# Supplementary Appendix for: Social Capital and Social Quilts: Network Patterns of Favor Exchange

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March 20, 2010

**Abstract**

This appendix contains supplementary theoretical and empirical results (including discussions of critical networks, asymmetric payoffs, robustness and an alternative definition of renegotiation-proofness) that accompany the paper "Social Capital and Social Quilts: Network Patterns of Favor Exchange."

Keywords: Social Networks, social capital, favor exchange, support, social quilts, renegotiation-proof

JEL Classification Codes: D85, C72, L14, Z13

## 1 Critical Networks and Renegotiation-Proofness

Here, we provide more discussion of critical networks and sufficient conditions for renegotiation-proof networks. Recall that $m$ is defined by

$$m\frac{\delta p(v-c)}{1-\delta} > c > (m-1)\frac{\delta p(v-c)}{1-\delta}. \quad (1)$$

and

$$G(m) = \{g \mid \forall i, d_i(g) \geq m \text{ or } d_i(g) = 0\}$$

is the set of networks in which each node has either at least $m$ links or 0 links.

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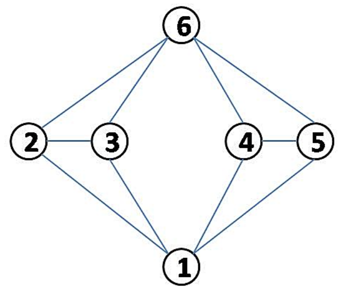



Figure 1: A critical network where two agents have an excess number of links

## 1.1 Critical Networks

Recall that a network $g$ is *$m$-critical* if

- $g \in G(m)$
- for any $i$ and $ij \in g$, there is no subnetwork $g' \subset g - ij$ such that $d_i(g') > d_i(g) - m$ and $g' \in G(m)$.

As the following example shows, it is also possible to have more than one node have more than $m$ links, as long as those two nodes are not adjacent.

Example 1 *A critical network such that two nodes have more than m links.*

Consider the following network, which is pictured in Figure 1:

$\{12, 13, 14, 15, 23, 45, 26, 36, 46, 56\}$.

In this network nodes 1 and 6 have degree four. This is critical and is a renegotiation-proof network when (1) holds for $m = 3$. If any node, including 1 or 6, drop a link, then some node's degree drops below 3 and there is no subnetwork that is sustainable.$\diamond$

### 1.1.1 Unions of Critical Networks

Let us explore the extent to which one can build richer classes of networks that are renegotiation-proof by agglomerating critical networks.

A first question is, "Are unions of critical networks renegotiation-proof networks?"

The first point is that one has to be careful as to how one builds a union of networks. To see this, suppose that $m = 2$ and we consider a union of two critical networks which are two triads. If the "union" is two disjoint networks with no intersecting nodes, then it is clear that the resulting network will be renegotiation-proof. However, if the union involves duplication of a link, then the resulting network might not be. For example, consider the network in Figure 2. This can be seen as the union of two triads where the link 13 is shared

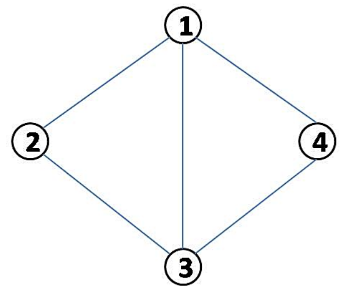



Figure 2: A five link network that is not sustained as a renegotiation-proof equilibrium

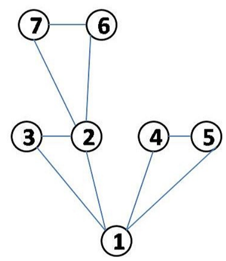



Figure 3: A tree union of critical networks that is not a critical network, but is still renegotiation-proof.

by the triads. We already know that this is not renegotiation-proof. Thus, we need to be careful that if the networks intersect, then they do not share links.

Recall that we defined "tree unions" of networks in the footnote of the main paper. A union of several networks $g_1, ..., g_K$ is called a *tree union* if the networks can be ordered in a way $g_1, ..., g_K$ such that successive unions

$$U_1 = g_1, \ldots, U_k = U_{k-1} \cup g_k, \ldots, U_K = \bigcup_{k=1\ldots K} g_k$$

are such that each additional network has at most one node in common with the preceding union: $|N(U_{k-1}) \cap N(g_k)| \leq 1$.

One thing to note is that a tree union of critical networks is not necessarily critical, as illustrated in the following example.

Example **2** *A Tree Union of Critical Networks*

Let $m = 2$ and consider the network of three linked triads $g = \{12, 23, 13, 14, 15, 45, 26, 27, 67\}$ as pictured in Figure 3. This is not critical since if 1 cuts the link 12, then all nodes in the sub-network still have at least 2 links. Nonetheless, (as we will verify below) this network is renegotiation-proof.$\diamond$

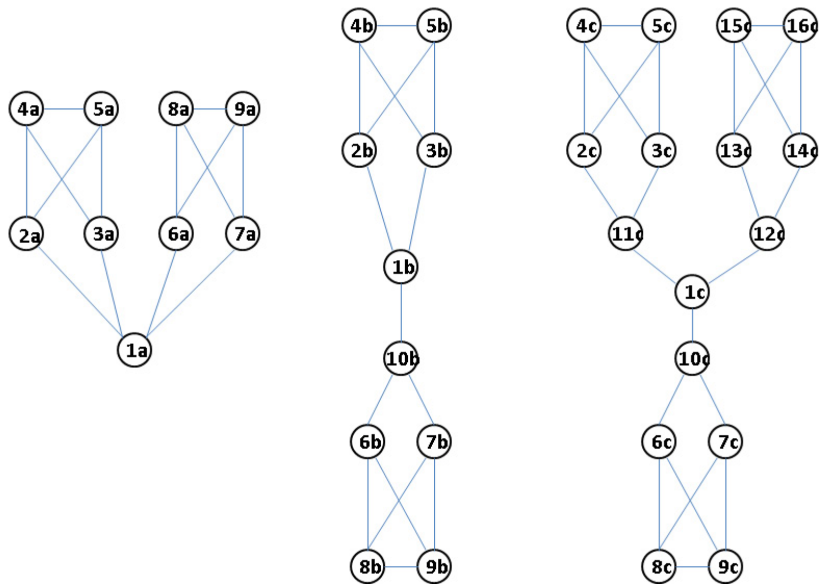



Figure 4: A tree union of critical networks that is not renegotiation-proof.

Although Example 2 shows that it is possible to have a network that is a tree union of critical networks not be critical, and yet still renegotiation-proof, that is not true of all tree unions, as the following example shows.

Example **3** *A tree union of critical networks that is not renegotiation-proof.*

Let $m = 3$ and consider the three critical networks $g_a$, $g_b$ and $g_c$ shown in Figure 4. Let $g'_a$ be a network with the same structure as $g_a$ but with a different group of agents $\{1a', ..., 9a'\}$, and define $g'_b$, $g''_b$ and $g'''_b$ similarly. Consider two tree unions of these critical networks:

- $U_1 = g_a \cup g'_a \cup g_c$. intersecting at the node $1a = 1a' = 1c$;
- $U_2 = g_b \cup g'_b \cup g''_b \cup g'''_b$, intersecting at the node $1b = 1b' = 1b'' = 1b'''$.

Structurally, $U_2 - \{6b, 7b, 8b, 9b, 10b\}$ is the same as $U_1$. The claim is $U_1$ and $U_2$ cannot both be both renegotiation proof networks. Otherwise, starting from $U_2$ if agent $1b$ refuses a favor to agent $10b$, the network played in the continuation has to be $U_2 - \{6b, 7b, 8b, 9b, 10b\}$ since it is renegotiation-proof (having the same structure as $U_1$) and noting that the nodes $\{6b, 7b, 8b, 9b, 10b\}$ must lose their links in any continuation. Thus, agent $1b$ only loses one link and would prefer not to do a favor for $10b$, contradicting the supposition that $U_2$ is a renegotiation-proof network.$\diamond$

One special character of networks in this example that potentially prevents the unions to be renegotiation-proof is there are some "critical" nodes in the networks such as $1a$, $1b$ and $1c$.

A node is called *critical* if deleting this node increases the number of components in the network.

In other words, a node is critical if it plays an essential role in connecting different agents who will be in different components without the critical node. For example, without $1a$, agents $2a$ and $6a$ won't be connected in $g_a$. Another way to present this character is by noting that any path between $2a$ and $6a$ has to contain $1a$ such that there is no simple cycle containing agents $2a$ and $6a$. The lemma below implies the equivalence of these two presentations of the special character of networks in Example 3.

LEMMA **1** *Consider a path-connected network involving links among at least three nodes. For each pair of path-connected nodes in the network there is a simple cycle containing them if and only if there is no critical node in the network.*

**Proof of Lemma 1:** Let us first argue that if there is a critical node, then there are at least two nodes that are path-connected but that do not lie on a simple cycle. Suppose that there is a critical node $i$ in the path connected network $g$, such that deleting $i$ results in at least two separate components. Pick nodes $j$ in one of those components and $k$ in another component. It follows that $i$ lies on all paths connecting $j$ and $k$ or else deleting $i$ would not have resulted in these nodes falling in separate components. Thus, there could not have been a simple cycle containing these two nodes in the original network.

For the other direction of the lemma, we consider any two path-connected nodes $i$ and $j$ that are embedded in a network with at least 3 nodes that has no critical nodes. We show that there exists a simple cycle containing $i$ and $j$. We proceed by induction on the distance between $i$ and $j$ (with the standard definition of distance being the number of links of the shortest path between them).

For the base case let the distance between $i$ and $j$ be 1 so that $i$ and $j$ are direct neighbors. There must exist some other node $k$ that is a neighbor of either $i$ or $j$ since the network involves at least 3 nodes and is path connected. Without loss of generality assume that it is adjacent to $i$. Since there are no critical nodes in the graph, $k$ and $j$ remain path-connected if we delete node $i$. Thus, let $P$ be a path that goes from $k$ to $j$ without passing through $i$. There is a simple cycle containing $i$ and $j$ given by $j - i - k$, and then taking $P$ from $k$ to $j$.

For the inductive step, suppose that the claim is true for any pair of nodes of distance $n$ or less, and consider some pair of nodes $i$ and $j$ at distance $n + 1$, and let $S$ be a path of length $n + 1$ between $i$ and $j$. There is a unique node $k$ adjacent to $i$ on $S$, at distance $n$ from $j$ and by the inductive hypothesis there exists a simple cycle containing $k$ and $j$. Let $P_1$ be a path from $k$ to $j$ contained in this simple cycle, and $P_2$ a path from $j$ to $k$ disjoint from $P_1$. Since the graph has no critical nodes there exists some path $P_3$ from $i$ to $j$ that does not go through $k$. If $P_3$ is disjoint from $P_1$ or $P_2$ we are done, since we then have a

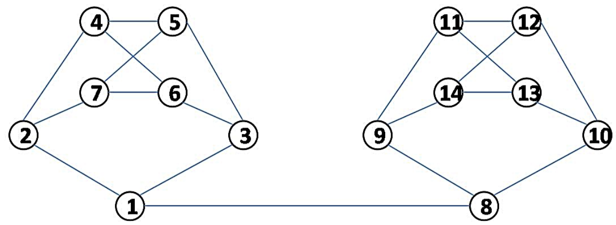



Figure 5: A critical network with a bridge

simple cycle given either by $i - k - P_1$ to $j$ and then back to $i$ via $P_3$, or by $i - k - P_2$ to $j$ and then back to $i$ via $P_3$. So assume that $P_3$ intersects both $P_1$ and $P_2$ and without loss of generality that it intersects $P_1$ first, at some node $m$ (since $P_1$ and $P_2$ are disjoint, this first-to-be-intersected order is strict). We now have a simple cycle including $i$ and $j$ given by $i \to m$ (via $P_3$), then $m \to j$ (via $P_1$) and then from $j$ to $k$ (via $P_2$), and then $k - i$ finally back to $i$. ▮

So in the following, we look at a nice subclass of critical networks that don't have these critical nodes. And it turns out that tree unions of networks in this subclass are renegotiation-proof.

### 1.1.2 Simply Critical Networks

A useful subclass of critical networks is the class in which any two nodes are connected via a simple cycle. Such networks can be agglomerated quite nicely.

A network $g$ is called *simply critical* if $d_i(g)$ equals $m$ or 0 for every $i$, and for any pair of nodes $i$ and $j$ there is a simple cycle containing them.

Clearly a simply critical network is critical. An obvious example of a simply critical network is a clique of $m + 1$ nodes.[1] To get a deeper feeling for what simplicity implies, see the network pictured in Figure 5 which is critical but not simply critical.

Example 4 *A Critical Network with a Bridge.*

Consider the network pictured in Figure 5:

[1] A clique is a completely connected (sub-)network

$g = \{12, 13, 24, 27, 35, 36, 45, 46, 67, 75, 18, 89, 8-10, 9-11, 9-14, 10-12, 10-13, 11-12, 11-13, 13-14, 14-12\}$

In this network every node has exactly 3 links. There is a bridge: the link 18. This network is renegotiation-proof when (1) holds for $m = 3$. If any node drops a link, then all links are dropped since there is no proper subnetwork where each node in the subnetwork has at least 3 links. However, this network is not simply critical$\diamond$

Thus, the idea of simply critical networks is that each each agent has exactly $m$ links and is cyclicly tied to every other agent. A nice feature of simply critical networks is that they make nice "building blocks" in that they can be agglomerated via tree unions to create renegotiation-proof networks.

PROPOSITION 1 *A tree union of simply critical networks is renegotiation-proof.*

**Proof of Proposition 1:** The proof proceeds by induction on the size of the tree union.

When $k = 1$, it is a single critical network, and so it is renegotiation proof. Suppose it is true for all $k' < k$. We show that a tree union of $k$ simply critical networks is renegotiation proof.

To establish the proposition, we show that tree unions of simply critical networks and some nonempty strict subnetworks of simply critical networks cannot be renegotiation-proof. This is enough to establish that tree unions of simply critical networks are renegotiation-proof, simply by deleting all links in any particular simply critical subnetwork of the tree union if some agent fails to perform a favor in that subnetwork.

Begin with a tree union of $k$ simply critical networks, $g_1, \ldots g_k$.

Let $g^0 = \left(\bigcup\limits_{h=1\ldots m_0-1} g_h\right) \cup \left(\bigcup\limits_{h=m_0\ldots k} g_h^0\right)$, with $m_0 \leq k$, $g_h^0 \subset g_h$, $g_h^0 \neq g_h$ $\forall h \geq m_0$ and at least one $g_h^0$ in the union is nonempty. So this is the tree union of simply critical networks and some nonempty strict subnetworks of simply critical networks. Suppose to the contrary that it is renegotiation-proof.

Note that $\bigcup\limits_{h=m_0\ldots k} g_h^0$ is a tree union of networks, and it must therefore have some leafs. Pick one such leaf and denote it $g_{h^*}^0$. Since $g_{h^*}^0$ is a strict subset of the simply critical network $g_{h^*}$ and a leaf of the subtree, there is some agent $i_0$ who has a positive number of links, less than $m$, in the subtree. Suppose this agent were to fail to provide a favor on a link $i_0 j_0$ in $g_{h^*}^0$. Since by supposition $g^0 \in RPN$, agent $i_0$ would have to lose at least $m$ links if he or she failed to provide a favor on any link $i_0 j_0$ in the subtree. Since the agent does not have enough links to lose in the subtree, he or she would have to lose links in $\bigcup\limits_{h=1\ldots m_0-1} g_h$. Denote the continuation by $g^1$ which must be renegotiation-proof. Note that $g^1$ cannot be a strict subset of $\bigcup\limits_{h=1\ldots m_0-1} g_h$, since by the inductive hypothesis $\bigcup\limits_{h=1\ldots m_0} g_h \in RPN$. Therefore $g^1$ must have some links from $\bigcup_{h=m_0...k} g_h^0$. In particular $g^1 = \left(\bigcup_{h=1...m_1-1} g_h\right) \cup \left(\bigcup_{h=m_1...k} g_h^1\right)$, where $g_h^1 \subset g_h$, $g_h^1 \neq g_h$ $\forall h \geq m_1$ and $m_1 < m_0$. This last inequality results from the fact that $i_0$ lost links in $\bigcup_{h=1...m_0} g_h$. Again, any agent who has fewer than $m$ links in $\bigcup_{h=m_1...k} g_h^1$ must have links in $\bigcup_{h=1...m_1-1} g_h$. We the derive a subnetwork $g^2$ from $g^1$ analogously to the way we derived $g^1$ from $g^0$. Proceeding in this fashion we produce a finite sequence of renegotiation proof networks $g^0, g^1, ..., g^\ell$, with $m_x < m_{x-1}$ at each iteration and there is always at least one link in $\bigcup_{h=m_x...k} g_h^x$. Continue until $m_\ell = 0$. Using the same argument with which we found $i_0$, it can be seen that we would find some node with less than $m$ links in total, contradicting $g^\ell \in RPN$. ∎

Before moving on, we note one useful observation for identifying simply critical networks.

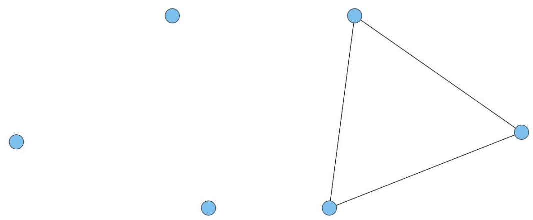
A *Hamiltonian network*[2] is a network with a simple cycle visiting all nodes. So a Hamiltonian network is sufficient, but not necessary, for there to be a simple cycle containing any given pair of nodes in a network. Thus, any critical network that is a Hamiltonian where each node has $m$ links is simply critical, but not vice versa.[3]

## 2 Some Renegotiation-Proof Networks

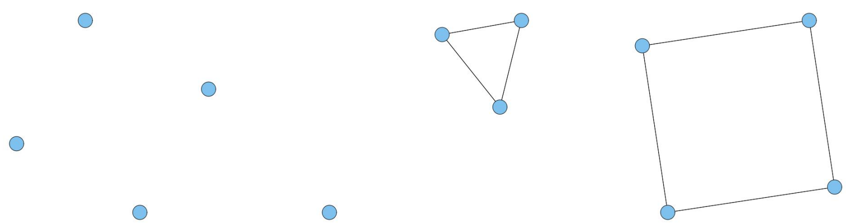
The equivalence between renegotiation proof and transitively critical works provides us with a straightforward algorithm for deciding whether a given network is renegotitation proof or not. Implementing this algorithm for large numbers of vertices ($n$), and high values of $m$ however, is currently not feasible due to the sheer size of the space of non-isomorphic graphs that must be traversed. Table 1 shows the number of non-isomorphic renegotation proof networks for a few small values of $m$ and $n$, along with the corresponding number of non-isomorphic subgame perfect networks. Note that the numbers in the first column ($m = 1$), also correspond to the number of non-isomorphic networks on the number of vertices associated to each row. Figure 6 shows the non-isomorphic renegotiation proof networks corresponding to the values of $n$ and $m$ shown in the table.

[2]See Jackson (2008) for more background.

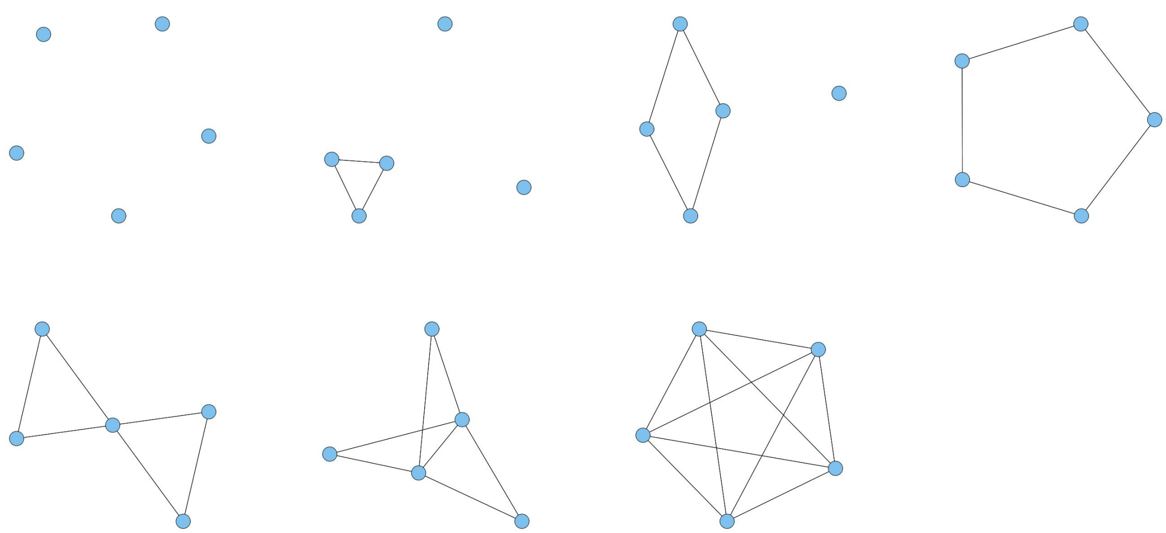
[3]Consider the network $g = \{13, 34, 35, 45, 46, 56, 62, 17, 78, 79, 89, 8-10, 9-10, 10-2, 1-11, 11-12, 11-13, 12-13, 12-14, 13-14, 14-2\}$ and $m = 3$. $g$ is critical since every node has exactly $m$ links and any pair of nodes has a cycle containing them. However, $g$ is not a Hamiltonian network since there are three "highways" connecting 1 and 2 such that there is no way a simple cycle can contain all nodes.



n=3,m=2

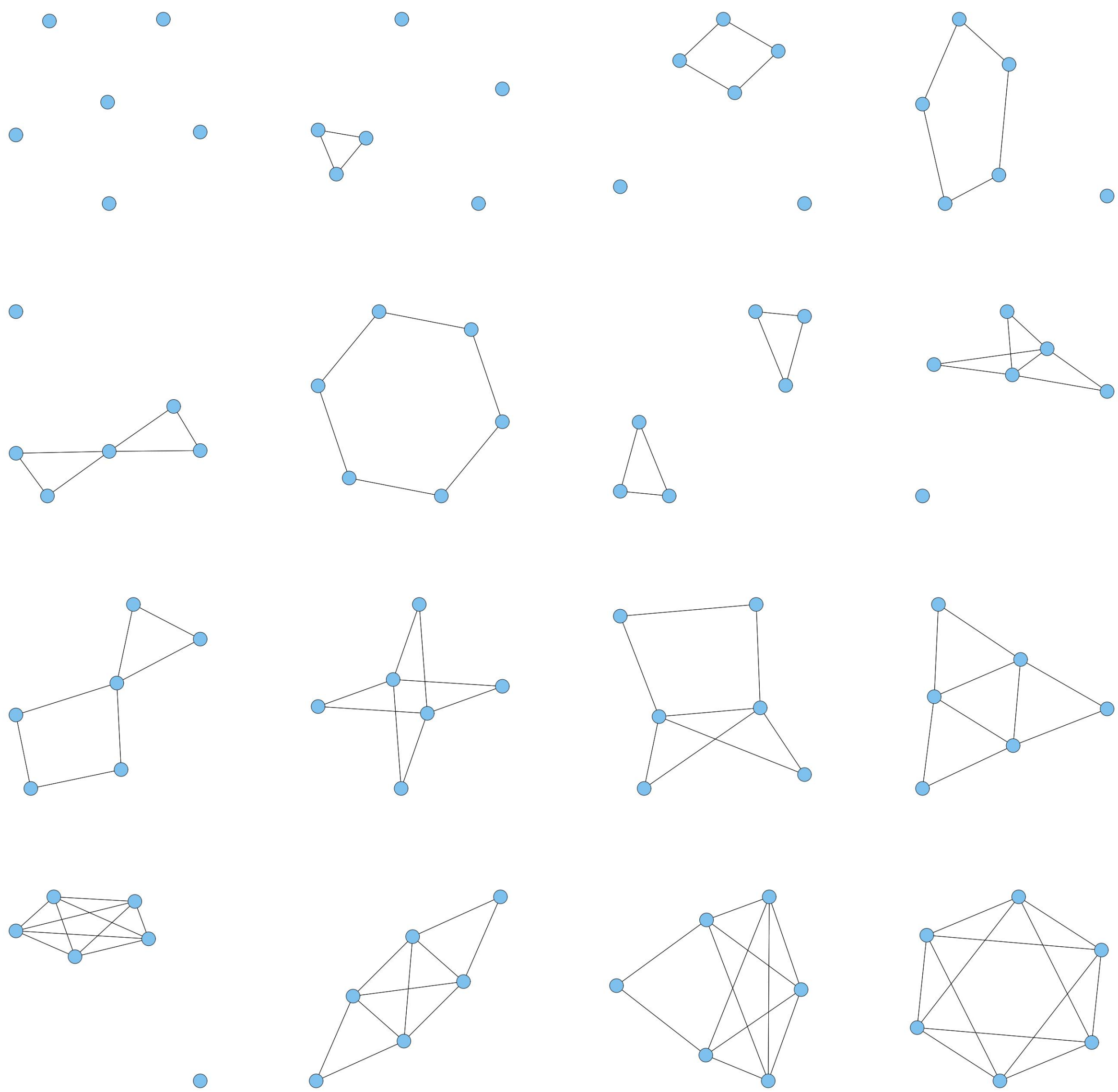


n=4,m=2



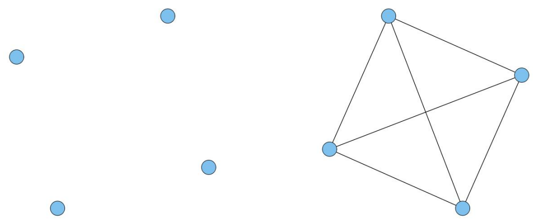
n=5,m=2

Figure 6: Non isomorphic networks for n=1-6, m=2 and m=3

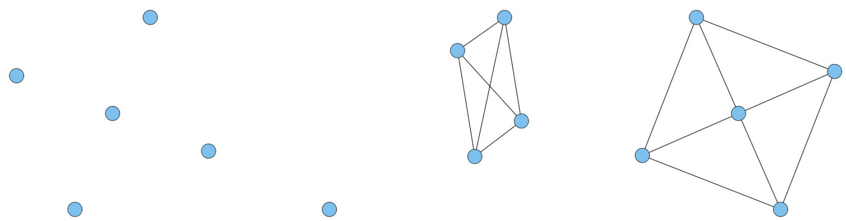


n=6,m=2

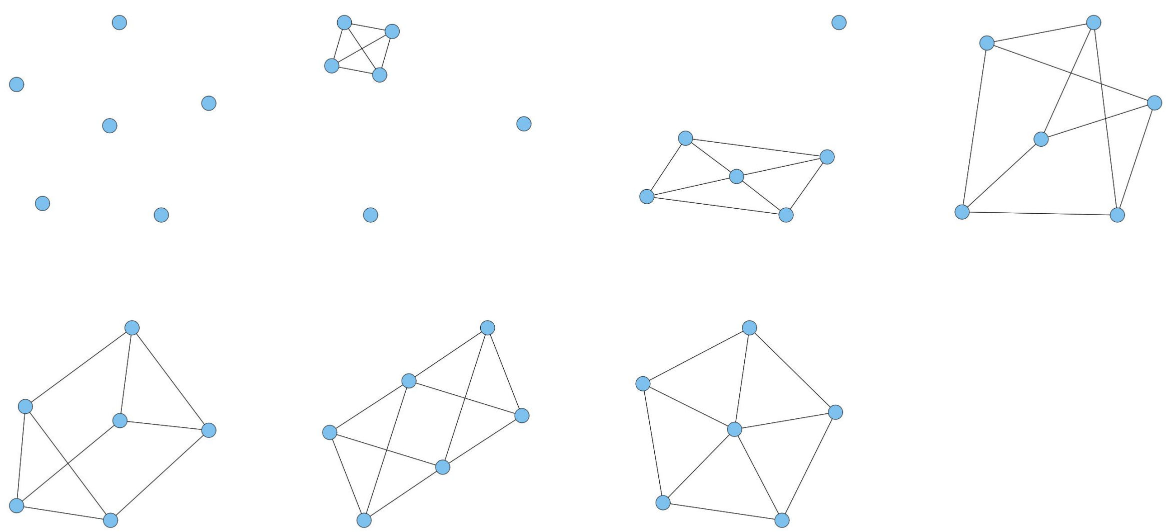
Figure 6 continued.



n=4,m=3



n=5,m=4



n=6,m=3

Figure 6 continued.

# 3 A Special Heterogeneous Case

An interesting case that generalizes the homogeneous case and yet is not as fully general as the heterogeneous case examined above is one where agents may have idiosyncratic values and costs to favors $v_i$ and $c_i$, and discount factors $\delta_i$, but where these values are not dependent upon to whom agents are linked and also where the favor probabilities are not agent dependent. In that case, each agent is characterized by his or her own $m_i$ such that[4]

$$m_i \frac{\delta_i p(v_i - c_i)}{1 - \delta_i} > c > (m_i - 1) \frac{\delta_i p(v_i - c_i)}{1 - \delta_i}. \tag{2}$$

For this case, our previous results have analogs.

We define *transitively critical* networks as before, simply changing the reference number of links to be agent specific.

Given $m = (m_1, \ldots, m_n)$, let $TC_k(m) \subset G_k$ denote the set of transitively critical networks with $k$ links.

- Let $TC_0(m) = \{\emptyset\}$.
- Inductively on $k$, $TC_k(m) \subset G_k$ is such that $g \in TC_k(m)$ if and only if for any $i$ and $ij \in g$, there exists $g' \subseteq g - ij$ such that $g' \in TC_{k'}(m)$, $d_i(g') \leq d_i(g) - m_i$, and there is no $g'' \in TC_{k''}(m)$ such that $g'' \subset g - ij$ and $D(g'') > D(g')$.

Next, in order to define the analog of social quilts we need to define an analog of a minimal clique. In the fully symmetric case a critical clique was simply one where each agent had $m$ links. Now, however, different agents may have different critical numbers of favor relationships that they must fear losing in order to give them incentives to exchange favors. There cannot be too much asymmetry in these critical numbers across the members of a clique or else some subset of the clique could sever some relationships and still have it be sustainable. For example if $m_i = 2$ for two agents and $m_i = 3$ for another two agents, making a clique from these four agents will not be renegotiation-proof. The first two agents could sever the link between them and end up with a (transitively) critical network.

A *critical clique* with $m'$ nodes is a clique that has $m'$ nodes and such that $m_i \leq m' - 1$ for each $i$ in the clique and $m_i < m' - 1$ for at most one $i$.

Thus, a critical clique has all but one agent having identical $m_i$'s and the remaining agent's $m_i$ being the lowest.

Next, in order to define social quilts we also need to be careful about how cliques are pieced together whenever there is a node involved in two of the cliques. A (tree) union of critical cliques is not always robust, as the following example illustrates.

[4] Again, we rule out indifference.

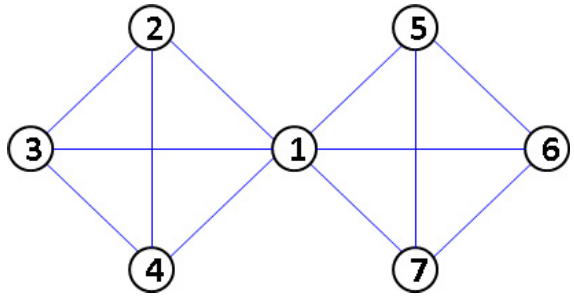



Figure 7: A tree union of critical cliques that is not robust

Example **5** *A tree union of critical cliques that is not robust.*

Consider two critical cliques with agents $\{1, 2, 3, 4\}$ and $\{1, 5, 6, 7\}$, respectively, where $m_i = 3$ for all $i$ except $m_4 = m_7 = 2$. The tree-union of these two cliques is denoted $g$ as in Figure 7. Then deleting links 14 and 17 leads to a subnetwork of $g$ that is critical and renegotiation-proof. Indeed, all agents end up with exactly their critical number of links except agent 1 who has one extra link. It then follows that $g$ is not robust, since this subnetwork would violate a local contagion condition. $\diamond$

Thus, we have to be careful in defining tree unions when some of the cliques have asymmetries in the agents' respective numbers of critical links. When we unite two critical networks at some agent like agent 1, we add extra links for that agent and some of them might become non-critical.

The example above suggests that if some agent has a lower $m_i$ than other agents in a clique, and we piece cliques together, then it should be that lowest agent who is the common agent in two cliques. Exactly how this works when there are various heterogeneities across cliques is somewhat subtle as the following example shows.

Example **6** *A tree union of heterogeneous critical cliques that is robust.*

Consider two critical cliques with agents $\{1, 2, 3\}$ and $\{1, 4, 5, 6\}$, respectively, where $m_1 = m_2 = 2$, $m_3 = 1$ and $m_4 = m_5 = m_6 = 3$. The tree-union of these two cliques is denoted $g$ as in Figure 8. Here, agent 1 has the lowest $m_i$ in one of the cliques, but not the other. This network is still robust, since as long as the "connecting" agent is minimal in at least one of the two cliques, then that clique remains completely critical, and so then has no interaction with the adjacent clique. Here it is impossible to remove any link in $\{1, 4, 5, 6\}$ without it losing all links. Then, $\diamond$

The insights from these two examples lead to the following definition.

| RNP | m | | | | |
|---|---|---|---|---|---|
| n | 1 | 2 | 3 | 4 | 5 |
| **1** | 1 | 1 | 1 | 1 | 1 |
| **2** | 2 | 1 | 1 | 1 | 1 |
| **3** | 4 | 2 | 1 | 1 | 1 |
| **4** | 11 | 3 | 2 | 1 | 1 |
| **5** | 34 | 7 | 3 | 2 | 1 |
| **6** | 156 | 16 | 7 | 3 | 2 |

| SP | m | | | | |
|---|---|---|---|---|---|
| n | 1 | 2 | 3 | 4 | 5 |
| **1** | 1 | 1 | 1 | 1 | 1 |
| **2** | 2 | 1 | 1 | 1 | 1 |
| **3** | 4 | 2 | 1 | 1 | 1 |
| **4** | 11 | 5 | 2 | 1 | 1 |
| **5** | 34 | 16 | 5 | 2 | 1 |
| **6** | 156 | 78 | 24 | 6 | 2 |

Table 1: Number of non-isomorphic renegotiation proof and subgame perfect networks for some values of $n$ and $m$.

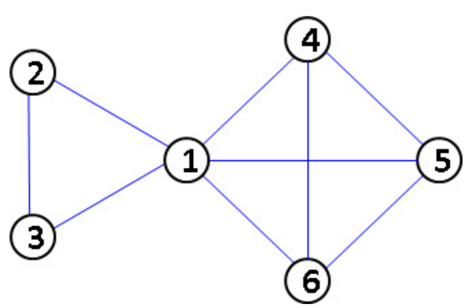



Figure 8: A tree union of critical cliques that is robust

Given a profile $m = (m_1, \ldots, m_n)$, we say that $g$ is an *ordered tree union* of networks $g_1, ..., g_K$ if the networks can be ordered in a way $g_1, ..., g_K$ such that

- successive unions

$$U_1 = g_1, \ldots, U_k = U_{k-1} \cup g_k, \ldots, U_K = \bigcup_{k=1\ldots K} g_k$$

  are such that each additional network has at most one node in common with the preceding union: $|N(U_{k-1}) \cap N(g_k)| \leq 1$, and

- in each step of the union $U_k = U_{k-1} \cup g_{(k)}$ the node in common (in $N(U_{k-1}) \cap N(g_k)$ if the intersection is nonempty) is the node with the smallest $m_i$ in $g_{(k)}$.

Of course, an ordered tree union is the same as a tree union in the case where all agents have the same critical number.

Now we define a *social quilt* to be a ordered tree union of critical cliques.

Social quilts thus defined are sufficient and necessary for robustness.

THEOREM 1 *In cases where each agent has an idiosyncratic $m_i$ defined by (2), a network is renegotiation-proof if and only if it is transitively critical, and a network is robust against social contagion if and only if it is a social quilt.*

Given our previous discussion of critical networks, it is a simple extension to see that transitive criticality characterizes renegotiation-proofness, and social quilts are renegotiation-proof. The critical cliques limit contagion to be local in nature. The subtle and difficult part of the proof of Theorem 1 is in showing that only social quilts are robust. For example, why is a complete network not robust? This requires an involved argument, which draws upon both the renegotiation-proofness and the local aspect of punishments. Roughly, the intuition is as follows. First, any robust network must contain some cliques, as an agent who cheats must lose some number of links, which must all be local. In terms of continuation equilibria, any smallest sustainable subnetwork of a given network must be a clique. This follows since any deviation must lead to the loss of all its links since it is the smallest, and by locality the agents must all be neighbors. Moreover, it must be of minimal size by renegotiation-proofness as otherwise the society could renegotiate to keep a minimal sized clique which would contradict this being the smallest sustainable subnetwork. The proof then works by using some graph theoretic reasoning to show that any network that is not a social quilt has some subnetwork that is a critical network, and hence a smallest sustainable subnetwork, but is not a clique. Thus, if a network is not a social quilt, there is some way in which it could be broken down so that the eventual contagion in a last stage of destruction would necessarily be non-local.

**Proof of Theorem 1:** We only prove that robustness implies that a network must be a social quilt, since the converse is an easy analog of the proof of Proposition 1, adapted to strong tree unions.

Suppose that $g$ is robust against social contagion. If there is a critical clique $g_c \subset g$ that has at most one node $i$ connected with nodes outside of the clique and $m_i$ is the smallest in $g_c$, then $g - g_c$ is also robust against social contagion. This follows since if any agent $j \neq i$ who is in $g_c$ deletes a link, he or she must lose all of his or her links, and then so must all other agents except $i$ in the clique, but by robustness no other links can be deleted. So, eliminate $g_c$ and continue with the network $g - g_c$. If repeating this process leads to an empty network, then $g$ must have been a social quilt. Suppose instead, that this elimination process leads to some nonempty $g'$. Note that since $g$ is robust, $g'$ is then also robust and hence sustainable. By the above process, $g'$ contains no critical cliques where at most one agent has links outside of the clique and that agent has the smallest $m_i$.

By the above process, any remaining cliques that are subnetworks of $g'$ and are such that each agent $i$ in the clique has at least $m_i$ links, must belong to at least one of the following sets:

(1) Cliques that are not critical.

(2) Cliques that are critical but such that some agent $j$ connected to another part of the network is not the agent $i$ with the smallest $m_i$ in the clique.

(3) Cliques that have at least two agents who have links outside of the clique.

So, identify some remaining clique that is a subnetwork of $g'$ and is such that each agent $i$ in the clique has at least $m_i$ links, and identify the first case of (1) to (3) that applies. Next, do the following depending on which case applied: In case (1) delete a link between the agents with the two smallest $m_i$s. In case (2) delete the link $ij$. In case (3), delete a link between a pair of agents who have links outside of the clique.

The remainder of the proof of the result follows the the logic in the end of the proof in Theorem 2 in the main body of the paper. ∎

# 4 Weak Robustness

It also turns out that we can weaken the definition of robustness and still end up with exactly the class of social quilts. In particular, we can weaken the notion of renegotiation-proofness in the definition. This is useful because it allows us to define robustness in a way that is not inductive, and can thus be easier to implement.

Let us say that a network $g$ is *weakly renegotiation-proof* if it is sustainable by a pure strategy subgame perfect equilibrium and at any subgame that starts with $g' \subset g$ that is critical, $g'$ is sustained in the equilibrium continuation.

Let $WPRN_k$ denote the set of all networks that have exactly $k$ links and can be sustained in perpetuity as part of a pure strategy weakly renegotiation-proof equilibrium.

LEMMA **2** *All renegotiation-proof networks are weakly renegotiation-proof; that is, $RPN_k \subset WRPN_k$ for all $k$.*

Weakly renegotiation-proof networks are a richer set than renegotiation-proof networks. This is not obvious, since the latter definition is inductive. Both definitions require that in any subgame starting with a network $g'$ that cannot degrade further without collapsing (thus being critical), $g'$ be played in perpetuity. Otherwise, weak renegotiation-proofness puts no additional restrictions on the networks in continuation whereas renegotiation-proofness does. Weak renegotiation-proofness allows richer punishments than renegotiation-proofness, while it still rules out things like grim trigger. While it may be on less solid ground as a solution concept, it is useful in proving some results, which then also hold a fortiori for the stronger concept of renegotiation-proofness.

We can also use weak renegotiation-proofness as a basis for a robustness definition. We say that a network $g$ is *weakly robust against social contagion* if it is weakly renegotiation-proof and sustained by a pure strategy subgame perfect equilibrium with $g_0 = g$ such that in any subgame continuation from some weakly renegotiation proof $g' \subset g$, and for any $i$ and $ij \in g'$, if $i$ does not perform the favor for $j$ when called upon, then the continuation leads to $g''$ such that if $h\ell \notin g''$ then $h \in N_i(g') \cup \{i\}$ and $\ell \in N_i(g') \cup \{i\}$.

Weak robustness turns out to be equivalent to robustness.

PROPOSITION **2** *A network is weakly robust against social contagion if and only if it is robust against social contagion.*

Proposition 2 follows easily from the observation that weak robustness against social contagion implies that a network must be a social quilt, which is then in turn robust against social contagion.

# 5 Maximal Equilibria

In this part, we consider a slight variation on the concept of renegotiation-proof equilibrium, called maximal equilibrium that is similar but does not require that a deviating agent be considered in the Pareto calculations. Thus, it allows agents to ostracize some deviating agent even when there would be some continuation that would make that agent better off without hurting any of the other agents.

*Maximal equilibria* are a subset of pure strategy subgame perfect equilibria and are defined as follows. Let $G_k$ denote the set of all networks that have $k$ links.

- Let $ME_0 = \{\emptyset\}$
- Let $ME_k$ denote the subset of $G_k$ such that $g \in ME_k$ if and only if beginning with $g_0 = g$ implies there exists a pure strategy subgame perfect equilibrium such that
  - on the equilibrium path $g$ is always sustained, and
  - in any subgame starting with some network $g' \in G_{k'}$ with $k' < k$ if $g''$ is played in perpetuity with some probability in the continuation then $g'' \in ME_{k''}$ for some $k''$ and there does not exist any $g'''$, $g'' \subset g''' \subset g'$ such that $g''' \in ME_{k'''}$.

These equilibria still embody a sort of renegotiation-proofness. For instance, they still rule out a full grim-trigger strategy where once any link is cut then all links are cut. At any point in time, if a given network is reached and that network can be sustained via some equilibrium (with the inductively defined continuations satisfying the same maximality condition), then it is sustained.

However, maximal equilibria allow agents to ostracize other agents, which is not always the case in renegotiation-proof equilibria as we illustrate in Example 7. Thus maximal equilibria might be appropriate in some social settings: they permit a society to punish an agent who does not abide by a social norm, and at the same time the society does not resort to arbitrarily drastic punishments but instead limits itself to punishments that minimize the damage to other agents.

Example **7** *Maximal Equilibria*

Let there be 4 nodes.

Consider a case where

$$2\frac{\delta p(v-c)}{1-\delta} > c > \frac{\delta p(v-c)}{1-\delta}$$

Here, no link is sustainable in isolation, since the value of providing a favor $c$ is greater than the value of the future expected stream of giving and receiving favors: $\frac{\delta p(v-c)}{1-\delta}$.

However, if an agent risks losing two links by not performing a favor, then links could be sustainable depending on the configuration of the network, since $c < 2\frac{\delta p(v-c)}{1-\delta}$.

What do the maximal equilibrium networks look like in this case?

Here, $ME_1 = \emptyset$ since no isolated links are sustainable.

Similarly, $ME_2 = \emptyset$ since any agent who only has one link will never perform a favor.

$ME_3 = \{g = \{ij, jk, ik\} :$ for some distinct $i, j, k\}$. Thus triads are sustainable, since if any agent severs a link, then that will lead to a two-link network which is not sustainable, and so becomes an empty network. Thus, not performing a favor leads to an empty network, and so it is a best response to perform a favor, anticipating favors by other agents.

$ME_4 = \{g = \{ij, jk, k\ell, \ell i\} :$ for some distinct $i, j, k, \ell\}$. This is an obvious extension of the logic from three-link networks.

The interesting difference between maximal and non-maximal equilibria come with $k = 5$ or more links.

Consider the network $g = \{12, 23, 34, 41, 13\}$ as pictured in Figure **??**. So, agents 1 and 3 each have three links and agents 2 and 4 have two links. There is a subgame perfect equilibrium sustaining this network: if any link is ever cut, then all agents cut every link in the future. However, there is no maximal equilibrium sustaining this network. To see this, suppose that agent 1 is called upon to do a favor for agent 3. If agent 1 does not do the favor, then the resulting network is $g' = \{12, 23, 34, 41\}$. Note that $g' \in ME_4$, and so there is a maximal equilibrium continuation sustaining $g'$. Thus, under any maximal equilibrium, $g'$ would be sustained in the equilibrium continuation. Thus, agent 1 can cut the link 13 and still expect the network $g'$ to endure, and so this is the unique best response for agent 1 and so $g$ is not part of any maximal equilibrium: $g \notin ME_5$. Thus, $ME_5 = \emptyset$.

Next, note that $ME_6 = G_6$, i.e., the complete network. To see this, consider an equilibrium as follows. If an agent $i$ severs a link, then the remaining agents never perform a favor for $i$ again and sustain the triad that excludes $i$. This continuation satisfies the requirements of maximal equilibrium, and is a subgame perfect equilibrium (with a fuller specification of all the off-equilibrium-path behaviors, which we provide in more detail in the results below).

Here we also point out the difference between maximal equilibria and renegotiation-proofness. The complete network is not renegotiation-proof. If agent 1 deletes the link 12, the above equilibrium calls for agents to then go to the triad that excludes agent 1. However, that equilibrium continuation is Pareto dominated by a continuation of the four link network 12, 23, 34, 41. Thus, it is a Pareto improvement for the society to forgive agent 1 and go to a four link network instead of the three link network. Interestingly, the other agents are indifferent and it is only the deviating agent who is helped. This is the aspect that if the other agents move to the other equilibrium where they are equally well off, they will help sustain the better six link network with a threat to ostracize agent 1.$\diamond$

## 5.1 Characterizing Maximal Networks

Before moving to the complete characterization of maximal networks, we consider various critical networks first. Recall the definition of criticality such that a network $g$ is *m-critical*, if

- $g \in G(m)$

- for any $i$ and $ij \in g$, there is no subnetwork $g' \subset g - ij$ such that $d_i(g') > d_i(g) - m$ and $g' \in G(m)$.

Any critical network $g$ is also sustainable as a pure strategy maximal equilibrium. Because criticality is defined such that if any agent $i$ delete a link, $i$ expects to lose at least $m$ links in the sequel. So criticality is sufficient, but not necessary for pure strategy ME. The network in Example 2 is sustainable in pure strategy ME, but it is not critical.

We also keep the definition of simply critical networks. Since criticality is not related to the utility of the agents. All the results on them can be generalized to maximal equilibria directly. Such as the following useful proposition, which ensures the social quilts are maximal networks:

PROPOSITION **3** *A tree union of simply critical networks is maximal.*

#### 5.1.1 Transitively Critical Networks

We define *transitively critical* networks for maximal equilibria as follows.

Given a whole number $m$, let $TC_k^M(m) \subset G_{k'}$ denote the set of transitively critical networks with $k$ links, where $m$ satisfies (1).

- Let $TC_0^M(m) = \emptyset$.

- Inductively on $k$, $TC_k^M(m) \subset G_k$ is such that $g \in TC_k^M(m)$ if and only if for any $i$ and $ij \in g$, there exists $g' \subseteq g - ij$ such that $g' \in TC_{k'}^M(m)$, $d_i(g') \leq d_i(g) - m$, and there is no $g'' \in TC_{k''}^M(m)$ for any $k''$ such that $g' \subset g'' \subset g - ij$.

Even though this is also an inductive definition (not surprisingly, given that maximal equilibria are so defined), it does not involve any incentive descriptions and is effectively an algorithm that can be run on any graph without any knowledge of agents, payoffs, etc.

Now let us examine maximal equilibria.

THEOREM **2** *Let $m$ satisfy (1). A network is sustainable as a pure strategy maximal equilibrium if and only if it is transitively critical.*

Proof of Theorem 2 is very similar to the Proof of the corresponding theorem in the paper. So we omit it here.

## 5.2 Social Quilts

Recall a *m-clique* is a complete network with $m+1$ nodes so that every node has exactly $m$ links. m-cliques are an important class of critical networks.

Note that different from pure strategy renegotiation-proof equilibria, a clique (completely connected subnetwork) $g$ with $m+2$ nodes (each having $m+1$ links) is sustainable as a pure strategy maximal equilibrium. To see this, have some $i$ delete a link $ij$. The continuation network is a m-clique $g'$ with all agents but $i$. It is not a valid continuation in pure strategy RPE since we can find some network $g''$ Pareto dominating it. However, it is a valid continuation in pure strategy ME. By the definition, there should not exist $g''$ such that $g'' \in ME_{k''}$ and $g' \subset g'' \subset g - ij$. Suppose such $g''$ does exist, $i$ should be in $g''$ with at least $m$ links. So the only possible $g'' = g - ij$, which is not sustainable in pure strategy ME. To see this, if any agent $j \neq i$ refuses a favor to $i$, the network in continuation should be $g'$ and $j$ only loses one link. So $g''$ is not sustainable.

We keep the definition that a network $g$ is an *m-quilt* if $g$ can be written as the union of m-cliques, such that any two of these cliques intersect in at most one node and there are no simple cycles involving more than $m+1$ nodes.

## 5.3 Robustness

Note the following observation:

OBSERVATION **1** *If (1) holds for $m \geq 2$, $g$ is sustainable as a pure strategy maximal equilibrium, and $ij \in g$, then $g - ij$ is not sustainable as a pure strategy maximal equilibrium.*

Thus, beginning from some ME network, if a link is deleted then the network will necessarily further degrade in terms of what is sustainable.

The definition of robustness remains for ME such that a network $g$ is *robust against social contagion* if it is renegotiation-proof and sustained as part of a pure strategy subgame perfect equilibrium with $g_0 = g$ such that in any subgame continuation from any renegotiation proof $g' \subset g$, and for any $i$ and $ij \in g'$, if $i$ does not perform the favor for $j$ when called upon, then the continuation leads to $g''$ such that if $h\ell \notin g''$ then $h \in N_i(g') \cup \{i\}$ and $\ell \in N_i(g') \cup \{i\}$.

Thus all theorems discussing the structures of the robust networks work for pure strategy maximal equilibria as well. Especially the following two:

THEOREM **3** *A network is robust against social contagion* if and only if *it is a social quilt.*

THEOREM **4** *In the asymmetric payoffs case, if a network $g$ is robust against social contagion then all links in $g$ are supported.*

# 6 Background Statistics on the Indian Village Networks

## 6.1 Descriptive Statistics

In this section we present some snapshots of the network data discussed in Section 6.

The graphs shown in Figure 9 summarize the distributions of the normalized degree, betweenness and eigenvalue centralities in each of the networks of relationships described in Tables 1 and 2 of Section 6.2. The distributions were computed by considering the network defined among all the surveyed people in the 75 villages in our sample by each relationship type[5]. Mores specifically, the aggregate sample is comprised by all the people that were surveyed and who reported at least one relationship of the types described in Table 1 of Section 6.2 with some other surveyed individual. The total number of people in the sample thus defined was 16855.

The first graph for each relationship type in Figure 9 shows the inverse cumulative distribution functions of normalized degree, betweenness and eigenvalue centralities. The second graph shows the inverse cumulative distribution function of normalized degree and the betweenness and eigenvalue centralities of the marginal village according to the normalized degree.

Figure 10 exhibits the distribution of the number of reported relationships by surveyed individuals for each relationship type described in Table 1 of Section 6.2. As shown in these graphs and discussed in Section 6.2, only a very small fraction of the surveyed population reported a number of relationships reaching the limits of 5 or 8 implied by the survey design.

[5] Note that relationships were restricted to lie within each village.

## 6.2 Support in the Data

Figure 11 shows the inverse cumulative distribution functions of support $S(g', g)$ in our sample of 75 villages for a number of combinations of the networks $g$ and $g'$ defined in Table 2 of Section 6.1. Each graph also includes the plots of the fraction of links supported by exactly $k$ other links in the marginal village (ordered according to their aggregate support levels). Note that the context network $g$ that defines in each case whether a given link in $g$ is supported or not, is always by construction a superset of $g'$.

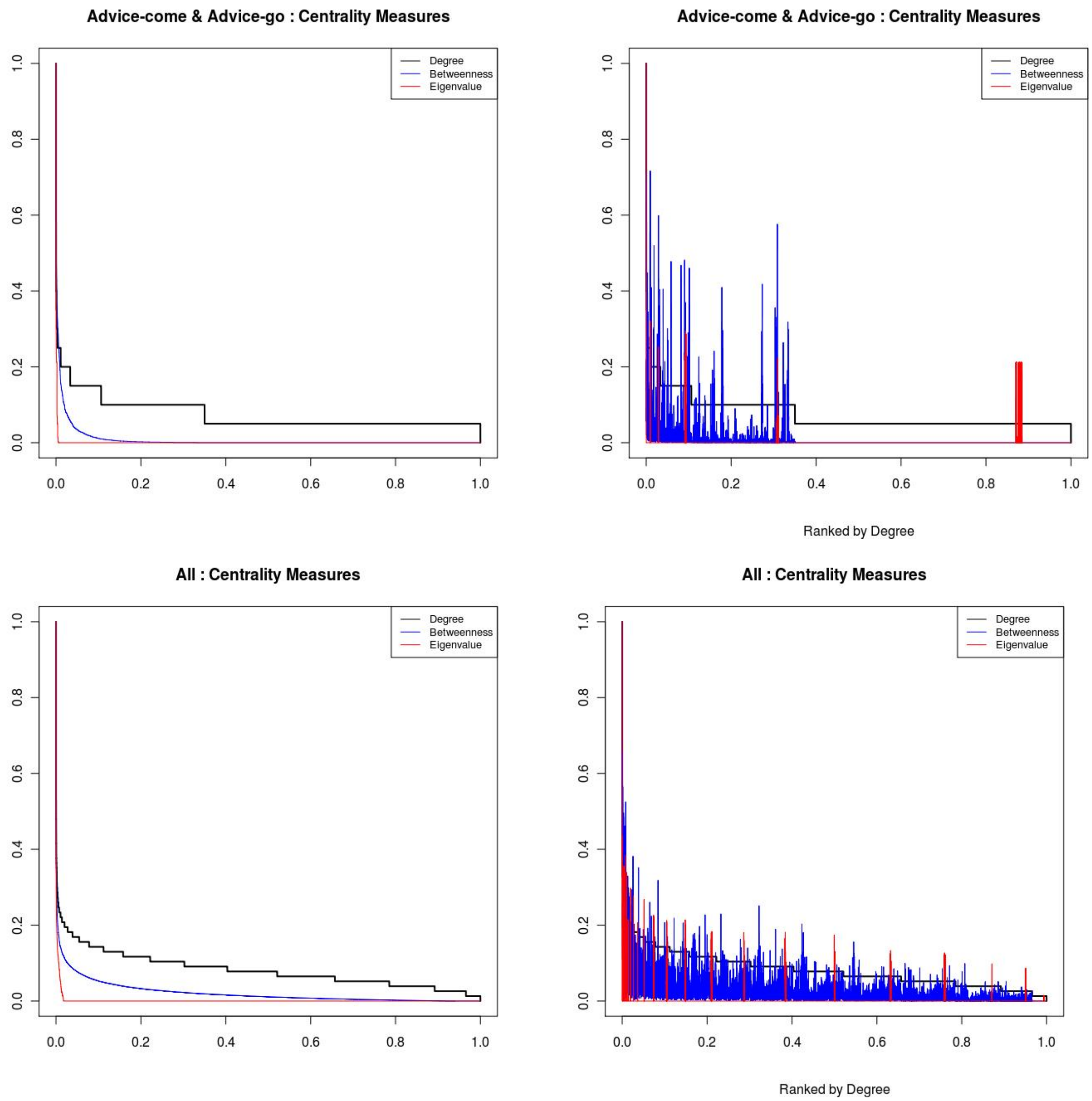



Figure 9: Centrality Measures

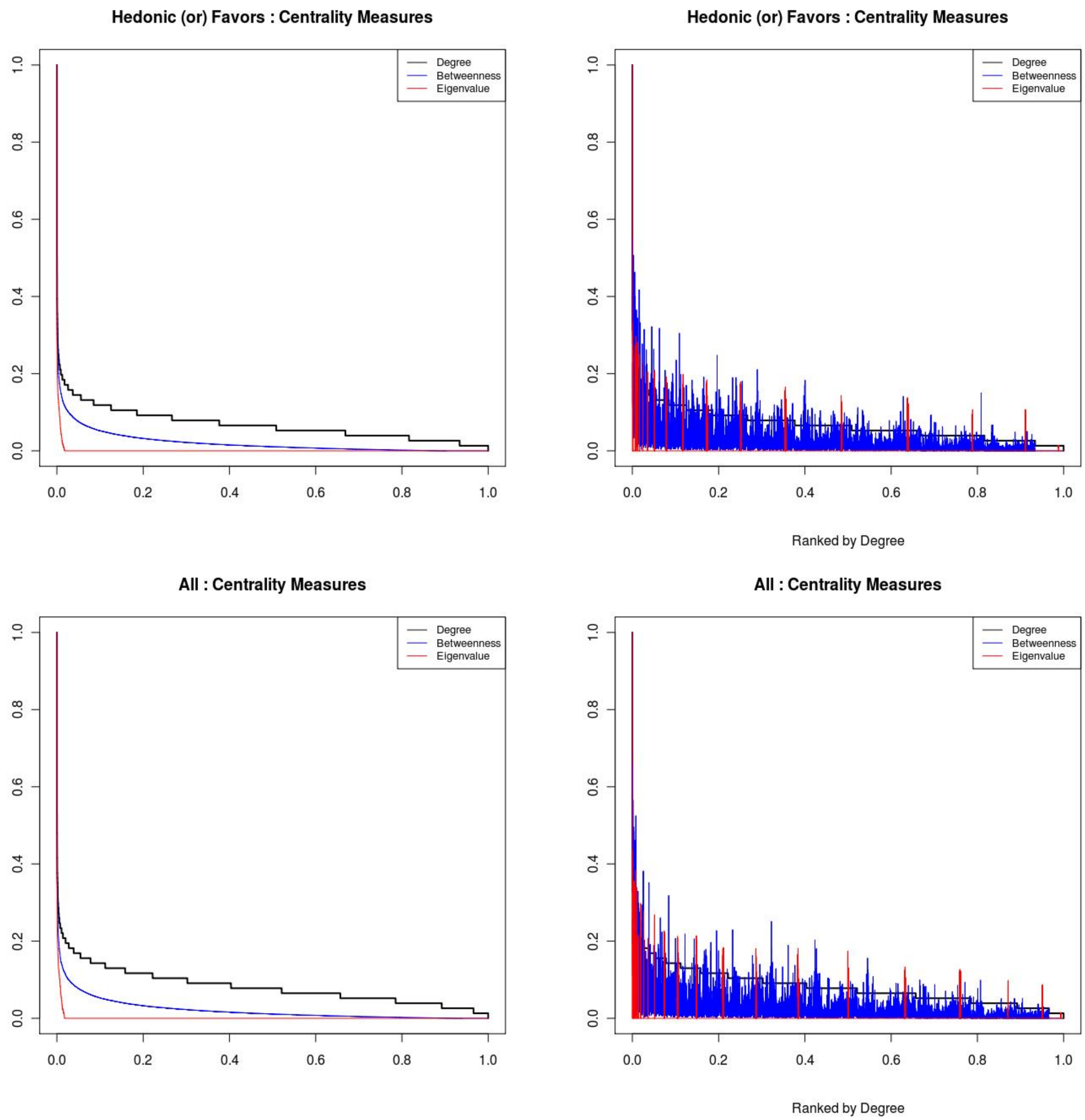



Figure 9 (continued)

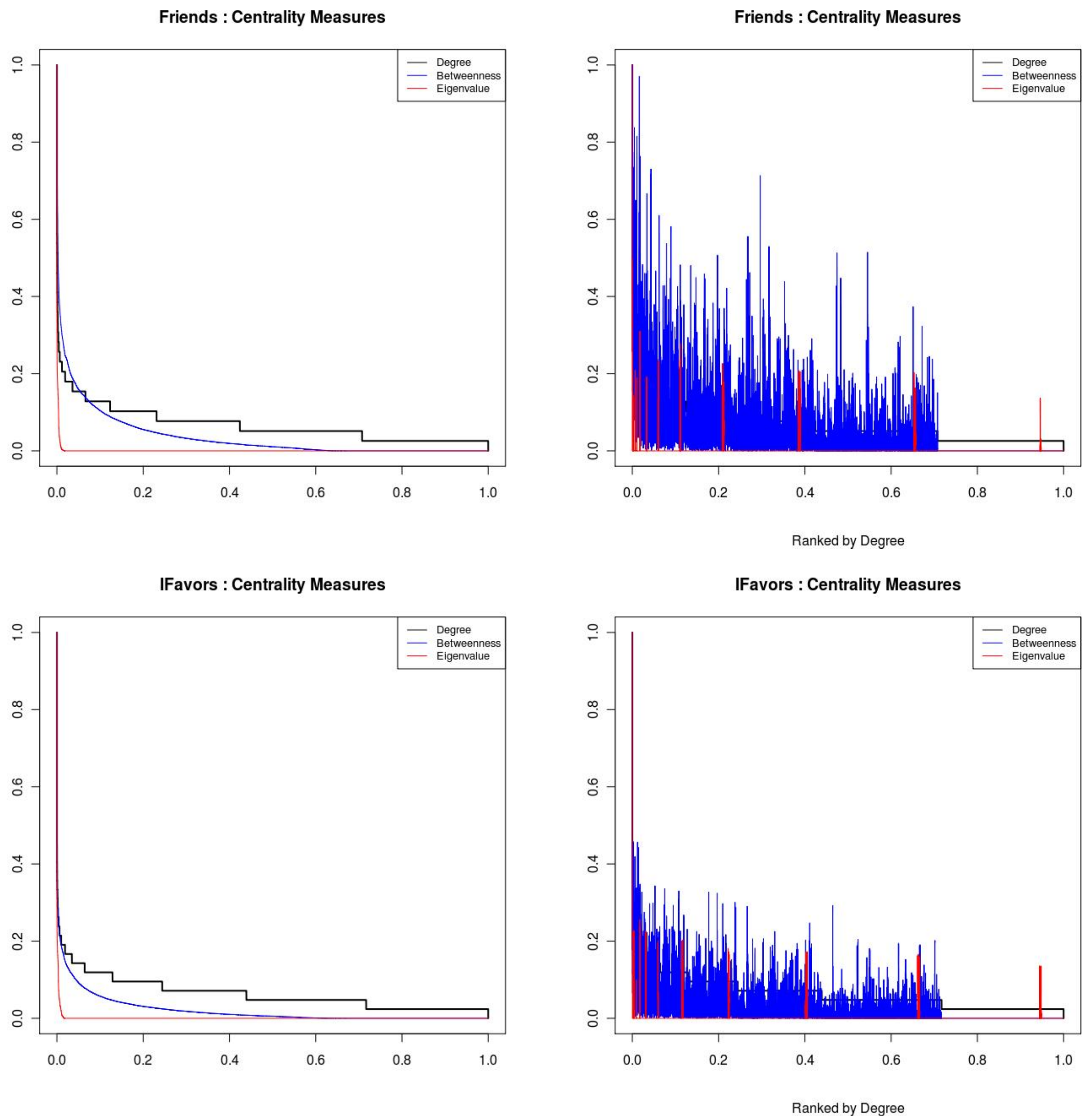



Figure 9 (continued)

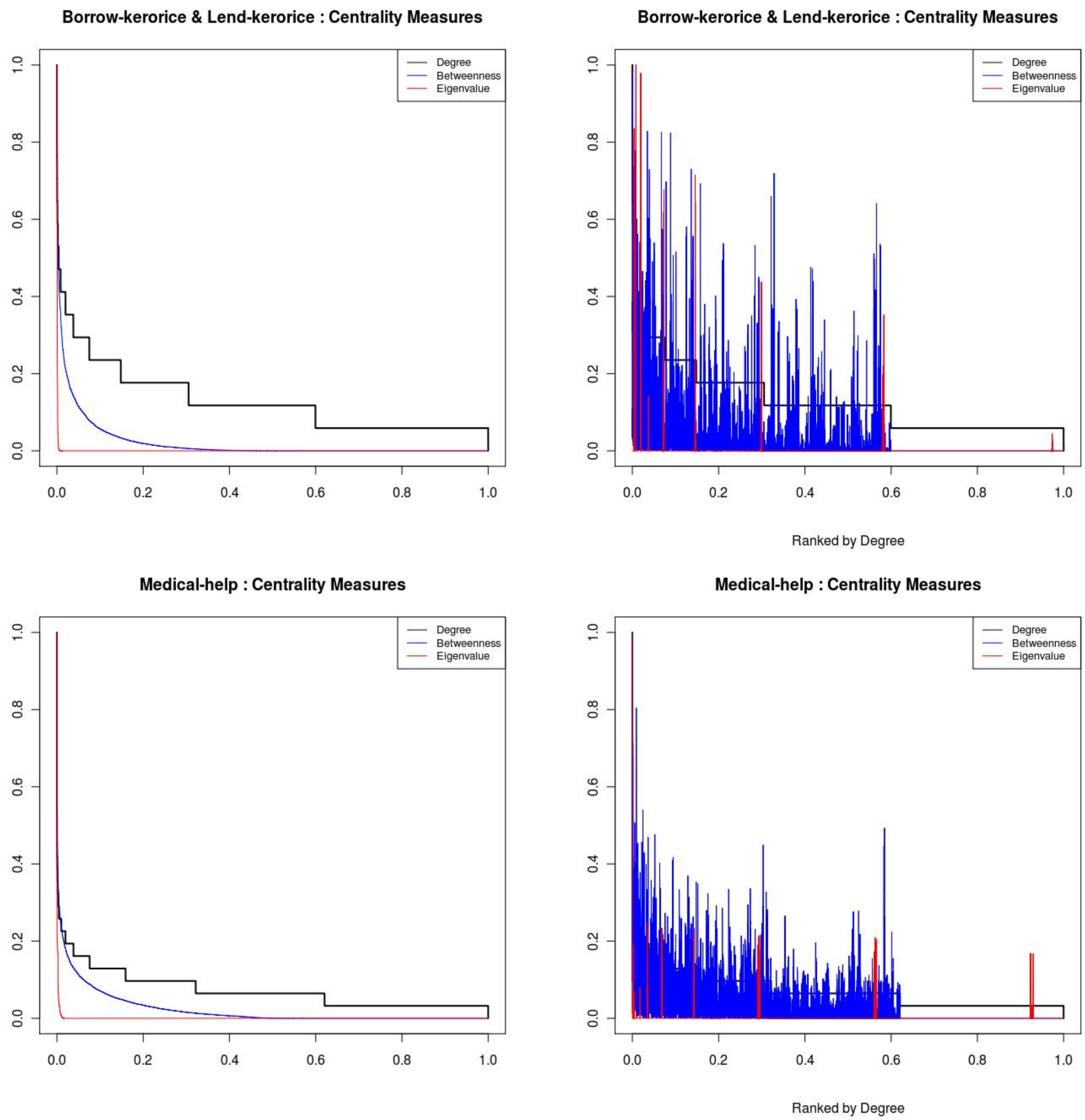



Figure 9 (continued)

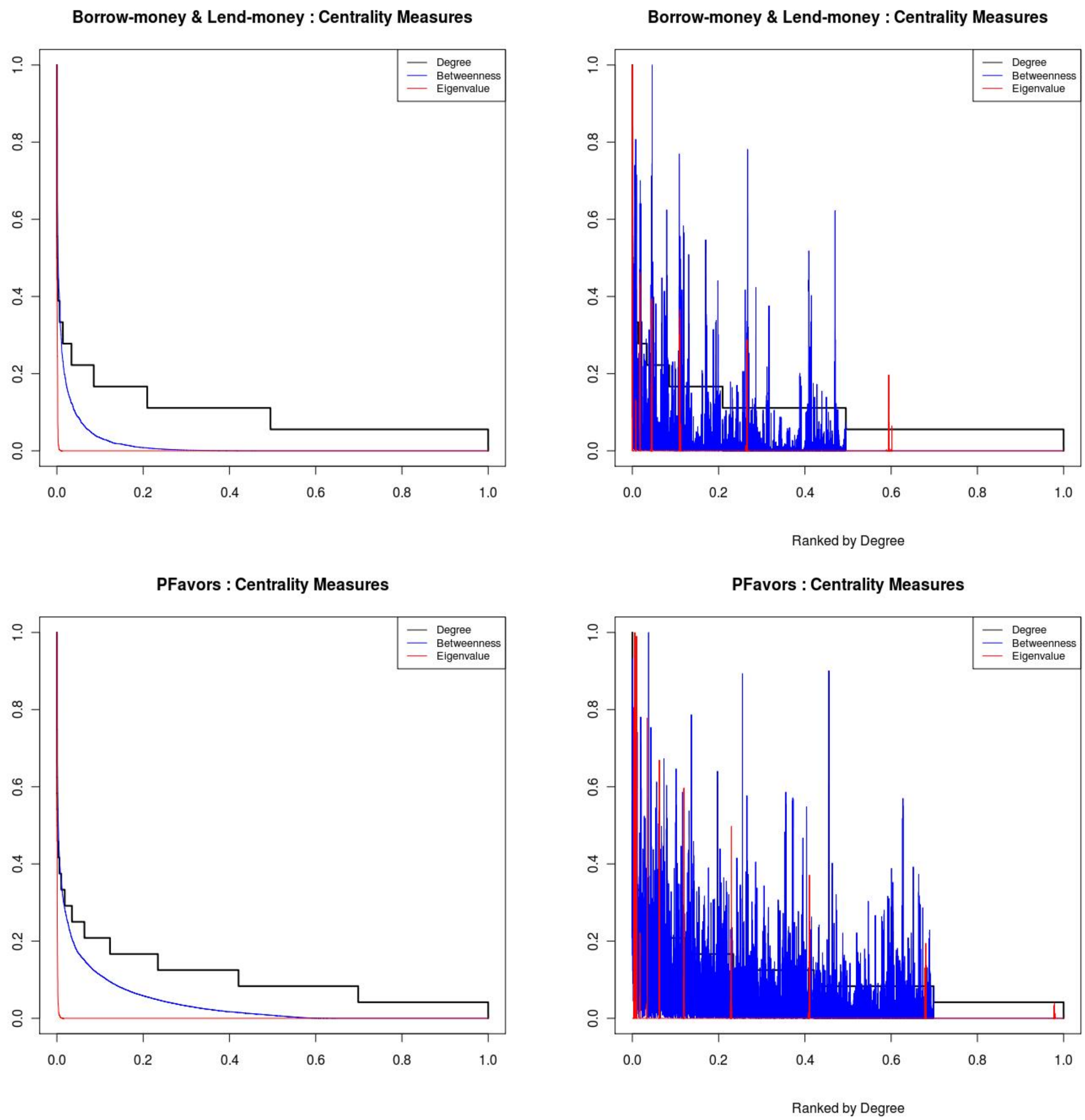



Figure 9 (continued)

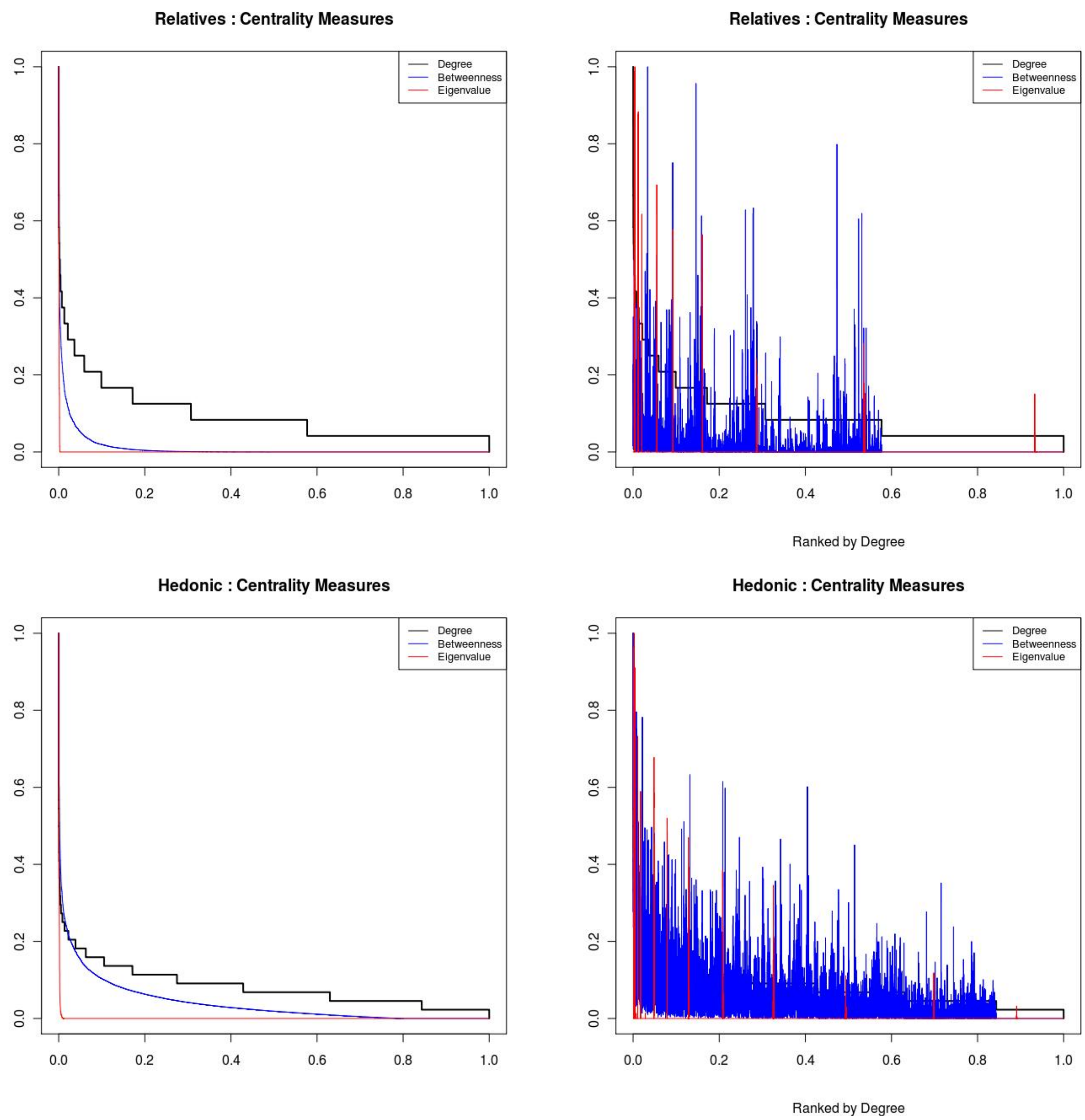



Figure 9 (continued)

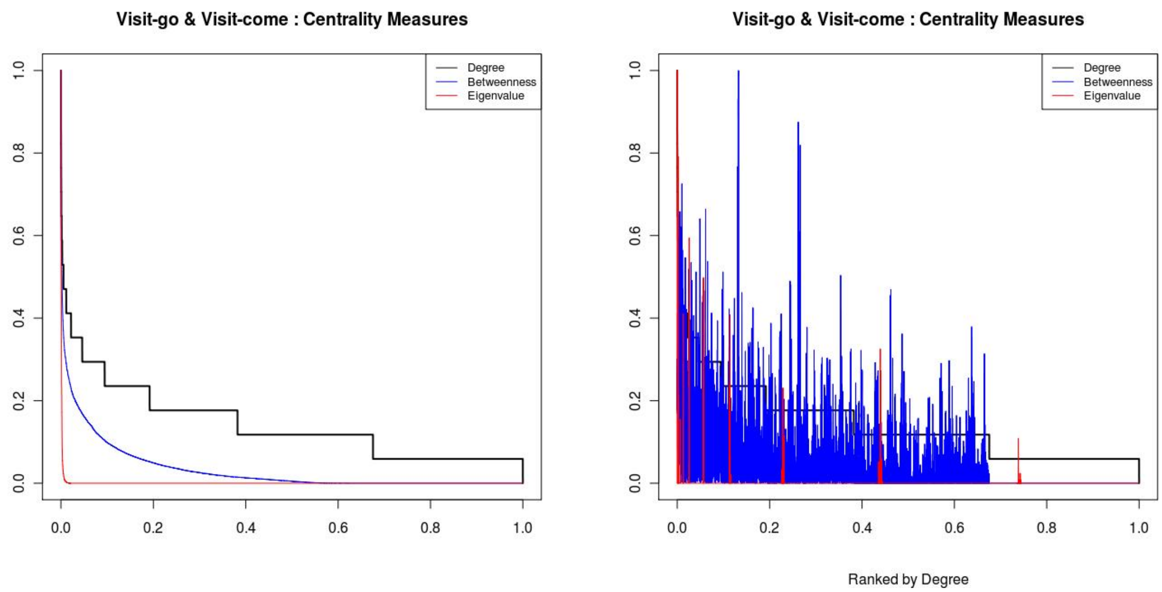



Figure 9 (continued)

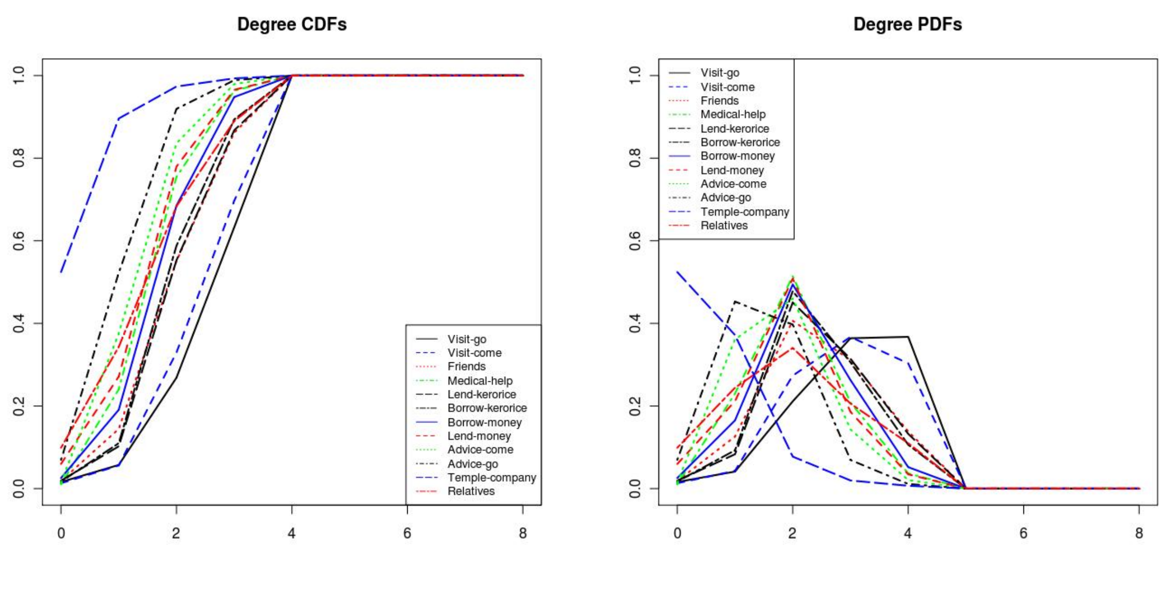



Figure 10: Distribution of the numer of reported relationships by surveyed individuals.

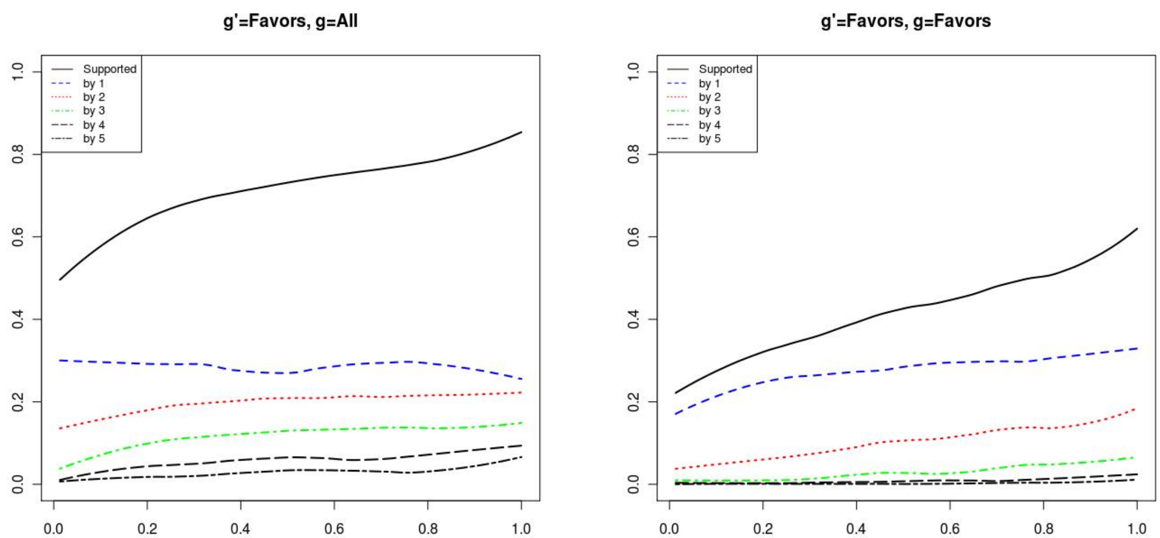



Figure 11: The inverse cumulative distribution function of support levels in the villages: The horizontal axis is the fraction of villages having support no more than the amount listed on the vertical axis. The upper-most curve is the inverse CDF of the fraction of supported $g'$ relationships in the $g$ network. The five curves below list the breakdown of the fraction for the marginal village by various levels of support: "by k" indicates the fraction of links in that village that are supported by exactly k other nodes (so that $i$ and $j$ have k friends in common), and so the five lines below sum to the curve above.

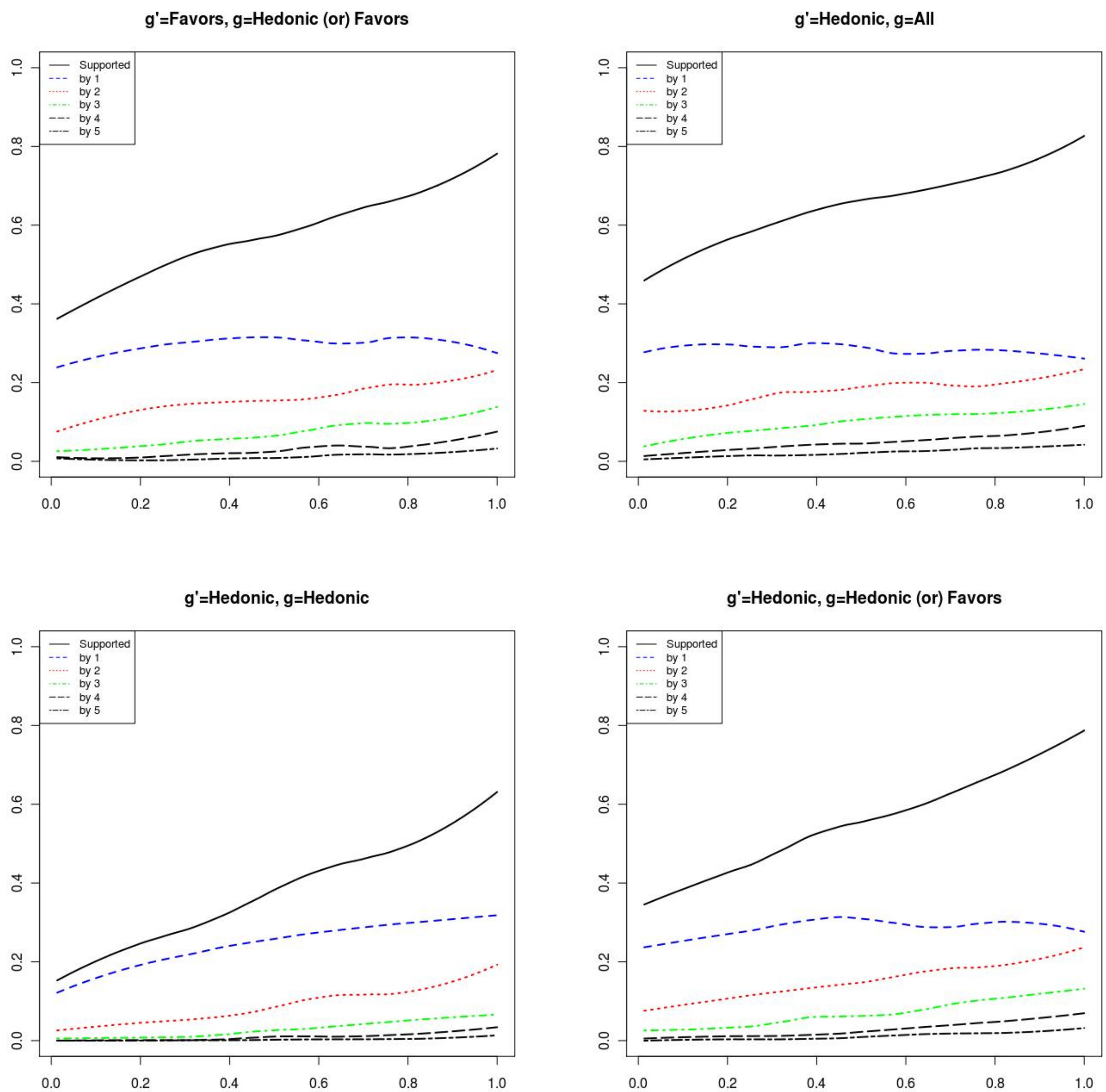



Figure 11 (continued)

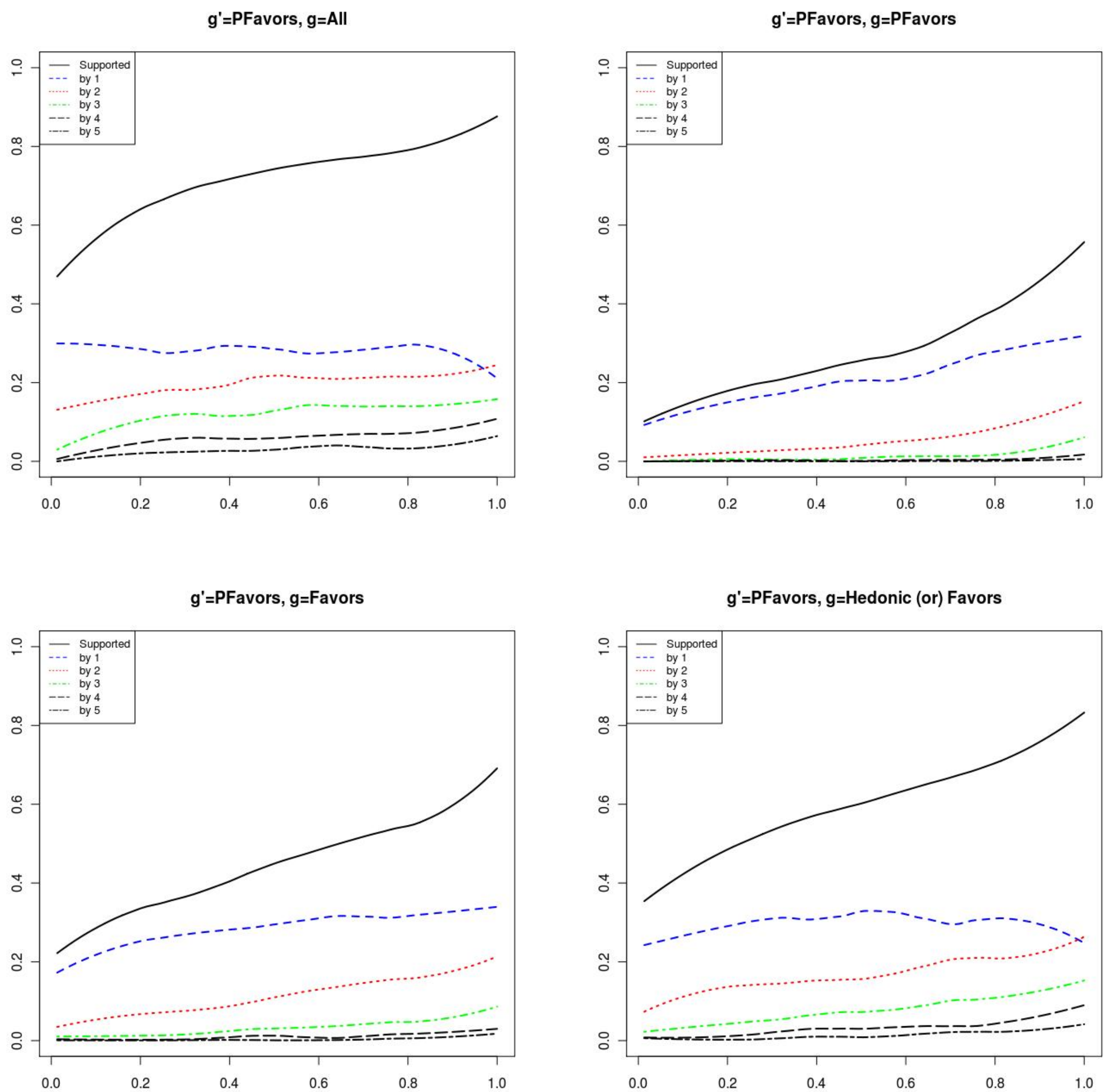



Figure 11 (continued)

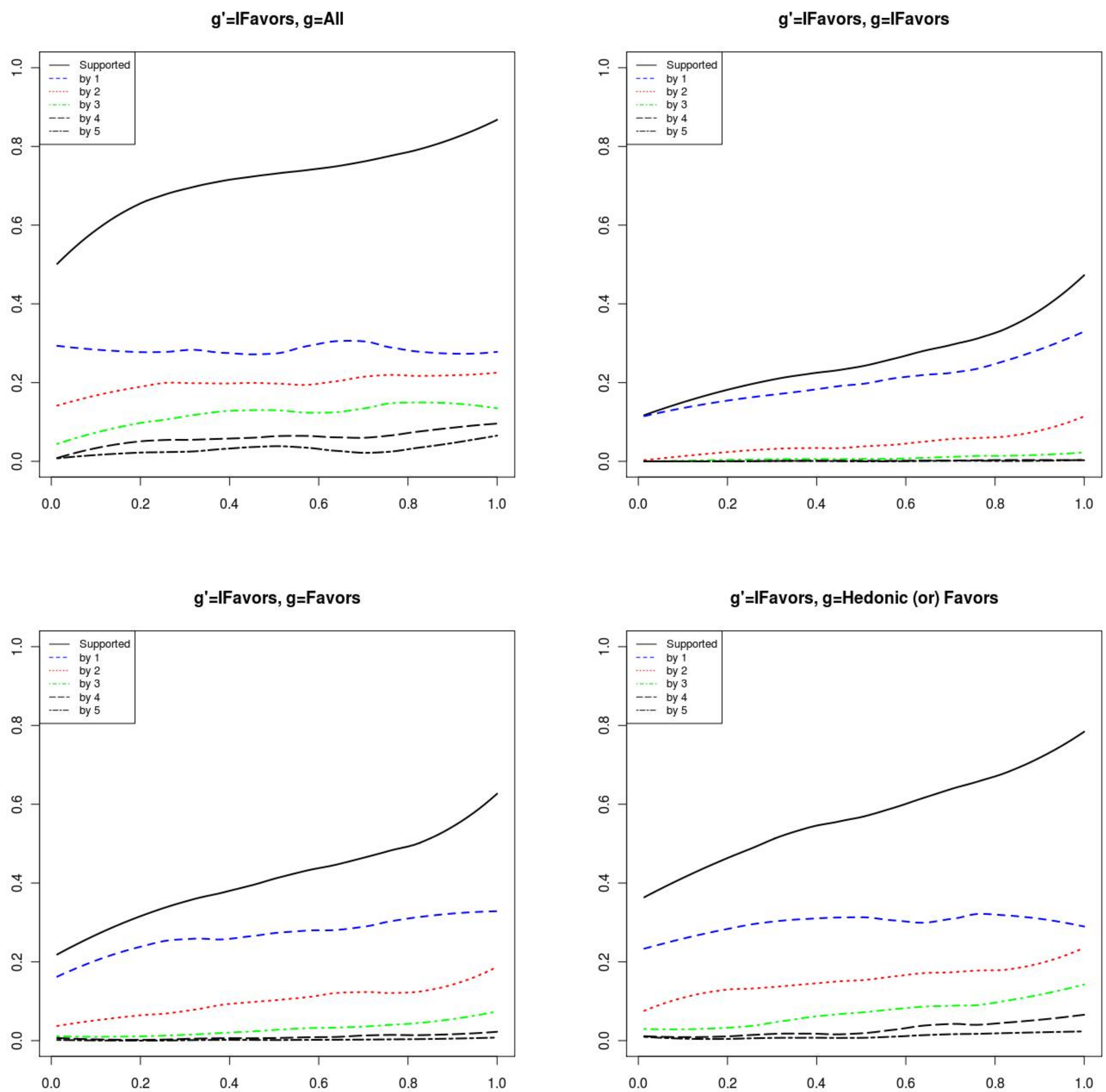



Figure 11 (continued)

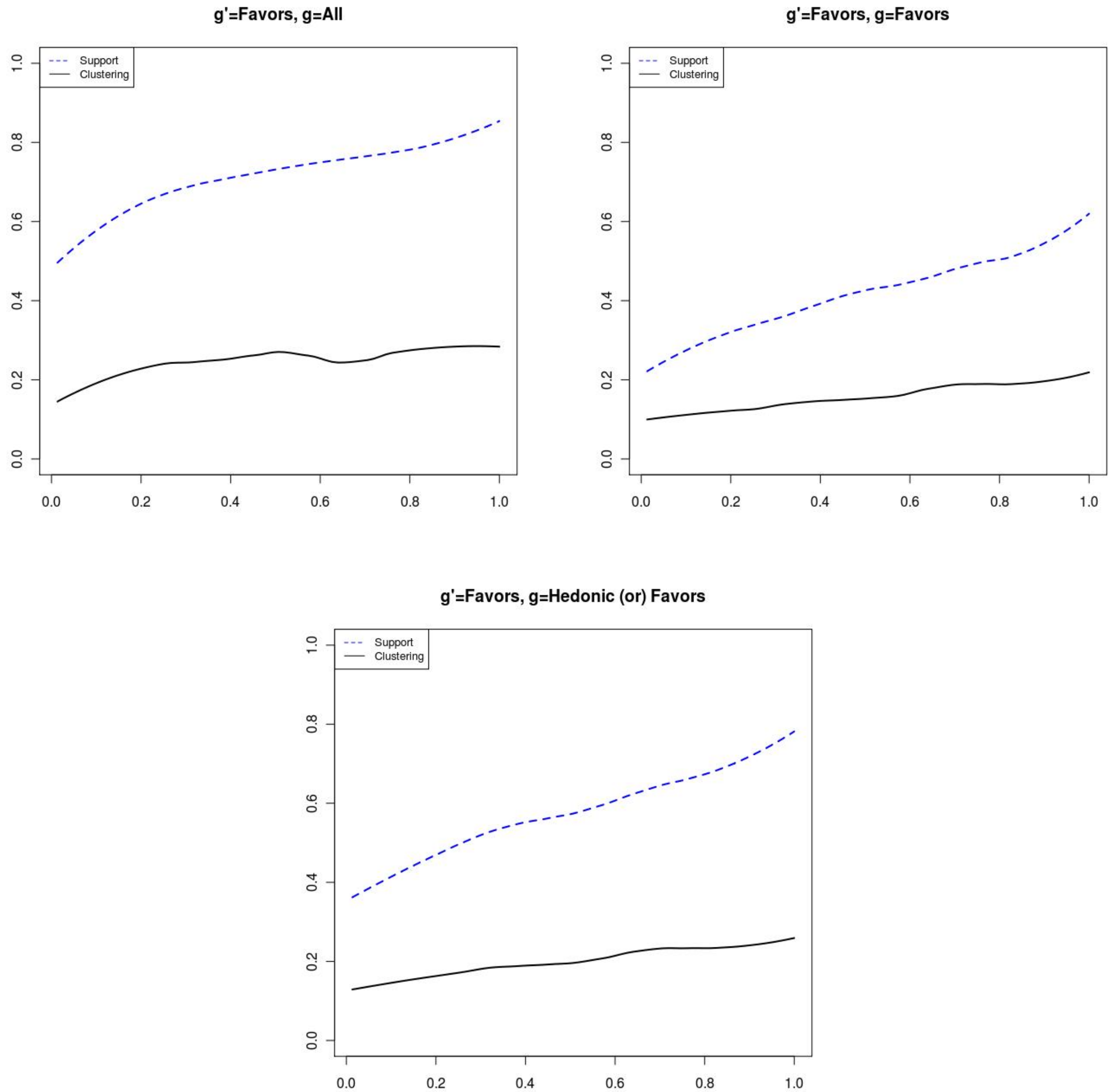



Figure 12: The inverse cumulative distribution function of support and clustering levels in the villages: The horizontal axis is the fraction of villages having support/clustering no more than the amount listed on the vertical axis. The upper-most curve is support and the lower-most is the clustering coefficient of the marginal village.

## 6.3 Comparing Support to Clustering

Figure 12 shows graphs comparing clustering and support in the various networks defined in Table 2 of Section 6.1.

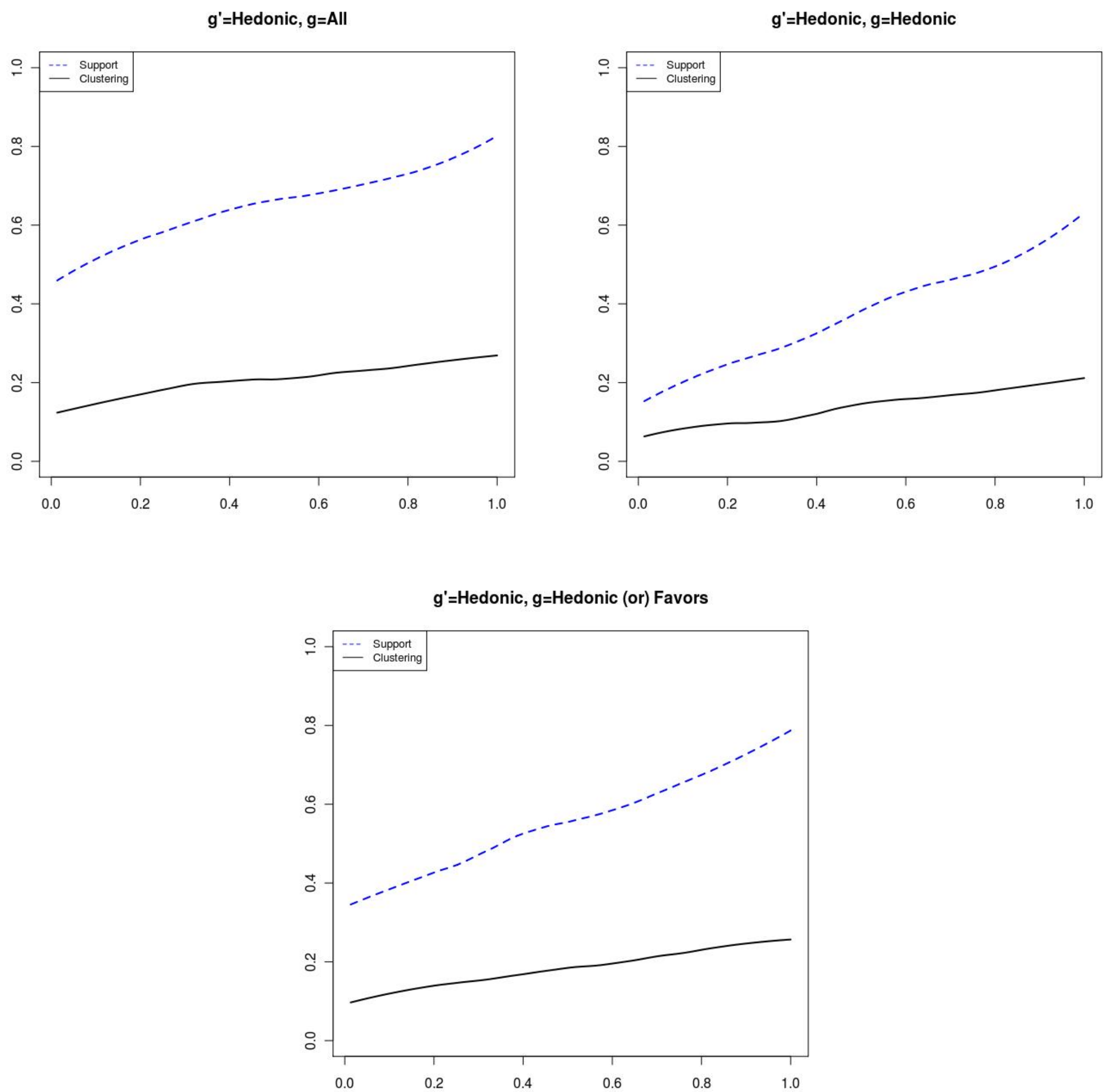



Figure 12 (continued)

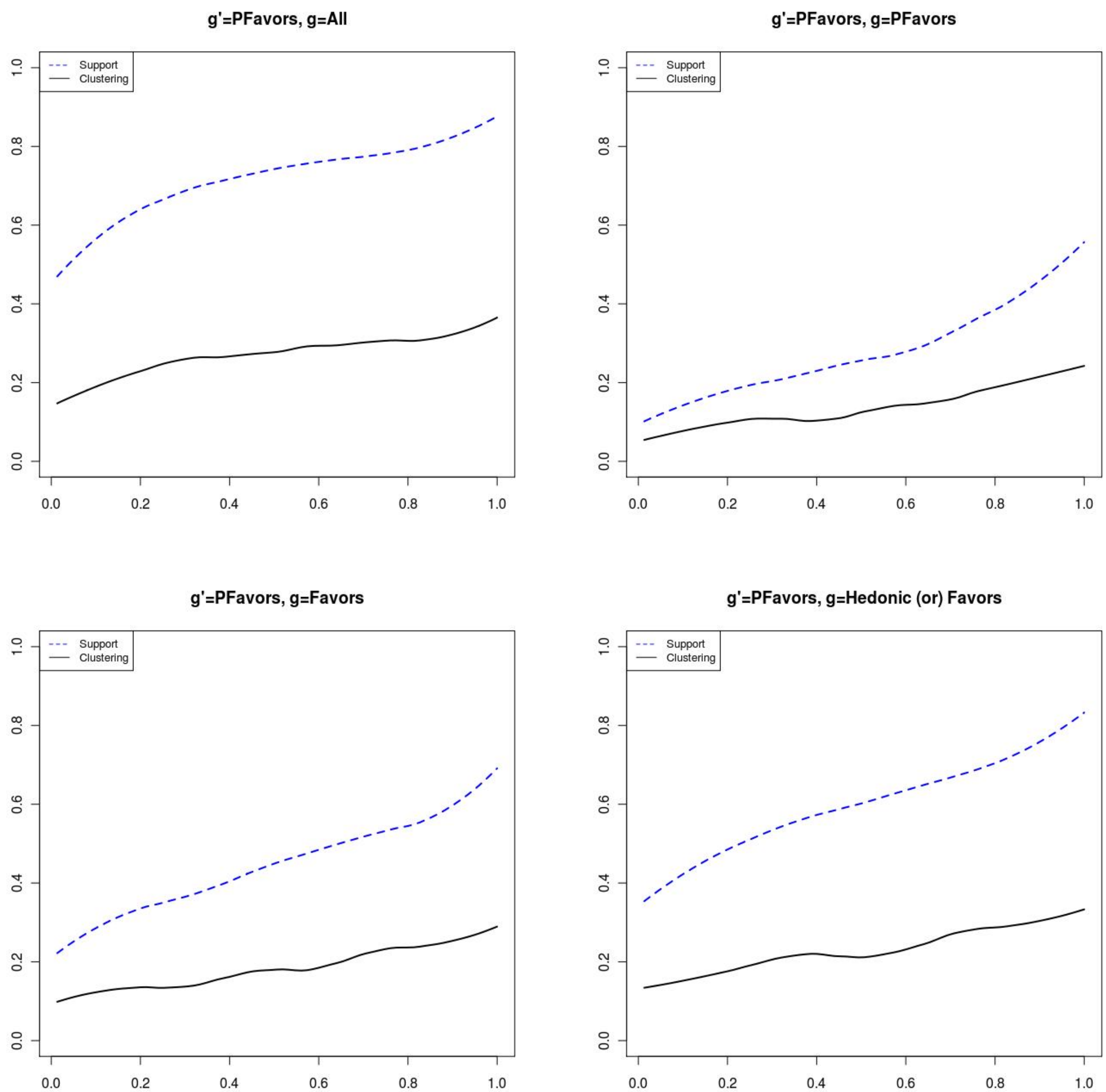



Figure 12 (continued)

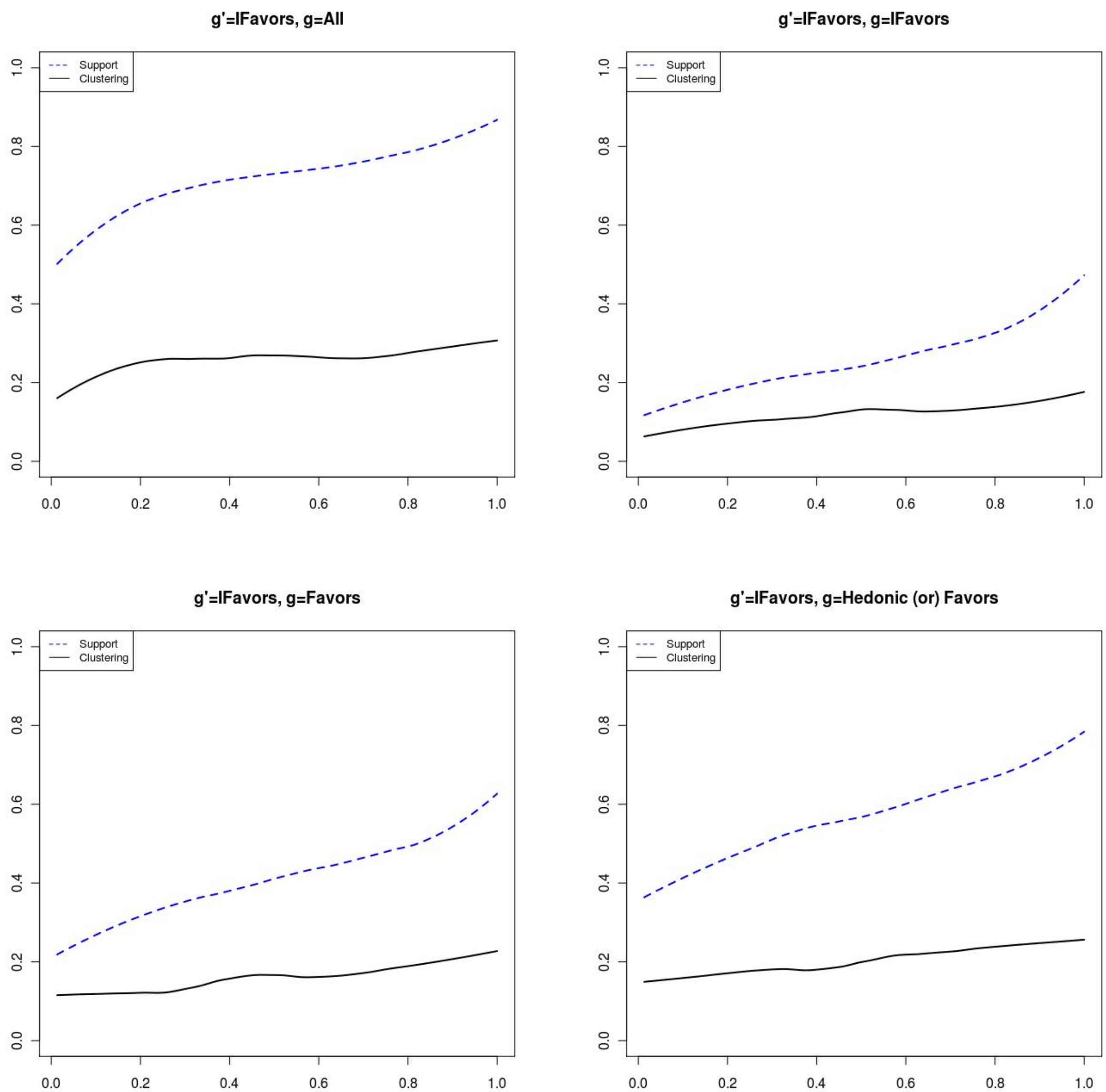



Figure 12(continued)

## 6.4 Comparing Support in Different Sorts of Relationships

In the main paper, we worked with a definition of an "All" network that included relatives. We also now include various calculations that include definitions that simply look at the union of "hedonic" and "favor" networks, $H \cup F$ in what follows.

Tables 3-5 show the comparison of support measures of various relationships. The entry $ij$ of the first table reports the number of villages for which $S(\mathbf{g}'_i, All)) > S(\mathbf{g}'_j, All))$. The entry $ij$ of Table 4 reports the number of villages for which $S(\mathbf{g}'_i, H(or)F)) > S(\mathbf{g}'_j, H(or)F))$, where $H(or)F$ is the union of the *Favor* network and the *Hedonic* network. Finally, Table 5 presents the comparison of self support measures. That is, the entry $ij$ reports the number of villages for which $S(\mathbf{g}'_i, \mathbf{g}'_i)) > S(\mathbf{g}'_j, \mathbf{g}'_j)$[6].

[6]The entries the self support of the Favors network with that of the Physical Favors and Intangible Favors networks are left blank because since they are 75 by definition (Since each of the latter are subnetworks of the favors network).

| **Support Measure** | **Self** | **H ∪ F** | **All** |
|---|---|---|---|
| Physical Favors | 0.2807 | 0.6002 | 0.7200 |
| Intangible Favors | 0.2587 | 0.5721 | 0.7198 |
| Hedonic | 0.3795 | 0.5569 | 0.6530 |
| H ∪ F | – | 0.5556 | – |
| All | – | – | 0.6931 |

Table 2: The Average Support Measure

| *Network* $\mathbf{g}'$ | Favors | Physical Favors | Intangible Favors | Hedonic |
|---|---|---|---|---|
| Favors | – | 30** | 24*** | 72*** |
| Physical Favors | 45** | – | 38 | 69*** |
| Intangible Favors | 51*** | 37 | – | 72*** |
| Hedonic | 3*** | 6*** | 3*** | – |
| *** significant difference at 1% level | | ** significant difference at 5% level | | |

Table 3: Comparison of Support Measures.
Entry $i, j$ is the number of villages for which $S(\mathbf{g}'_i, All)) > S(\mathbf{g}'_j, All))$.

| *Network* $\mathbf{g}'$ | Favors | Physical Favors | Intangible Favors | Hedonic |
|---|---|---|---|---|
| Favors | – | 15*** | 40*** | 46 ** |
| Physical Favors | 60*** | – | 57*** | 63*** |
| Intangible Favors | 35 | 18*** | – | 38 |
| Hedonic | 29 ** | 12*** | 37 | – |
| *** significant difference at 1% level | | ** significant difference at 5% level | | |

Table 4: Comparison of Support Measures.
Entry $i, j$ is the number of villages for which $S(\mathbf{g}'_i, H \cup F)) > S(\mathbf{g}'_j, H \cup F))$.

## 6.5 How Observed Support Compares to that Expected in a Random Network

One way to get a feeling for how much support we observe is to compare the observed level of support with that which would arise if the same number of links were instead distributed purely at random.

| *Network* $\mathbf{g}'$ | Favors | Physical Favors | Intangible Favors | Hedonic |
|---|---|---|---|---|
| Favors | – | – | – | 55*** |
| Physical Favors | – | – | 48*** | 8*** |
| Intangible Favors | – | 27*** | – | 11*** |
| Hedonic | 20*** | 8*** | 64*** | – |
| *** significant difference at 1% level | | ** significant difference at 5% level | | |

Table 5: Comparison of Support Measures.
Entry $i, j$ is the number of villages for which $S(\mathbf{g}'_i, \mathbf{g}'_i)) > S(\mathbf{g}'_j, \mathbf{g}'_j))$.

In a random network with a the probability $p$ of a link and $n$ as the population of the network, the expected support measure can be approximated as follows: the average degree is $D = p \cdot (n-1)$, and the chance any given link is supported is roughly $S = 1-(1-p)^{(D-1)}$, since the chance a link $ij$ is not supported is that none of the other $D-1$ friends of agent $i$ are friends of agent $j$ which is $(1-p)^{(D-1)}$.[7] From the data, we can calculate $p$ and $n$ for each village and estimate what the support measure $S$ would be if the network were generated uniformly at random. Table 6 shows the average support measure in real networks are substantially larger than those expected in random networks.

| | **S(PF, PF)** | **S(IF, IF)** | **S(H, H)** | **S(H∪ F, H ∪ F)** | **S(All, All)** |
|---|---|---|---|---|---|
| Observed network | 0.2807 | 0.2587 | 0.3795 | 0.5556 | 0.6931 |
| Random network | 0.0137 | 0.0172 | 0.0400 | 0.0960 | 0.1487 |

Table 6: Average Support Measures for Observed and Random Networks

While these numbers are suggestive, we now provide a more detailed statistical test to see if the support is higher than would be generated at random, where random also allows for geographic biases.

### 6.5.1 Geography and Support

We proceed in two different manners. First, we build an explicit random network model that incorporates geography directly. Next, we work with an exponential random graph model.

[7]This approximates things since not all nodes are expected to have degree $D$. Slightly more accurate estimates could be obtained by working with the specific degree distribution that would be generated, or some other degree distribution. We do not pursue those, since we provide a more rigorous test with the geographic based networks in any case.

Some of the relationships in these data are bound to be at least partly correlated by geographic closeness, since it is natural to expect some sorts of favor exchange among geographic neighbors , and geographic closeness is a transitive relation. Therefore networks that we observe may inherit some support from this geographic determinacy in a manner unrelated to the network structure based favor exchange that we have examined. In order to address this issue, we examine a geographically-biased-random network formation model and then see whether the support measures from that model differ in a statistically significant way from the observed support measures.

We proceed as follows.

- For each village we decomposed the observed links of each type into deciles according to the geographic proximity of the members of the pair in question, as measured by the households' GPS coordinates. Based upon this decomposition we constructed an empirical link distribution for each specific relationship and each village.

- We then carried out 50 simulations for each village and relationship. In each case we constructed a random graph based on the corresponding empirical distribution of links by geographic location. In order to guarantee that each simulated base network was a subset of the context network, we produced the context network by augmenting the simulated base network with the appropriate number of randomly drawn links according to the appropriate conditional distribution.

- We measured the support of each simulated network, and by comparing it to the observed support for the corresponding village and relationship, created a realization of a random variable with value 1 if the simulated support measure exceeded the observed support and 0 otherwise.

- Pooling all the random variables generated according to this method for a given relationship across all villages, we performed a one sided test of the null hypothesis that the random variable was binomially distributed with equal probability of being 1 or 0.[8] As shown in Table 7, for every one of relationships, the observed support is significantly greater (with p-values smaller than 0.0001 in all cases) than the one generated by the geographically biased random graph models.

[8]The variances of the random variable are likely to be different for different villages. Not taking into account heteroskedasticity biases the test against rejection of the null hypothesis.

| Base-Context | p-value |
|---|---|
| PF-PF | 0.000 |
| PF-HR | 0.000 |
| PF-All | 0.000 |
| IF-IF | 0.000 |
| IF-HR | 0.000 |
| IF-All | 0.000 |
| HR-HR | 0.000 |

Table 7: Binomial one sided test

### 6.6 How Observed Support Compares to that Expected via an Exponential Random Graph Model Incorporating Geography

Table 8 shows the estimated coefficients and the standard errors in the collection of models:

$$log(Pr(G = \mathbf{g})) = \beta_0 + \beta_1 \sum_{i<j} \mathbf{g}_{ij} + \beta_2 \sum_{i<j} \mathbf{g}_{ij}\mathbf{s}(\mathbf{g}, \mathbf{g}')_{ij} + \beta_3 \sum_{i<j} d(i,j)\mathbf{g}_{ij} \quad (3)$$

We have fit one such model for each of the *Favors* networks in each of the 75 villages in the sample. The indicator function $\mathbf{s}(\mathbf{g}, \mathbf{g}')_{ij}$ is 1 if and only if link $ij$ is supported in the *All* networks of the corresponding village.

We used version 2.1-1 of the R package *statnet* developed by Handcock et.al. to estimate the exponential random graph models.

## 6.7 Bounding Measurement error

Another thing that we do is examine how much measurement error there would have to be in order to see support measures of the level that we observed if the true support level were really 100 percent. In particular, what fraction of links would have to be missing to get the observed relationships?[9] Specifically, the types of errors that are likely to arise in our data are one-sided: while people are quite likely to forget relationships, it is less likely that they "imagine" ones given the way in which these questions were designed.[10] To address this issue, there are various ways in which one might proceed, and here we followed a fairly simple one where we simulated the survey process 100 times, proceeding as follows in each iteration:

For each village in our sample and type combination of base-context networks in question we consider the closest network to the context network that leads the base network to have full support; where closest is defined as having the least number of additional links.[11] We then remove each link in the augmented network with a measurement error probability, and calculate the support of the resulting base-context pair. Figure 13 shows for each measurement error in the $x$ axis the mean fraction of villages in the sample that ended with a support fraction of at most the level observed in the survey. It should be noted that the number of simulations (100) is such that any differences in expectation for different measurement errors in a given base-context pair or for different base-context-pairs are statistically significant. Table 9 provides a closer look at a small segment of Figure 13.

## 6.8 The Relation of Support to other Characteristics of Links, Households and Individuals

### 6.8.1 Link Level Predictors of Support

The first collection of statistics, presented in Tables 10-14, builds upon the fact that support is firstly a property of relationships themselves, rather than a property of agents. We look at the likelihood that a a pair of agents have at least one friend in common conditional on them being similar/disimilar according to a number of individual characteristics: education, age, caste, gender and participation in the microfinance program[12]. Specifically we break the set of pairs of agents in each village into similarity/dissimilarity classes for each of characteristic

[9] This test is biased against us since we are not considering missing nodes.

[10] By asking questions regarding actual actions (borrowing or lending money or rice, visiting someone's home, etc.) rather than asking about perceived relationships (who is your friend), we eliminate many problems with misperceived or asymmetric sorts of relationships.

[11] We randomly draw one network from the set of closest networks.

[12] As discussed above, the data that we use was collected as part of the deployment of a micro-finance program (see Banerjee et al. (2010)).

| **Village** | $\hat{\beta}_1$ | $\hat{\beta}_2$ | $\hat{\beta}_3$ |
|---|---|---|---|
| 1 | -3.535 | 1.951 | -2.64591 |
| | (0.103) | (0.094) | (0.15248) |
| 2 | -3.411 | 2.031 | -2.99944 |
| | (0.108) | (0.102) | (0.18072) |
| 3 | -5.741 | 3.228 | -0.0239 |
| | (0.055) | (0.066) | (0.00903) |
| 4 | -5.416 | 3.138 | -0.06672 |
| | (0.064) | (0.077) | (0.01876) |
| 5 | -5.283 | 3.287 | -0.10529 |
| | (0.098) | (0.111) | (0.02784) |
| 6 | -4.562 | 2.435 | 0.02029 |
| | (0.099) | (0.126) | (0.01198) |
| 7 | -3.831 | 2.5 | -2.26155 |
| | (0.143) | (0.129) | (0.14617) |
| 8 | -4.6 | 2.305 | -0.00219 |
| | (0.109) | (0.125) | (0.01419) |
| 9 | -5.57 | 2.845 | -0.01145 |
| | (0.071) | (0.086) | (0.00866) |
| 10 | -4.599 | 2.256 | -0.28231 |
| | (0.161) | (0.163) | (0.09628) |
| 11 | -5.045 | 2.58 | 0.02389 |
| | (0.1) | (0.114) | (0.01656) |
| 12 | -4.975 | 2.435 | -0.07023 |
| | (0.074) | (0.088) | (0.03028) |
| 13 | -5.047 | 2.684 | -0.03246 |
| | (0.094) | (0.11) | (0.01584) |
| 14 | -5.027 | 2.509 | -0.14136 |
| | (0.072) | (0.084) | (0.03267) |
| 15 | -5.218 | 2.84 | -0.09215 |
| | (0.09) | (0.101) | (0.02527) |
| 16 | -5.06 | 2.613 | -0.08345 |
| | (0.077) | (0.087) | (0.03251) |
| 17 | -5.573 | 3.062 | -0.01536 |
| | (0.061) | (0.076) | (0.00996) |
| 18 | -4.167 | 2.366 | -1.39615 |
| | (0.089) | (0.082) | (0.08668) |
| 19 | -5.203 | 2.964 | -0.1163 |
| | (0.103) | (0.112) | (0.03037) |
| 20 | -5.411 | 2.786 | -0.03094 |
| | (0.079) | (0.094) | (0.01605) |
| 21 | -5.716 | 3.273 | 0.01757 |
| | (0.066) | (0.076) | (0.00286) |
| 22 | -5.295 | 2.635 | -0.0024 |
| | (0.074) | (0.088) | (0.00612) |
| 23 | -5.233 | 2.597 | -0.17919 |
| | (0.055) | (0.071) | (0.03192) |
| 24 | -5.364 | 3.144 | 0.02495 |
| | (0.107) | (0.12) | (0.00619) |
| 25 | -4.921 | 2.371 | 0.00219 |
| | (0.073) | (0.097) | (0.00424) |
| 26 | -5.811 | 3.006 | -0.00087 |
| | (0.048) | (0.064) | (0.00364) |
| 27 | -5.615 | 3.11 | -0.00811 |
| | (0.058) | (0.07) | (0.00394) |
| 28 | -5.119 | 2.658 | 0.00057 |
| | (0.085) | (0.099) | (0.00611) |
| 29 | -5.011 | 2.67 | -0.04409 |
| | (0.071) | (0.084) | (0.02404) |
| 30 | -5.284 | 2.303 | 0.01208 |
| | (0.05) | (0.069) | (0.00419) |
| 31 | -4.469 | 2.508 | -1.15936 |
| | (0.106) | (0.093) | (0.10727) |
| 32 | -3.564 | 2.173 | -2.78001 |
| | (0.114) | (0.11) | (0.18851) |
| 33 | -5.523 | 3.35 | 0.01532 |
| | (0.08) | (0.089) | (0.01093) |
| 34 | -4.566 | 2.498 | -1.5863 |
| | (0.081) | (0.079) | (0.07984) |

Table 8: Estimated Coefficients of the ergm the model for the Favors network. The model is $log(Pr(G=\mathbf{g})) = \beta_0 + \beta_1 \sum_{i<j} \mathbf{g}_{ij} + \beta_2 \sum_{i<j} \mathbf{g}_{ij}\mathbf{s}(\mathbf{g},\mathbf{g}')_{ij} + \beta_3 \sum_{i<j} d(i,j)\mathbf{g}_{ij}$

| Village | $\hat{\beta}_1$ | $\hat{\beta}_2$ | $\hat{\beta}_3$ |
|---|---|---|---|
| 35 | -2.929 | 2.064 | -2.42283 |
| | (0.139) | (0.122) | (0.13827) |
| 36 | -4.69 | 2.003 | -0.01248 |
| | (0.062) | (0.091) | (0.00822) |
| 37 | -5.593 | 2.937 | -0.00046 |
| | (0.047) | (0.06) | (0.00348) |
| 38 | -5.627 | 3.448 | 0.00809 |
| | (0.067) | (0.076) | (0.00535) |
| 39 | -2.806 | 2 | -2.84171 |
| | (0.108) | (0.093) | (0.13161) |
| 40 | -3.439 | 2.12 | -2.23149 |
| | (0.099) | (0.09) | (0.12461) |
| 41 | -5.035 | 2.717 | -0.0257 |
| | (0.062) | (0.072) | (0.01106) |
| 42 | -5.359 | 2.951 | 0.00402 |
| | (0.063) | (0.071) | (0.00778) |
| 43 | -5.436 | 2.862 | -0.00495 |
| | (0.061) | (0.08) | (0.00828) |
| 44 | -5.381 | 2.601 | 0.00614 |
| | (0.055) | (0.078) | (0.0034) |
| 45 | -4.982 | 2.596 | 0.02517 |
| | (0.081) | (0.1) | (0.00517) |
| 46 | -4.915 | 2.327 | -0.14076 |
| | (0.065) | (0.08) | (0.02759) |
| 47 | -5.042 | 2.584 | -0.08957 |
| | (0.085) | (0.092) | (0.03094) |
| 48 | -5.274 | 3.073 | -0.31429 |
| | (0.077) | (0.077) | (0.0453) |
| 49 | -5.536 | 2.857 | -0.00476 |
| | (0.057) | (0.065) | (0.00529) |
| 50 | -4.595 | 2.495 | -1.49029 |
| | (0.063) | (0.06) | (0.06625) |
| 51 | -3.623 | 1.986 | -1.67765 |
| | (0.097) | (0.097) | (0.10073) |

| Village | $\hat{\beta}_1$ | $\hat{\beta}_2$ | $\hat{\beta}_3$ |
|---|---|---|---|
| 52 | -2.565 | 1.231 | -2.87712 |
| | (0.121) | (0.118) | (0.18388) |
| 53 | -5.638 | 3.609 | -0.00381 |
| | (0.064) | (0.073) | (0.00457) |
| 54 | -3.705 | 2.396 | -2794.07224 |
| | (0.113) | (0.105) | (230.7352) |
| 55 | -3.626 | 1.914 | -2.00195 |
| | (0.089) | (0.084) | (0.10752) |
| 56 | -4.961 | 2.273 | 0.00665 |
| | (0.059) | (0.082) | (0.00469) |
| 57 | -5.879 | 3.36 | 0.00788 |
| | (0.051) | (0.063) | (0.00269) |
| 58 | -5.17 | 2.816 | -0.46967 |
| | (0.05) | (0.055) | (0.04399) |
| 59 | -4.598 | 2.793 | -0.65429 |
| | (0.119) | (0.118) | (0.08063) |
| 60 | -4.517 | 2.522 | -1.20137 |
| | (0.09) | (0.083) | (0.11496) |
| 61 | -4.762 | 2.937 | -0.53211 |
| | (0.093) | (0.095) | (0.08401) |
| 62 | -4.097 | 2.544 | -2.07602 |
| | (0.077) | (0.073) | (0.12048) |
| 63 | -5.718 | 3.244 | 0.00255 |
| | (0.055) | (0.064) | (0.00411) |
| 64 | -4.123 | 2.399 | -1.59247 |
| | (0.117) | (0.106) | (0.13163) |
| 65 | -5.59 | 2.95 | -0.03898 |
| | (0.078) | (0.094) | (0.01971) |
| 66 | -5.064 | 2.771 | -0.17221 |
| | (0.094) | (0.105) | (0.0409) |
| 67 | -3.857 | 2.363 | -1.96435 |
| | (0.102) | (0.091) | (0.10493) |
| 68 | -4.909 | 2.281 | -0.16078 |
| | (0.061) | (0.072) | (0.02437) |

Table 8 (continued)

| **Village** | $\hat{\beta}_1$ | $\hat{\beta}_2$ | $\hat{\beta}_3$ |
|---|---|---|---|
| 69 | -5.692 | 3.216 | -0.00852 |
| | (0.062) | (0.072) | (0.00561) |
| 70 | -5.503 | 2.947 | -0.00275 |
| | (0.072) | (0.084) | (0.00429) |
| 71 | -5.491 | 3.029 | -0.00194 |
| | (0.08) | (0.09) | (0.00335) |
| 72 | -5.151 | 3.098 | 0.00485 |
| | (0.073) | (0.087) | (0.00472) |
| 73 | -5.331 | 2.899 | -0.01296 |
| | (0.076) | (0.088) | (0.01131) |
| 74 | -5.403 | 3.251 | -0.0077 |
| | (0.059) | (0.071) | (0.00875) |
| 75 | -4.654 | 2.59 | -0.20904 |
| | (0.077) | (0.087) | (0.04137) |

Table 8 (continued)

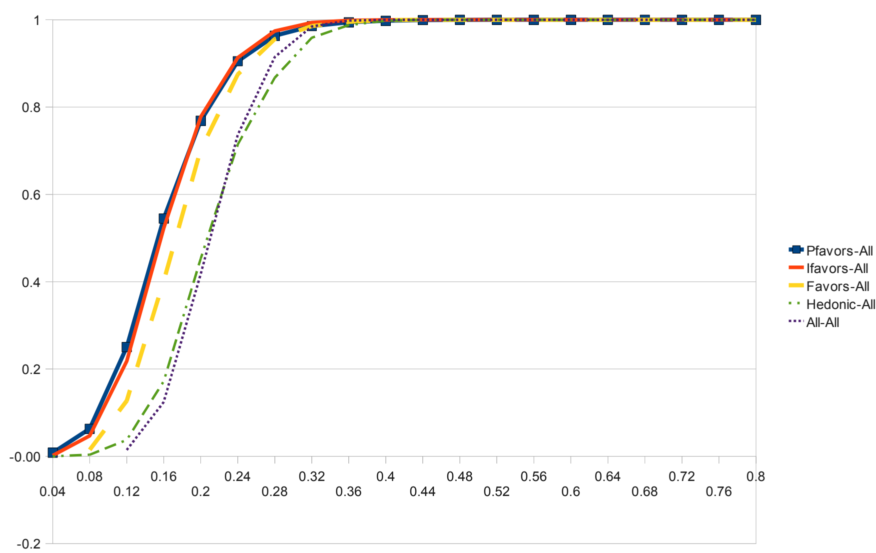



Figure 13: Fraction of villages with at most the observed support as a function of measurement error

| Support Measure | Measurement Error | | | |
|---|---|---|---|---|
| | 16% | 24% | 32% | 40% |
| PFavors-All | 0.54 | 0.91 | 0.99 | 0.999 |
| IFavors-All | 0.52 | 0.91 | 0.99 | 0.999 |
| Favors-All | 0.40 | 0.87 | 0.99 | 0.999 |
| Hedonic-All | 0.17 | 0.71 | 0.96 | 0.999 |
| All-All | 0.12 | 0.74 | 0.98 | 0.999 |

Table 9: A closer look at Figure 13

and for each class compute the mean across the 75 villages of the fraction of pairs of agents in the class who have at least one friend in common[13] in the *All* network. We present one table for each characteristic. In addition to the mean for each similarity/dissimilarity class, each table shows the number of villages in which the class is the one with the highest fraction of pairs of agents with a friend in common. Note that in each of the 5 individual characteristics considered the presence of a link among dissimilar agents is associated with a significantly greater probability of a friend in common *relative* to the classes of similar agents, with respect to situations in which the agents are not linked. This is a property which holds for links in the favors networks and also in the hedonic networks.

In addition to the mean for each similarity/dissimilarity class, each table shows the number of villages in which the class is the one with the highest fraction of pairs of agents with a friend in common. Note that in each of the 5 individual characteristics considered the presence of a link among dissimilar agents is associated with a significantly greater probability of a friend in common *relative* to the classes of similar agents, with respect to situations in which the agents are not linked. This is a property which holds for links in the favors networks and also in the hedonic networks.

#### 6.8.2 Household Level Predictors of Support

Here, we examine links at a household level, where we a pair of households to be linked in a given network if there exists a agent in each household who are linked to each other.

Table 16 presents the coefficients associated to two probit regressions relating the likelihood of having a "household friend" in common in the *All* network to some household characteristics: maximum education, mean age, rooms per person and household size. In the first one an observation corresponds to a randomly chosen friend in the favors network of a randomly chosen household in one of the 75 villages in our sample. In the second regression

[13] A link is supported when the linked agents have at least one friend in common; as above, speaking of *having a friend in common* lets us refer to linked agents as well as to agents that are not linked.

| **Education** | $ij \notin Favors$ | | $ij \in Favors$ | | $ij \notin Hed$ | | $ij \in Hed$ | |
|---|---|---|---|---|---|---|---|---|
| | Mean | # Max | Mean | # Max | Mean | # Max | Mean | # Max |
| both below median | 0.139 | 26 | 0.707 | 25 | 0.140 | 25 | 0.651 | 23 |
| # total pairs in class | 600724 | | 9507 | | 600616 | | 9615 | |
| above/below median | 0.131 | 3 | 0.719 | 25 | 0.132 | 2 | 0.663 | 28 |
| # total pairs in class | 1003084 | | 11795 | | 1003210 | | 11669 | |
| both above median | 0.149 | 46 | 0.714 | 25 | 0.150 | 48 | 0.637 | 24 |
| # total pairs in class | 469260 | | 7617 | | 469222 | | 7655 | |

Table 10: Mean fraction of pairs of agents with at least one friend in common in the *All* network, by similarity/dissimilarity in formal education, when the agents have a link in the favors network compared to when they do not have a link in the favors network. The column labeled $\#Max$ depicts the number of villages (out of 75) in which the fraction of pairs with at least one friend in common is the highest among the 3 education categories. The education scale has 15 different levels, ranging from no formal education to graduate degree. The median level of education in the scale is 5.

| **Gender** | $ij \notin Favors$ | | $ij \in Favors$ | | $ij \notin Hed$ | | $ij \in Hed$ | |
|---|---|---|---|---|---|---|---|---|
| | Mean | # Max | Mean | # Max | Mean | # Max | Mean | # Max |
| both male | 0.203 | 75 | 0.707 | 16 | 0.202 | 75 | 0.637 | 16 |
| # total pairs in class | 404266 | | 10343 | | 402374 | | 12235 | |
| female/male | 0.123 | 0 | 0.728 | 30 | 0.126 | 0 | 0.690 | 43 |
| # total pairs in class | 1033918 | | 7572 | | 1038232 | | 3258 | |
| both female | 0.117 | 0 | 0.713 | 29 | 0.116 | 0 | 0.649 | 17 |
| # total pairs in class | 634884 | | 11004 | | 632442 | | 13446 | |

Table 11: Mean fraction of pairs of agents with at least one friend in common in the *All* network, by similarity/dissimilarity in gender, when the agents have a link in the favors network compared to when they do not have a link in the favors network. See the caption of Table 10 for more details.

an observation corresponds to a randomly chosen household unrelated to a randomly chosen household in one of the 75 villages.

| **Age** | $ij \notin Favors$ | | $ij \in Favors$ | | $ij \notin Hed$ | | $ij \in Hed$ | |
|---|---|---|---|---|---|---|---|---|
| | Mean | # Max | Mean | # Max | Mean | # Max | Mean | # Max |
| both belong | 0.116 | 3 | 0.700 | 23 | 0.117 | 3 | 0.633 | 17 |
| # total pairs in class | 552166 | | 7753 | | 551759 | | 8160 | |
| belong/not belong | 0.134 | 0 | 0.727 | 30 | 0.136 | 0 | 0.670 | 36 |
| # total pairs in class | 1031445 | | 12236 | | 1032157 | | 11524 | |
| neither belong | 0.167 | 72 | 0.705 | 22 | 0.168 | 72 | 0.644 | 22 |
| # total pairs in class | 489457 | | 8930 | | 489132 | | 9255 | |

Table 12: Mean fraction of pairs of agents with at least one friend in common in the *All* network, by similarity/dissimilarity in their age, when the agents have a link in the favors network compared to when they do not have a link in the favors network. See the caption of Table 10 for more details.

| **Caste** | $ij \notin Favors$ | | $ij \in Favors$ | | $ij \notin Hed$ | | $ij \in Hed$ | |
|---|---|---|---|---|---|---|---|---|
| | Mean | # Max | Mean | # Max | Mean | # Max | Mean | # Max |
| different castes | 0.091 | 0 | 0.578 | 3 | 0.090 | 0 | 0.540 | 7 |
| # total pairs in class | 1432149 | | 7638 | | 1429094 | | 10693 | |
| same caste | 0.239 | 75 | 0.755 | 72 | 0.244 | 75 | 0.707 | 68 |
| # total pairs in class | 640919 | | 21281 | | 643954 | | 18246 | |

Table 13: Mean fraction of pairs of agents with at least one friend in common in the *All* network, by whether they belong to the same caste or not, when the agents have a link in the favors network compared to when they do not have a link in the favors network. See the caption of Table 10 for more details.

| **Micro Finance** | $ij \notin Favors$ | | $ij \in Favors$ | | $ij \notin Hed$ | | $ij \in Hed$ | |
|---|---|---|---|---|---|---|---|---|
| | Mean | # Max | Mean | # Max | Mean | # Max | Mean | # Max |
| Neither belong | 0.103 | 7 | 0.696 | 14 | 0.102 | 7 | 0.629 | 13 |
| # total pairs in class | 336948 | | 5018 | | 335703 | | 6263 | |
| belong/not belong | 0.100 | 5 | 0.691 | 8 | 0.099 | 5 | 0.637 | 7 |
| # total pairs in class | 69383 | | 1087 | | 69092 | | 1378 | |
| both belong | 0.159 | 26 | 0.635 | 16 | 0.158 | 26 | 0.610 | 18 |
| # total pairs in class | 4862 | | 186 | | 4836 | | 212 | |

Table 14: Mean fraction of pairs of agents with at least one friend in common in the *All* network, by whether they participate or not in the micro finance program, when the agents have a link in the favors network compared to when they do not have a link in the favors network. This table was computed only considering pairs agents eligible for participation in the program. That is, women of age 15 or older, living in one of the 38 villages in the treatment group.

| | *Linked* | *Not Linked* |
|---|---|---|
| Mean Age | 0.244 | 0.005 |
| Mean Education | 0.034 | $-0.010$ |
| Rooms per person | $-0.84^{**}$ | 0.16 |
| Household size | $-0.064$ | 0.041 |
| Intercept | $1.54^{*}$ | $-0.56$ |

(**) Significant at 5%, (*) Significant at 10%

Table 15: Probit regression of support in the households all network of linked and unlinked pairs in the households favor network. In the regression shown in the first column an observation corresponds to a randomly chosen agent linked in the favors network to a randomly chosen household in one of the 75 villages in our sample. In the regression shown in the second column an observation corresponds to a randomly chosen agent unlinked to a randomly chosen household in one of the 75 villages.

| | *Linked* | *Not Linked* |
|---|---|---|
| Mean Age | 0.205 | 0.008 |
| Max Education | 0.017 | 0.007 |
| Rooms per person | $-0.78^{**}$ | 0.13 |
| Household size | $-0.063$ | 0.029 |
| Intercept | $1.70^{**}$ | $-0.66^{*}$ |

(**) Significant at 5%, (*) Significant at 10%

Table 16: Probit regression of support in the households all network of linked and unlinked pairs in the households favor network. In the regression shown in the first column an observation corresponds to a randomly chosen household linked in the favors network to a randomly chosen household in one of the 75 villages in our sample. In the regression shown in the second column an observation corresponds to a randomly chosen household not linked to a randomly chosen household in one of the 75 villages.