APPPHYS383 Thursday 14 January 2010

Summary points

1. Discrete-to-discrete transitions have first-order probabilities $\sim T^2$
   a. Turns out to be leading order of a sinusoid
   b. Exact calculations possible for isolated two-dimensional subspaces
2. Discrete-to-continuum transitions have first-order probabilities $\sim T$
   a. Turns out to be leading order of an exponential
   b. Interesting to see where this comes from - requires integration over diffraction functions $[\delta^{(T)}(E-E_i)]^2$
   c. Valid for T long enough to justify flat continuum approximation, short enough to justify first-order perturbation expression
3. Collapse-and-revival dynamics sits between the extremes
4. Inelastic scattering: Figure 2(e) versus Figure 3

General perturbation of a two-level system

Without loss of generality we can write

$$H_0 = \begin{pmatrix} E_i & 0 \\ 0 & E_f \end{pmatrix}, \quad V = \begin{pmatrix} 0 & V_{fi}^* \\ V_{fi} & 0 \end{pmatrix}.$$

Hence

$$H = \begin{pmatrix} E_i & V_{fi}^* \\ V_{fi} & E_f \end{pmatrix}.$$

Since H is Hermitian it is diagonalizable,

$$H = S^{-1}\Lambda S,$$

with a real diagonal matrix of eigenvalues. If we want to consider the $|\varphi_i\rangle \to |\varphi_f\rangle$ transition probability, then we would ideally like to compute

$$T_{fi} = \begin{pmatrix} 0 & 1 \end{pmatrix} S^{-1} \exp(-i\Lambda t/\hbar) S \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

At this point we resort to computer algebra to compute the Jordan decomposition of

$$H \to \begin{pmatrix} a & c+id \\ c-id & b \end{pmatrix},$$

from which we obtain the exact result,

$$T_{fi} = \frac{c-id}{\sqrt{a^2-2ab+b^2+4c^2+4d^2}}\{\exp(-i\theta t/\hbar) - \exp(-i\lambda t/\hbar)\},$$

where θ and λ are the eigenvalues of H,

$$\theta = \frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}\sqrt{a^2-2ab+b^2+4c^2+4d^2},$$
$$\lambda = \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}\sqrt{a^2-2ab+b^2+4c^2+4d^2}.$$

If we switch back to $a \leftrightarrow E_i,\ b \leftrightarrow E_f,\ c+id \leftrightarrow V_{fi}^*$ and define

$$\Omega \equiv \frac{1}{2}\sqrt{a^2-2ab+b^2+4c^2+4d^2}$$
$$\leftrightarrow \frac{1}{2}\sqrt{(E_f-E_i)^2+4|V_{fi}|^2},$$

then

$$
\begin{aligned}
T_{fi} &= \frac{V_{fi}\exp(-i(E_a+E_b)t/2\hbar)}{\sqrt{(E_f-E_i)^2+4|V_{fi}|^2}}\{\exp(i\Omega t/\hbar)-\exp(-i\Omega t/\hbar)\} \\
&= \frac{V_{fi}\exp(-i(E_a+E_b)t/2\hbar)}{\sqrt{(E_f-E_i)^2+4|V_{fi}|^2}}2i\sin(\Omega t/\hbar),
\end{aligned}
$$

and

$$
P_{fi} = \frac{4|V_{fi}|^2}{(E_f-E_i)^2+4|V_{fi}|^2}\sin^2\left(\frac{t}{2\hbar}\sqrt{(E_f-E_i)^2+4|V_{fi}|^2}\right).
$$

Literature assignment #1

- Validity of electric dipole approximation with one compact and one extended state; non-E1 effects in Rydberg atoms
- Non-E1 effects in nanophotonics
- Dipole-type approximations in resonant energy transfer
- Relativistic origin of non-E1 effects
- Emergence of electric quadruople and magnetic dipole terms in next-to-highest order?