

Exercise 1.1.1.1a: Given locality, causality, Lorentz invariance, and known physical data since 1860, show that the Lagrangian describing all observed physical processes (sans gravity) can be written:

$$\begin{aligned}
& -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e + \frac{1}{2}ig_s^2(\bar{q}_i^\sigma \gamma^\mu q_j^\sigma)g_\mu^a + \bar{G}^a \partial^2 G^a + \\
& g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \\
& \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h \left[\frac{2M^2}{g^2} + \frac{2M}{g} H + \right. \\
& \left. \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right] + \frac{2M^4}{g^2} \alpha_h - igc_w [\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - Z_\nu^0 (W_\mu^+ \partial_\nu W_\mu^- - \\
& W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0 (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] - ig s_w [\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - \\
& W_\mu^- \partial_\nu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ + \\
& g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - Z_\mu^0 Z_\nu^0 W_\mu^+ W_\nu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\nu W_\mu^+ W_\nu^-) + \\
& g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-] - g\alpha [H^3 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-] - \\
& \frac{1}{8}g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] - gM W_\mu^+ W_\mu^- H - \\
& \frac{1}{2}g \frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H - \frac{1}{2}ig [W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \frac{1}{2}g [W_\mu^+ (H \partial_\mu \phi^- - \\
& \phi^- \partial_\mu H) - W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)] + \frac{1}{2}g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - ig \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - \\
& W_\mu^- \phi^+) + ig s_w M A_\mu (W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \\
& \phi^- \partial_\mu \phi^+) - \frac{1}{4}g^2 W_\mu^+ W_\mu^- [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \frac{1}{4}g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-] - \\
& \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) + \\
& \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \\
& \bar{\nu}^\lambda \gamma \partial \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma \partial + m_d^\lambda) d_j^\lambda + ig s_w A_\mu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)] + \\
& \frac{ig}{4c_w} Z_\mu^0 [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5) u_j^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (1 - \\
& \frac{8}{3}s_w^2 - \gamma^5) d_j^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^+ [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d_j^\kappa)] + \frac{ig}{2\sqrt{2}} W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1 + \\
& \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger \gamma^\mu (1 + \gamma^5) u_j^\lambda)] + \frac{ig}{2\sqrt{2}} \frac{m_e^\lambda}{M} [-\phi^+ (\bar{\nu}^\lambda (1 - \gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1 + \gamma^5) \nu^\lambda)] - \\
& \frac{g}{2} \frac{m_e^\lambda}{M} [H(\bar{e}^\lambda e^\lambda) + i\phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}} \phi^+ [-m_d^\kappa (\bar{u}_j^\lambda C_{\lambda\kappa} (1 - \gamma^5) d_j^\kappa) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 + \gamma^5) d_j^\kappa) + \\
& \frac{ig}{2M\sqrt{2}} \phi^- [m_d^\lambda (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 + \gamma^5) u_j^\kappa) - m_u^\kappa (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 - \gamma^5) u_j^\kappa) - \frac{g}{2} \frac{m_u^\lambda}{M} H(\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2} \frac{m_d^\lambda}{M} H(\bar{d}_j^\lambda d_j^\lambda) + \\
& \frac{ig}{2} \frac{m_u^\lambda}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_d^\lambda}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \\
& \bar{Y} \partial^2 Y + igc_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + ig s_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + igc_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\
& \partial_\mu \bar{X}^0 X^+) + ig s_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + igc_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + ig s_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
& \partial_\mu \bar{X}^- X^-) - \frac{1}{2}gM [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2} \bar{X}^0 X^0 H] + \frac{1-2c_w^2}{2c_w} igM [\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \\
& \frac{1}{2c_w} igM [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + igM s_w [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2}igM [\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
\end{aligned}$$