An Expected-Efficient Status Quo Allows Efficient Bargaining

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Abstract

In a general model of asymmetric-information bargaining with independent private values and quasilinear utilities, there exists an efficient incentive-compatible and individually rational bargaining mechanism when the status-quo allocation is the expected efficient allocation. The only assumption needed for this is that the total surplus is convex in the allocation (which holds as long as randomized allocations are allowed).

1 Introduction

Myerson and Satterthwaite (1983) first demonstrated, in a bilateral trading setting, that private information and property rights may make it impossible for parties to bargain towards fully efficient trade. (Mailath and Postlewaite (1990) showed a similar result for public good provision.) Cramton et al. (1987) showed that in the problem of allocating a divisible good to the highest-value agent, efficiency can be achieved with symmetric agents when the status-quo allocation of the good is close enough to equal shares. Schmitz (2002) demonstrated other examples in which efficient bargaining is possible, while Neeman (1999) considered a pollution model in which the existence of an efficiency-permitting status-quo allocation depended on some parameters. Schweizer (2006, Proposition 2) established the existence of an efficiency-permitting status-quo allocation in a model that generalized the preceding ones.

This note contributes to the literature by describing a status-quo allocation that allows efficient bargaining in a very general model, which strictly generalizes all the preceding models for which existence has been shown. In contrast to Schweizer's result, we explicitly describe a status-quo allocation that permits efficient bargaining – it is the expectation of the ex post efficient allocation. The only assumption needed (in addition to the quasilinearity of payoffs and independent private values) is that the total surplus is convex in the allocation.

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2 The Model and Result

Consider I expected-utility maximizing agents whose utilities are quasilinear in money. The set of non-monetary allocations X is a subset of a measurable topological vector space. The agents' privately observed types are independently distributed random variables $\tilde{\theta}_1, \ldots, \tilde{\theta}_I$ with values in measurable spaces $\Theta_1, \ldots, \Theta_I$ respectively. The state space is thus $\Theta = \Theta_1 \times \ldots \times \Theta_I$. The utility of each agent i is given by $u_i(x, \theta_i) + t_i$, where $t_i \in \mathbb{R}$ is the payment to the agent, $x \in X$ is the nonmonetary allocation, and $\theta_i \in \Theta_i$ is the agent's type. The functions $u_i : X \times \Theta_i \to \mathbb{R}$ are measurable and uniformly bounded (i.e., $\sup_{i,x,\theta} |u_i(x, \theta_i)| < \infty$).[1]

Proposition 1 *Suppose that the total surplus* $s(x, \theta) \equiv \sum_i u_i(x, \theta_i)$ *is convex in* x *for all* θ. *Suppose that* $\chi^* : \Theta \to X$ *is an efficient allocation rule, i.e.,* $s(\chi^*(\theta), \theta) = S(\theta) \equiv \max_{x \in X} s(x, \theta)$ *for all* θ, *and that the status-quo allocation is* $\widehat{x} \equiv \mathbb{E}[\chi^*(\tilde{\theta})] \in X$. *Then there exists a Bayesian incentive-compatible and interim individually rational mechanism implementing allocation rule* χ^* *in which the total expected payment to the agents is nonpositive.*

Proof. Consider the direct mechanism $\langle \chi^*, \tau \rangle$ that, given announcements $\theta = (\theta_1, \ldots, \theta_I)$, implements allocation $\chi^*(\theta)$ and makes a payment to each agent i of

$$\tau_i(\theta) = \sum_{j \neq i} u_j(\chi^*(\theta), \theta_j) - \inf_{\widehat{\theta}_i \in \Theta_i} \left(\mathbb{E}_{\tilde{\theta}_{-i}} \left[S(\widehat{\theta}_i, \tilde{\theta}_{-i}) \right] - u_i(\widehat{x}, \widehat{\theta}_i) \right). \quad (1)$$

First, note that this is a VCG mechanism, hence the mechanism is dominant-strategy and therefore Bayesian incentive compatible. Next, the interim net expected utility of type θ_i of agent i in the mechanism (that is, this agent's surplus over his utility from the status quo) is

$$\begin{aligned} & \mathbb{E}_{\tilde{\theta}_{-i}} \left[u_i(\chi^*(\theta_i, \tilde{\theta}_{-i}), \theta_i) + \tau_i(\theta_i, \tilde{\theta}_{-i}) \right] - u_i(\widehat{x}, \theta_i) \\ = \; & \mathbb{E}_{\tilde{\theta}_{-i}} \left[S(\theta_i, \tilde{\theta}_{-i}) \right] - u_i(\widehat{x}, \theta_i) - \inf_{\widehat{\theta}_i \in \Theta_i} \left(\mathbb{E}_{\tilde{\theta}_{-i}} \left[S(\widehat{\theta}_i, \tilde{\theta}_{-i}) \right] - u_i(\widehat{x}, \widehat{\theta}_i) \right) \geq 0. \end{aligned}$$

Finally, the ex ante expected payment to agent i is bounded above as follows

$$\begin{aligned} \mathbb{E}\left[\tau_i(\tilde{\theta})\right] &= \sup_{\widehat{\theta}_i \in \Theta_i} \mathbb{E} \left[\sum_{j \neq i} u_j \left(\chi^*(\tilde{\theta}), \tilde{\theta}_j \right) - S(\widehat{\theta}_i, \tilde{\theta}_{-i}) + u_i(\widehat{x}, \widehat{\theta}_i) \right] \\ &\leq \sup_{\widehat{\theta}_i \in \Theta_i} \mathbb{E} \left[\sum_{j \neq i} u_j \left(\chi^*(\tilde{\theta}), \tilde{\theta}_j \right) - s\left(\chi^*(\tilde{\theta}), (\widehat{\theta}_i, \tilde{\theta}_{-i}) \right) + u_i(\widehat{x}, \widehat{\theta}_i) \right] \\ &= \sup_{\widehat{\theta}_i \in \Theta_i} \mathbb{E} \left[u_i(\widehat{x}, \widehat{\theta}_i) - u_i(\chi^*(\tilde{\theta}), \widehat{\theta}_i) \right], \end{aligned}$$

[1] The last two assumptions ensure that expectations exist and that the inf in the proof of the Proposition is finite.

where the inequality holds by definition of $S(\widehat{\theta}_i, \tilde{\theta}_{-i})$. Adding up over i and using Jensen's inequality yields

$$\mathbb{E}\left[\sum_i \tau_i(\tilde{\theta})\right] \leq \sup_{\widehat{\theta}\in\Theta} \mathbb{E}\left[s(\widehat{x}, \widehat{\theta}) - s(\chi^*(\tilde{\theta}), \widehat{\theta})\right] \leq 0. \tag{2}$$

■

Corollary 1 *Under the assumptions of the Proposition, for the status-quo allocation $\widehat{x} = \mathbb{E}[\chi^*(\tilde{\theta})]$ there exists a Bayesian incentive-compatible and interim individually rational mechanism $\langle \chi^*, \psi \rangle$ implementing the efficient allocation rule χ^* in which the total payment to the agents is zero in all states.*

Proof. For each i, let

$$\psi_i(\theta) \equiv \mathbb{E}_{\tilde{\theta}_{-i}}\left[\tau_i(\theta_i, \tilde{\theta}_{-i})\right] - \frac{1}{I-1}\sum_{j\neq i}\left(\mathbb{E}_{\tilde{\theta}_{-j}}\left[\tau_j(\theta_j, \tilde{\theta}_{-j})\right] - \mathbb{E}\left[\tau_j(\tilde{\theta})\right]\right) - \frac{1}{I}\sum_j \mathbb{E}\left[\tau_j(\tilde{\theta})\right].$$

By construction, $\sum_i \psi_i(\theta) = 0$ for all $\theta \in \Theta$. Also by construction, for all i, θ_i,

$$\mathbb{E}_{\widetilde{\theta}_{-i}}\left[\psi_i(\theta_i, \widetilde{\theta}_{-i})\right] = \mathbb{E}_{\tilde{\theta}_{-i}}\left[\tau_i(\theta_i, \tilde{\theta}_{-i})\right] - \frac{1}{I}\mathbb{E}\left[\sum_j \tau_j(\tilde{\theta})\right],$$

hence mechanism $\langle \chi^*, \psi \rangle$ inherits Bayesian incentive compatibility of mechanism $\langle \chi^*, \tau \rangle$, and it also inherits interim individual rationality since we know that $\mathbb{E}\left[\sum_j \tau_j(\tilde{\theta})\right] \leq 0$ for all j. ■

3 Remarks

3.1 Role of Convexity

When the allocation space X is not convex, an expected-efficient allocation $\mathbb{E}[\chi^*(\tilde{\theta})]$ need not lie in X, and if it does not then Proposition 1 does not apply:

Example 1 (Bilateral Trade) *$I = 2$, each $\tilde{\theta}_i$ is distributed on $\Theta_i = [0, 1]$ according to a continuous strictly increasing c.d.f. F_i, $X = \{(1, 0), (0, 1)\}$, and $u_i(x, \theta_i) = \theta_i x_i$. In this setting, the efficient allocation rule has $\chi_i^*(\theta) = 1$ whenever $\theta_i > \theta_{-i}$. By the Myerson-Satterthwaite Theorem, neither possible status-quo allocation allows efficient bargaining.*

Even with a convex allocation space X, the assumption of convexity of the total surplus $s(x, \theta)$ in the allocation x is also crucial. Neeman (1999, p.685) offers an example of pollution in which the convexity assumption fails and there does not exist an efficiency-permitting status-quo allocation.

On the other hand, both convexity assumptions are trivially ensured by allowing randomized allocations, in which case we take X to be the space of probability distributions over a primitive set of pure allocations, and the agents' expected utilities are linear in x. Allowing randomizations does not affect the efficiency of a deterministic allocation rule. but a randomized status quo allowing efficient bargaining will exist by Proposition 1. In Example 1, the space of randomized allocations is the simplex $X = \left\{(x_1, x_2) \in \mathbb{R}_+^2 : x_1 + x_2 = 1\right\}$, and the utilities are linear in x, hence Proposition 1 applies.

3.2 Comparison to Schweizer's (2006) Proposition 2

Schweizer's approach may be described as follows: Consider an intermediary who offers a mechanism $\langle \chi^*, \tau \rangle$ with the efficient allocation rule $\chi^*$and payments to each agent i of the form

$$\tau_i(\theta) = \sum_{j \neq i} u_j(\chi^*(\theta), \theta_j) - \mathbb{E}_{\tilde{\theta}_{-i}}\left[S(\widehat{\theta}_i, \tilde{\theta}_{-i})\right] + u_i(\widehat{x}, \widehat{\theta}_i) \tag{3}$$

for some $\widehat{x} \in X$ and $\widehat{\theta} = (\widehat{\theta}_1, \ldots, \widehat{\theta}_I) \in \Theta$. Note that this is a VCG mechanism and that it makes the participation constraints of "critical types" $\widehat{\theta}_i$ bind given the status-quo allocation $\widehat{x}$. The intermediary's expected profit in this mechanism is

$$\pi(\widehat{x}, \widehat{\theta}) = -\mathbb{E}\left[\sum_i \tau_i(\tilde{\theta})\right] = -(I-1)\,\mathbb{E}\left[S(\tilde{\theta})\right] + \sum_i \mathbb{E}\left[S(\widehat{\theta}_i, \tilde{\theta}_{-i})\right] - s(\widehat{x}, \widehat{\theta}).$$

Now consider the maxmin program $\max_{\widehat{x} \in X} \min_{\widehat{\theta} \in \Theta} \pi(\widehat{x}, \widehat{\theta})$ in which the intermediary first chooses the status-quo allocation $\widehat{x}$ and then an adversary chooses the critical types $\widehat{\theta}$. If the program has a saddle point $(\widehat{x}, \widehat{\theta}) \in X \times \Theta$, then it is characterized by

$$\widehat{\theta}_i \in \arg\min_{\theta_i \in \Theta_i} \mathbb{E}_{\tilde{\theta}_{-i}}\left[S(\theta_i, \tilde{\theta}_{-i})\right] - u_i(\widehat{x}, \theta_i) \text{ for all } i, \tag{4}$$

$$\widehat{x} \in \arg\min_{x \in X} s(x, \widehat{\theta}) \tag{5}$$

Condition (4) says that each agent i's critical type $\widehat{\theta}_i$ is his worst-off type in the mechanism given the status-quo allocation $\widehat{x}$. Condition (5) says that the status-quo allocation $\widehat{x}$ minimizes the total surplus in the state in which the agents have their critical types.

Schweizer imposes sufficient assumptions to ensure that a saddle point $(\widehat{x}, \widehat{\theta})$ indeed exists – namely, he assumes that the sets of allocations X and of types $\Theta_1, \ldots, \Theta_I$ are convex and compact subsets of Euclidean spaces, that the total surplus $s(x, \theta)$ is linear in types θ, convex in allocations x, and continuous in (x, θ).[2] Then it can be seen that given status quo $\widehat{x}$, and the payments described by (3), $\langle \chi^*, \tau \rangle$ is a Bayesian incentive-compatible and interim individually rational mechanism in which the total expected payment to the agents is nonpositive.

[2] Schweizer also assumes in footnotes 1 and 2 that the payoffs are continuously differentiable in (x, θ) and the efficient allocation rule χ^* is continuous a.e., but these assumptions do not seem to play any role in his Proposition 2.

To see this, first observe that under condition (4) the transfer rules (1) and (3) coincide. Then follow the proof of Proposition 1, with the only difference being that inequality (2) now follows from condition (5) rather than from $\widehat{x}$ being an expected-efficient allocation and Jensen's inequality.[3]

One advantage of our approach is that it allows us to dispense with Schweizer's assumptions needed to ensure the existence of a saddle point — in particular, we allow for infinite-dimensional allocations or types, non-convex or non-compact types spaces, and utility functions being nonlinear or discontinuous. The only indispensable assumption proves to be convexity of the total surplus in the allocation. A second and perhaps more important advantage is our explicit description of a natural status-quo allocation that permits efficiency.

It is interesting to note that, even when a saddle-point status-quo allocation does exist, it generally differs from an expected-efficient allocation (i.e., the expected-efficient status-quo allocation need not maximize the intermediary's expected profits). For example, consider the bilateral trade setting of Example 1, with the randomized allocation space $X = \{(x_1, x_2) \in \mathbb{R}^2_+ : x_1 + x_2 = 1\}$. This setting has a unique saddle point, which is found as follows. Given a status-quo allocation $\widehat{x} = (\widehat{x}_1, \widehat{x}_2)$, condition (4) is satisfied only by the types $\widehat{\theta}_i$ whose expected efficient consumption equals $\widehat{x}_i$ (any other type can obtain a greater interim net expected utility by pretending to be type $\widehat{\theta}_i$), thus we must have $F_{-i}(\widehat{\theta}_i) = \widehat{x}_i$. To satisfy condition (5), we must have $\widehat{\theta}_1 = \widehat{\theta}_2$ (unless $\widehat{x}_i = 1$ for some i and $\widehat{\theta}_i < \widehat{\theta}_{-i} \leq 1$, but this contradicts the previous condition $F_{-i}(\widehat{\theta}_i) = \widehat{x}_i = 1$). The equation $F_1(\widehat{\theta}) + F_2(\widehat{\theta}) = 1$ then uniquely defines the saddle-point types $\widehat{\theta}_1 = \widehat{\theta}_2 = \widehat{\theta}$ and the status-quo allocation $\widehat{x} = (F_1(\widehat{\theta}), F_2(\widehat{\theta}))$. [This allocation was used by Schmitz (2002) in proving his Proposition 3.] Note that this status-quo allocation does not change if we perturb F_1 and F_2 in ways that keep $F_1(\widehat{\theta})$ and $F_2(\widehat{\theta})$ fixed, but such perturbations generally alter the expected-efficient allocation $\mathbb{E}[\chi^*(\tilde{\theta})]$.

3.3 Relation to Hold-Up Models

Suppose agents choose ex ante investments and then each agent i's investment determines the distribution of his type θ_i. Then if bargaining occurs using the mechanism constructed in our Corollary with our status-quo allocation, an efficient investment profile will emerge as an equilibrium of the ex ante game. This follows from the analysis of Rogerson (1992), since the mechanism constructed in the Corollary is an expected-externality mechanism.

The use of expected-efficient allocation as the status quo is reminiscent of the result of Edlin and Reichelstein (1996), who show that in the hold-up model

[3] Schweizer's saddle-point status quo $\widehat{x}$ allows one advantage: if instead of (3) we let the transfers be $\tau_i(\theta) = \sum_{j \neq i} u_j(\chi^*(\theta), \theta_j) - S(\widehat{\theta}_i, \theta_{-i}) + u_i(\widehat{x}, \widehat{\theta}_i)$, then following the argument in Proposition 1 and the saddle-point condition (5) we can obtain inequalities (2) not just in expectation but for each state of the world. That is, we obtain $\sum_i \tau_i(\theta) \leq 0$ for all $\theta \in \Theta$, so the mechanism never requires a monetary infusion. The mechanism is also dominant-strategy incentive compatible (note, however, that participation is not a dominant strategy).

in which investments in private values are followed by symmetric-information Nash bargaining such status quo sustains efficient investments in equilibrium, provided that payoffs satisfy a separability condition. Our result is formally quite distinct: e.g. our bargaining mechanism is quite different from Nash bargaining and the assumptions of convexity and separability are non-nested. Yet, we think that the parallel merits further investigation.

References

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