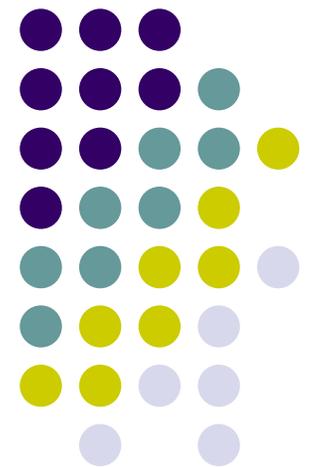


Multi-Unit Auctions

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Econ 285, Market Design
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Examples

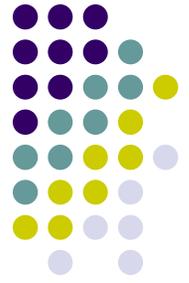


- Treasury auctions
- Auction-rate securities
- IPO auctions
- Privatization
- Electricity markets
- Asset sales
- Condominium sales
- Wine/Art/Antiques
- Auto auctions
- Natural resources
- Radio spectrum
- Emissions permits
- Airport landing slots
- Bus routes
- Procurement contracts
- Sponsored search
- Internet display ads
- eBay marketplace



Plan for next few lectures

- Sealed bid and multi-round designs
 - Spectrum auctions
 - Sponsored search auctions
 - Package bidding
-
- Plus guest appearances by Jeremy Bulow and Paul Milgrom



Selling identical goods

- With identical goods, it is often desirable to use an auction in which all buyers pay the same price.
 - Perceived as “fair”; achieves “price discovery”
 - Does the Vickrey auction have this property?
- Sealed bid auction: participants bid a price-quantity schedule and bids are used to determine the uniform market-clearing price.
- Clock auction: the auctioneer announces a sequence of prices and bidders name quantities until a market-clearing price is found and auction ends.

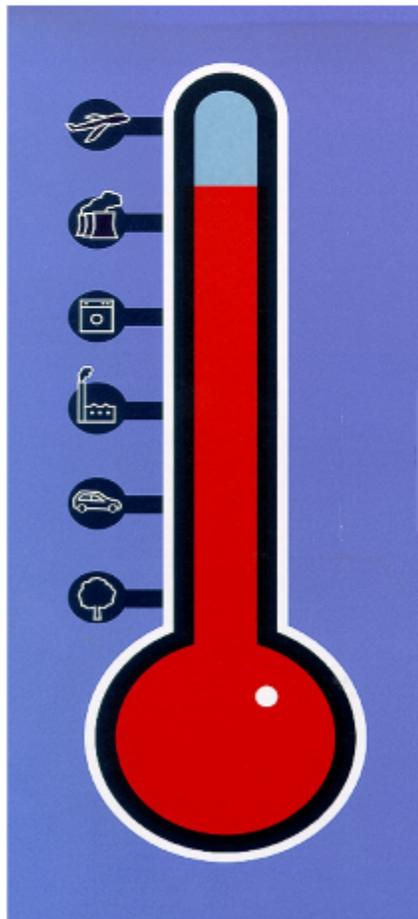


British CO₂ Auctions

- Problem: UK government sets budget to reward firms that reduce CO₂ emissions.
 - But what price to pay per unit? And which firms to reward?
 - Solution: run an auction to find the “market price”
- Greenhouse Gas Emissions Trading Scheme Auction, United Kingdom, 2002
 - 215 million British pounds
 - 4 million metric tons of CO₂ emission reductions
 - 38 bidders, and 34 winners

Greenhouse Gas Emissions Trading Scheme Auction

United Kingdom – March 2002

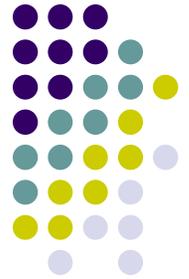


The UK Greenhouse Gas Emissions Trading Scheme (ETS) was introduced by the UK Government as part of the UK Climate Change Programme. Emissions trading is an approach designed to allow greenhouse gas emission reductions to be made in the most economically efficient way. At the time, emissions trading was already being developed internationally – as part of the Kyoto Protocol – and the European Commission had proposed that EU-wide trading would start in 2005. The UK emissions trading scheme started in April 2002. Direct participants wishing to enter into the UK ETS did so through the ETS Auction.

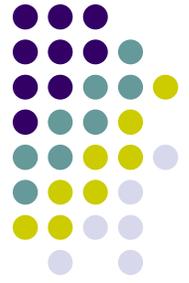
The UK Government made available £215 million over 5 years as an incentive for direct participants to enter into the scheme and to reduce their greenhouse gas emissions. The incentive payments were allocated by a descending-clock auction. The organisations that were interested in obtaining incentive payments for making emission reductions participated in the auction during which they offered a quantity of emission reduction in exchange for a given price per tonne of carbon dioxide equivalent (tCO₂e).

The UK ETS Auction was operated on the PowerAuction™ software system in March 2002. Further details about the UK Greenhouse Gas Emissions Trading Scheme can be found on the [DEFRA website](#).

Greenhouse Auction Rules



- Auctioneer calls out price
 - Price starts high and decreases each round.
- Each round, bidders state tons of CO₂ they will abate
 - Tons abated can only decrease as prices decrease.
- Auctioneer multiplies tons of abatement times price.
 - If total cost exceeds budget, lowers the price
 - When total cost first falls short of budget, auction ends and that allocation is implemented



Strategic Equivalence

- Suppose bidders in the auction observe *only the prices* and that prices decline *in a fixed sequence*.
- A pure strategy maps current price and the bidder's own past quantities into a current quantity, or equivalently current price into current quantity.
- So the clock auction is *strategically equivalent* to a sealed bid auction in which
 - a bid is a “supply curve”
 - the auctioneer posts its demand curve
 - the price is determined so that supply = demand.
- What if more information is revealed each round?



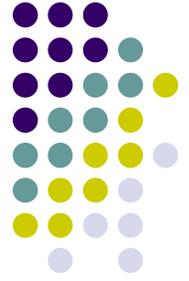
Sealed Bid Design

- Seller sets supply $Q(p)$
 - Initially, assume an inelastic supply Q .
- Each bidder i
 - Has a value function $V_j(q)$ for goods acquired
 - Submits price-quantity schedule: $(p_{jk}, q_{jk}), k=1, 2, \dots, K_j$.
- Seller “clears the market”
 - Allocates goods to the Q highest bids
 - Sets the price equal to the highest rejected bid



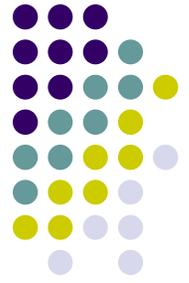
Demand Reduction

- Bidder who wants just one unit has a dominant strategy: set $b_{i1} = v_{i1}$ and $b_{ik} = 0$ for all $k > 1$.
- Bidder who wants multiple units may want to *strategically reduce demand*.
- Example with two units for sale
 - Bidder 1 wants one unit, value w
 - Bidder 2 wants two units, values $v_1 > v_2 > 0$
 - Strategic behavior...
 - Bidder 1 bids his true value.
 - Bidder 2 bids v_1 for first unit and *zero* for the second unit!



Demand Reduction

- Suppose N bidders, K items, where $2 \leq k < N$.
 - Each bidder has positive value for two items.
 - Bidder values have support $\{v \in [0, 1]^{2N} : v_{1j} \geq v_{2j}\}$
- **Theorem.** *In the $k+1$ price auction, it is weakly dominant to bid true value for first unit. However:*
 - (1) *there is no equilibrium in which bidders all bid full value for both items, and*
 - (2) *there is no equilibrium in undominated strategies that is efficient for all realized valuations.*



“Low price” Equilibria

- With fixed supply, uniform price auction often has equilibria with low prices due to demand reduction.
- Example: five units, five bidders.
 - Bidders value units at 10, want as many as possible.
 - Each bidder offers to buy one unit for 10, and no additional units at any price above zero.
 - Bidders split the units, price is zero!
 - A bidder who wants to purchase additional units has to pay ten for every unit he buys: not desirable.



Making supply elastic

- Suppose the seller offers
 - To sell 10 at any price
 - To sell 11 at price of 4
 - To sell 12 at price of 6
 - To sell 13 at price of 7
 - To sell 14 at price of 8
- Then any equilibrium involves selling 14 at a clearing price of at least 8!
- Somewhat surprisingly, seller has managed to *increase supply* and yet also *increase prices*.



Market Power in Practice

- Borenstein, Bushnell and Wolak (*AER*, 2003) and others argue that market power problems have been pervasive in some deregulated electricity markets, e.g. California.
- Joskow: “California electricity crisis is what happens when a vertical supply curve intersects a vertical demand curve.”
- Remedies?
 - Create elasticity in electricity demand (how?).
 - Restrict slope of submitted supply curves.
 - Encourage build-out of additional capacity (how?).
 - Forward contracts (unravel the market!).

Note on Increasing Returns...

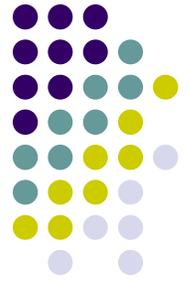


- Discussion implicitly focused on bidders with decreasing marginal values who submit downward-sloping demand curves.
- What if there are scale economies?
 - Two units for sale
 - Bidder 1 offers 10 for one unit.
 - Bidder 2 offers 5 for first unit and 11 for second.
 - There is no uniform price that clears the market!
- Not much is known about performance of uniform price auctions where there are scale economies.



Multiple Kinds of Goods

- Can we extend the sealed bid market clearing mechanism to allow for multiple kinds of goods?
 - Electricity delivered from/to different places.
 - Different kinds of financial assets.
 - Emissions reductions in different years.
 - Spectrum licenses covering different cities.
- The challenge
 - Ideally want bidders to be able to express substitution and maybe even complementarity.
 - Without restrictions, bidding gets complicated and market clearing prices may not exist.



Simple Substitution

- Many applications seem to exhibit roughly one-for-one substitution between “versions” of a good
 - Commodity in different locations or at different times.
 - Loan backed by different types of collateral.
 - Comparably sized road repair projects.
- Milgrom assignment auction design and (much simpler) Klemperer central bank design provide extension of mkt clearing mechanism.



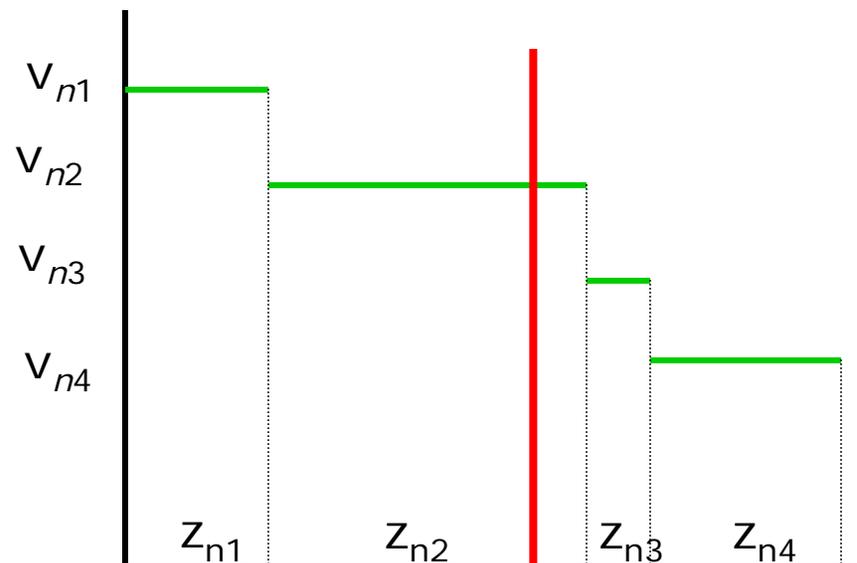
Assignment Auction

- In a ***basic assignment auction***,
 - Each bidder n makes one *or more* bids (indexed by ni) to express demand.
 - Items are assigned to maximize the total bid value.
 - (Minimum) market clearing prices are calculated.
- Each bid is a vector consisting of
 - A desired quantity z_{ni} of the product, and
 - A maximum price v_{nij} to be paid for each version j of the product.



Single-Product Version

- In the one-product case, the accumulated bids describe a step demand function which can be used to find a market clearing price.





How Goods are Assigned

- Treat bids as truthful reports of value, and find efficient assignment, with value π :

$$\pi = \max_{x_{nik}} \sum_{n,i,k} v_{nik} x_{nik} \text{ subject to}$$

$$\sum_{n,i} x_{nik} \leq q_k \text{ for all product versions } k (p)$$

$$\sum_k x_{nik} \leq z_{ni} \text{ for all bids } ni (\lambda)$$

$$x_{nik} \geq 0 \text{ for } n,i,k$$

- Let π be the maximal value.
- λ is interpretable as “profit” on a bid.



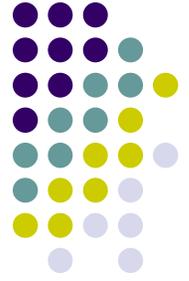
Substitution via “Constraints”

- Seller is offers natural gas (in millions of Btus) delivered at locations A and B.
 - Duke Energy wants 10 bn Btus, and is willing to pay \$5 per million Btu, but it will cost Duke \$0.10 per unit to transport from A and \$0.40 from B.
 - Duke offers to pay \$4.90 for 10bn units at A, and \$4.60 for 10 bn units at B, but adds the constraint that it wants no more than 10bn units.
- Suppose prices are p_A, p_B . Duke will get allocated gas at A only if $p_A \leq \$4.90$ and $p_A - p_B \leq \$0.30$.



How much substitution?

- Can limit the total amount of variety k
- Can limit purchases of subsets of goods, so long as subsets aren't overlapping.
 - Suppose restrict $q_1+q_2 \leq A$.
 - Can add $q_1+q_2+q_3 \leq B$ but not $q_2+q_3 \leq B$. Why?
- Can limit “high price” purchases
 - Eg. I really want 50 units of A or B and am willing to pay \$95. If the price of either drops below \$75, I'll take 100 in total, of whichever is cheaper.



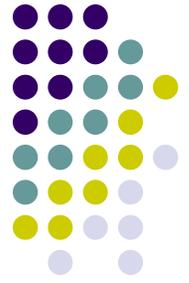
Main Results

- **Theorem.** There is a vector of minimum (and also maximum) market clearing prices.
- **Theorem.** If each q_k and z_{ni} is an integer, then there is an optimal solution x of the LP such that each x_{nik} is an integer.
 - Milgrom's lecture will explain the math...



Assignment Exchanges

- Buyers are submitting bids $(q_{nj}, p_{nj1}, \dots, p_{njK})$, where n is buyer identity, q_{nj} a quantity and p_{njK} the offered price for that quantity if it's "variety k ".
- Sellers can also submit bids $(q_{mj}, c_{mj1}, \dots, c_{mjK})$, where q is quantity and c a cost to supply.
 - Just think of this as an offer to buy negative quantity.
- Auctioneer can also impose constraints
 - No more than 50% of items to bidder A, or A and B together.



Troubled Asset Auctions

- Two examples from the financial crisis
 - Central bank “credit facilities”: pool of funds available as loans to banks that pledge different types of collateral in exchange.
 - US Treasury has considered purchasing large quantities of “toxic assets” under TARP.
- Both programs require placing a value on risky and illiquid financial assets, including some assets that are closely held.



Some TARP alternatives

- 1. Treasury offers to buy a fraction of each security, and runs separate clock or sealed bid auctions.
 - Problem: if there are only a few sellers of each security, not much competition and prices likely to be high.
- 2. Treasury offers to buy a fraction of the overall securities, and looks for a single market clearing price --- induces competition between owners.
 - Problem: unless Treasury groups securities to be close to homogenous, it will end up purchasing crummy ones.



TARP alternatives, cont.

- 3. Treasury scores each asset and sellers bid discounts: e.g. A's base is \$1.00, and B's base is \$0.90, owners of A/B bid percent discounts.
- Using assignment ideas we've discussed
 - Can allow sellers to express substitution.
 - Can include [private buyers](#) in an exchange.