

Erratum:
Endogenous Games and Mechanisms: Side Payments
Among Players
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We thank Tore Ellingsen and Elena Paltseva for pointing out that there is an error in the proof of Theorem 5 in the above-referenced paper. In fact, there is not just an error in the proof, but also in the theorem. The following example shows that Theorem 5, which applies to three or more players, is incorrect. The error in the proof is that it does not properly account for potential mixed strategy deviations: it only considers pure deviations and erroneously claims that the proof extends to mixed deviations. The other theorems in the paper are unaffected and still hold, including Theorem 6. However, it is not clear whether Corollaries 1 and 2 that follow Theorem 5 are true as they are not ruled out by the example below.

The main conclusions of the original paper are modified as follows. The basic messages that efficient play in games is not always supportable when pre-game side contracting is available, and that conditions under which efficient play is supportable can be subtle, still hold. What is modified is the message that with three or more players, profiles of pure actions that Pareto improve upon pure strategy Nash equilibria can be supported with pre-game side contracting. The example below shows this is not always true, at least with unilateral side contracts that condition only upon pure strategies. Thus, there may be a less dramatic distinction between games with two players and those with three or more players than we originally thought. Part of the difficulty in achieving efficiency, is that each player would like to use pre-game side contracting to distort a game in a way such that the resulting equilibrium is most favorable to him or herself. In a two player game, a player can effectively refuse transfers from another player (even if transfers do not require consent to be received)

simply by promising to make transfers back that cancel out any receipts. This can impede supporting efficient play. The incentives are different with three or more players, since players can make promised payments to more than one other player at once, and by making large promises conditional on certain actions can effectively commit not to play certain actions and thus steer the game towards actions that they would like to see played. The difficulty that arises is that when several players are doing this at the same time, the combination of constraints on the transfers needed to support an efficient profile as an equilibrium can be quite complex. The hole in the original proof was that such combinations can be vulnerable to deviations in transfer functions by some players that induce mixed strategy equilibria in the game that are better for those players than the pure strategy profile that other players are trying to support. It may still be that Corollary 2 is correct, and being able to make side payments to completely disinterested third parties can help in achieving efficiency as they have less incentive to distort the game, but that is still an open question.

Theorem 5 would hold if one worked with definitions that did not allow for mixed strategies in the play of the game. However, that would not be very satisfactory since players could still promise transfers that would result in nonexistence of pure strategy equilibria in resulting game, and then it would not be clear what players should expect. Instead, Tore Ellingsen and Elena Paltseva (2011) make important progress along a different route in a manuscript “Non-cooperative Contracting.” They consider transfer functions that can depend explicitly on mixed strategies, rather than just pure strategies. So players can promise to pay other players conditional upon certain mixed strategies being played in the game. Ellingsen and Paltseva show that efficient play can be supported with such transfer schemes. They also provide some sufficient conditions on games for which only transfers conditional upon pure strategies are needed. They thus make important progress, since given the numerous applications where some side-payments are available to players it is essential to have a fuller understanding of which types of side-contracting result in efficient play in games and for which settings.

The notation and terminology below all follow from the above-referenced Jackson and Wilkie (2005) paper.

Consider the three player game where player 1 chooses the row action, player 2 chooses the column action, and player 3 has only one action, and payoffs are:

	<i>C</i>	<i>D</i>
<i>C</i>	1, 1, 0	0, 2, 4
<i>D</i>	2, 0, 4	0, 0, 0

where the first entry is that of player 1, the second of player 2, and the third of player 3.

Suppose that the transfers t support C, C as an equilibrium with payoffs 1,1,0 to the

players on the equilibrium path, as would be possible if Theorem 5 were true. Let us show that we reach a contradiction of equilibrium and thus have a counter-example to Theorem 5.

Claim 1. It must be that $t_{21}(C, D) + t_{31}(C, D) \leq 2$ and similarly $t_{12}(D, C) + t_{32}(D, C) \leq 2$.

Proof of Claim 1: Suppose to the contrary that, for instance, $t_{21}(C, D) + t_{31}(C, D) > 2$. Now suppose that 1 deviates from t_1 and sets all transfers from 1 to other players to 0. It follows that 1's payoffs conditional on t_2, t_3 (which are nonnegative) and the play of the game are at least:

	C	D	
C	1	a	
D	2	0	

where $a > 2$. In the second period, by mixing evenly on C and D , player 1 is guaranteed a payoff of more than 1 regardless of what other players do. Thus any equilibrium in the subgame following 1's deviation must lead to a payoff of more than 1 to player 1. This contradicts the support of C, C as an equilibrium with a payoff of 1 to player 1. ■

Claim 2. Player 3 has an improving deviation away from t to some \hat{t} .

Proof of Claim 2: Note that either $-t_{12}(D, D) + t_{21}(D, D) \leq 0$ or $t_{12}(D, D) - t_{21}(D, D) \leq 0$. Without loss of generality, assume it is the first (or else simply switch the labels of 1 and 2, and swap C, D with D, C in what follows).

Have player 3 deviate from the prescribed equilibrium t and offer transfers \hat{t} in the first period, where \hat{t} is defined as follows.

$$\hat{t}_{31}(C, C) = t_{31}(C, C) + \varepsilon.$$

$$\hat{t}_{31}(D, C) = t_{31}(D, C).$$

$$\hat{t}_{31}(C, D) = \max[0, t_{13}(C, D) + t_{12}(C, D) - t_{21}(C, D)] + \varepsilon.$$

$$\hat{t}_{31}(D, D) = 0.$$

$$\hat{t}_{32}(C, D) = \max[0, t_{23}(C, D) + t_{21}(C, D) - t_{12}(C, D) - 1] + \varepsilon.$$

For all other entries, $\hat{t}_3 = t_3$.

We now show that under t_1, t_2, \hat{t}_3 there is a unique (strict) equilibrium of C, D and that the payoff to player 3 under these transfers and equilibrium is larger than 0, which contradicts the support of (C, C) with t .

First, note that, given the specifications of $\hat{t}_{31}(C, C)$ and $\hat{t}_{31}(D, C)$, under t_1, t_2, \hat{t}_3 it a strict best response for player 1 to play C if C is played by player 2 (since C was a best response to C under t).

Second, note that under t_1, t_2, \hat{t}_3 it is a strict best response for player 1 to play C if D is played by player 2. This follows since we are in a case where $-t_{12}(D, D) + t_{21}(D, D) \leq 0$ (from above), which then implies that the payoff to player 1 from D, D is $-t_{13}(D, D) -$

$t_{12}(D, D) + t_{21}(D, D) + 0 \leq 0$, while the payoff to player 1 from C, D is $-t_{13}(C, D) - t_{12}(C, D) + t_{21}(C, D) + \hat{t}_{31}(C, D) \geq \varepsilon$.

So, under t_1, t_2, \hat{t}_3 it is a strictly dominant strategy for player 1 to play C .

Next, note that the payoff to player 2 from C, D under t_1, t_2, \hat{t}_3 is $2 + \hat{t}_{32}(C, D) - t_{23}(C, D) - t_{21}(C, D) + t_{12}(C, D) \geq 1 + \varepsilon$. Thus, if player 1 plays C , then it is a strict best response for 2 to play D (given that the payoff to (C, C) for 2 is still 1).

Thus, under t_1, t_2, \hat{t}_3 there is a unique (strict) equilibrium of C, D .

We complete the proof by showing that the payoff to player 3 for C, D under t_1, t_2, \hat{t}_3 is larger than 0, which contradicts the support of C, C under t with payoffs 1, 1, 0.

Player 3's payoff for C, D under t_1, t_2, \hat{t}_3 is

$$4 - \hat{t}_{31}(C, D) - \hat{t}_{32}(C, D) + t_{13}(C, D) + t_{23}(C, D).$$

Since C, C is supported under t with a payoff of 1 to player 2, it must be that 2 would not gain by deviating to D under t , and so

$$2 + t_{32}(C, D) - t_{23}(C, D) - t_{21}(C, D) + t_{12}(C, D) \leq 1.$$

Since $t_{32}(C, D) \geq 0$, this implies that

$$1 \leq t_{23}(C, D) + t_{21}(C, D) - t_{12}(C, D),$$

and so

$$\max[0, t_{23}(C, D) + t_{21}(C, D) - t_{12}(C, D) - 1] = t_{23}(C, D) + t_{21}(C, D) - t_{12}(C, D) - 1.$$

Thus, the payoff to player 3 for C, D under t_1, t_2, \hat{t}_3 is

$$4 - \max[0, t_{13}(C, D) + t_{12}(C, D) - t_{21}(C, D)] - [t_{23}(C, D) + t_{21}(C, D) - t_{12}(C, D) - 1] + t_{13}(C, D) + t_{23}(C, D) - 2\varepsilon.$$

or

$$5 - \max[0, t_{13}(C, D) + t_{12}(C, D) - t_{21}(C, D)] - t_{21}(C, D) + t_{12}(C, D) + t_{13}(C, D) - 2\varepsilon.$$

If $t_{13}(C, D) + t_{12}(C, D) - t_{21}(C, D) > 0$, then the expression simplifies to $5 - 2\varepsilon$, which is larger than 0 for small ε .

Otherwise, the payoff is

$$5 - t_{21}(C, D) + t_{12}(C, D) + t_{13}(C, D) - 2\varepsilon.$$

By Claim 1 it follows that $t_{21}(C, D) \leq 2$. Thus the payoff is at least

$$3 - 2\varepsilon,$$

which is again, larger than 0 for small ε . ■