive a set of moment conditions from the firm's optimal pricing problem and describe the method used to estimate the (single) supply-side parameter.

When the number of payments made,  $S_i = s_i T_i$ , is known, profit per sale is given by

$$\text{(B.23)} \quad \pi_i = D_i + \frac{\frac{1}{\kappa}(1 - e^{-\kappa S_i})}{\frac{1}{z_i}(1 - e^{-z_i T_i})} (p_i - D_i) + e^{-\kappa S_i} k_i(S_i) - C_i.$$

The first term in the equation is the borrower's down payment, the second term is the present value of loan payments assuming the lender discounts payments at a rate  $\kappa$ , the third term is the present value of expected vehicle recovery,

and the fourth term is the cost of the loan. The expected recovery value is estimated from the data using the specification described in Section 5.1. The cost of the loan is  $C_i = c_i + \psi$ , where  $c_i$  is the (observed) cost of the car and  $\psi$  is the (unobserved) indirect cost of originating the loan. The latter cost is estimated below.

When  $S_i$  is unknown, we can integrate over  $S_i$  to calculate expected profit per sale conditional on  $\varepsilon_{Qi}$  and  $\varepsilon_{Di}$ . Expected profit per sale conditional on  $\varepsilon_{Qi}$  and  $\varepsilon_{Di}$  is

$$(B.24) \quad \mathbb{E}[\pi_i | \varepsilon_{Qi}, \varepsilon_{Di}] = p_{S_i=0} \mathbb{E}[\pi_i | S_i = 0] + p_{S_i=1} \mathbb{E}[\pi_i | S_i = 1]$$

$$(B.25) \quad + \int_0^1 p_{S_i=S_i^*} \mathbb{E}[\pi_i | S_i^*] dS_i^* \\ = p_{S_i=0} (D_i + k_i - C_i) \\ + p_{S_i=1} \left( D_i + \frac{\frac{1}{z_i} (1 - e^{-\kappa T_i})}{\frac{1}{z_i} (1 - e^{-z_i T_i})} (p_i - D_i) - C_i \right)$$

$$(B.26) \quad + \int_0^1 p_{S_i=S_i^*} \left( D_i + \frac{\frac{1}{z_i} (1 - e^{-\kappa S_i^* T_i})}{\frac{1}{z_i} (1 - e^{-z_i T_i})} (p_i - D_i) \right. \\ \left. + e^{-\kappa S_i^* T_i} k_i - C_i \right) dS_i^*,$$

where  $p_{S_i=0}$ ,  $p_{S_i=S_i^*}$ , and  $p_{S_i=1}$  are defined in Section B.1 above. The first term on the right-hand side of the equation is equal to the probability of zero payments times the net revenue from zero payments, the second term is equal to the probability of full payment times the net revenue from full payment, and the third term is equal to the expected net revenue from between 1 and  $T_i - 1$  payments.

The above expression for expected revenue can be used when  $\varepsilon_{Qi}$  and  $\varepsilon_{Di}$  are known. However,  $\varepsilon_{Qi}$  is a latent variable that is never observed and  $\varepsilon_{Di}$  is observed only if a borrower puts down more than the minimum. To compute the expected profit conditional on sale for a borrower who purchases and puts down the minimum, we must integrate over the region of sale and the region of minimum down:

$$(B.27) \quad \mathbb{E}[\pi_i | U_i^* \geq 0] = \int_{-(x_i' \alpha_x + \alpha_p p_i + \alpha_d d_i)}^{\infty} \int_{-\infty}^{d_i - x_i' \beta_x + \beta_p p_i} \mathbb{E}[\pi_i | \varepsilon_{Di}, \varepsilon_{Qi}] d\varepsilon_{Di} d\varepsilon_{Qi}.$$

For borrowers who purchase and put down more than the minimum, the second integral is removed and the lower limit of integration over  $\varepsilon_{Q_i}$  changes to account for the lack of a constraining minimum down payment.

Expected profits per applicant are equal to the probability of sale times the expected revenue conditional on sale, or

$$(B.28) \quad \Pi_i(x_i, p_i, d_i; \psi) = \Pr[Q(x_i, p_i, d_i, \varepsilon_i) = 1] \\ \times \mathbb{E}[\pi_i(x_i, p_i, d_i; \psi) | Q(x_i, p_i, d_i, \varepsilon_i) = 1].$$

This is the quantity the firm seeks to maximize. To compute supply-side estimates, we compute the first-order condition of this equation with respect to uniform (across all credit categories and time periods) changes in the lender's required minimum down payment and find the value of  $\psi$  that makes this first-order condition equal to zero. That is, we find  $\psi$  such that

$$(B.29) \quad \frac{\partial \Pi_i}{\partial d_i} = \sum_i \left\{ \frac{\partial \Pr[Q(x_i, p_i, d_i, \varepsilon_i) = 1]}{\partial d_i} \mathbb{E}[\pi_i(\psi) | Q(x_i, p_i, d_i, \varepsilon_i) = 1] \right. \\ \left. + \Pr[U_i^* \geq 0] \frac{\partial \mathbb{E}[\pi_i(\psi) | Q(x_i, p_i, d_i, \varepsilon_i) = 1]}{\partial d_i} = 0 \right\}.$$

To compute indirect cost estimates, we assume that the lender chooses prices that satisfy one of the two inequalities described in Section 5.2:

$$(B.30) \quad \sum_i \Pi(x_i, p_i, d_i; \psi) \geq \sum_i \Pi(x_i, p_i, d_i + a; \psi) \quad \text{for all } a \neq 0$$

or

$$(B.31) \quad \sum_{i \in I_\tau} \Pi(x_i, p_i, d_i; \psi) \geq \sum_{i \in I_\tau} \Pi(x_i, p_i, d'_i; \psi) \quad \text{for all } \tau = 2, \dots, 23.$$

The first inequality assumes that the firm's chosen prices are more profitable on average than prices that are higher or lower by any fixed amount. The second inequality assumes that the firm's updated prices after any pricing change represent an improvement over previous prices. To estimate the indirect cost of lending based on these inequalities, we compute the sum of squared violations of the inequalities over all pricing periods:

$$(B.32) \quad \Omega_d[d_i, \psi] = \left[ \min \left\{ 0, \sum_i \Pi(x_i, p_i, d_i; \psi) - \sum_i \Pi(x_i, p_i, d_i + a; \psi) \right\} \right]^2$$

or

$$(B.33) \quad \Omega_d[d_i, \psi] = \sum_t \left[ \min \left\{ 0, \sum_{i \in I_\tau} \Pi(x_i, p_i, d_i; \psi) - \sum_{i \in I_\tau} \Pi(x_i, p_i, d'_i; \psi) \right\} \right]^2.$$

Our estimate of  $\psi$  minimizes this objective function. In practice, the estimate is found by grid search over a grid with increments of \$100.

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