## 1-bit memory using one electron: Parametric oscillations in a Penning trap

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F. L. Walls
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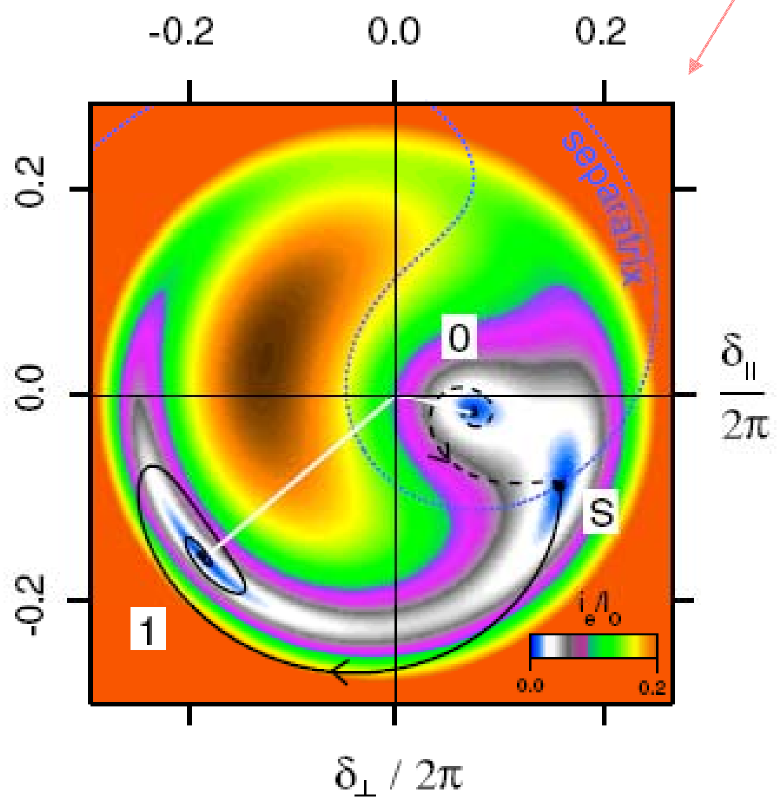
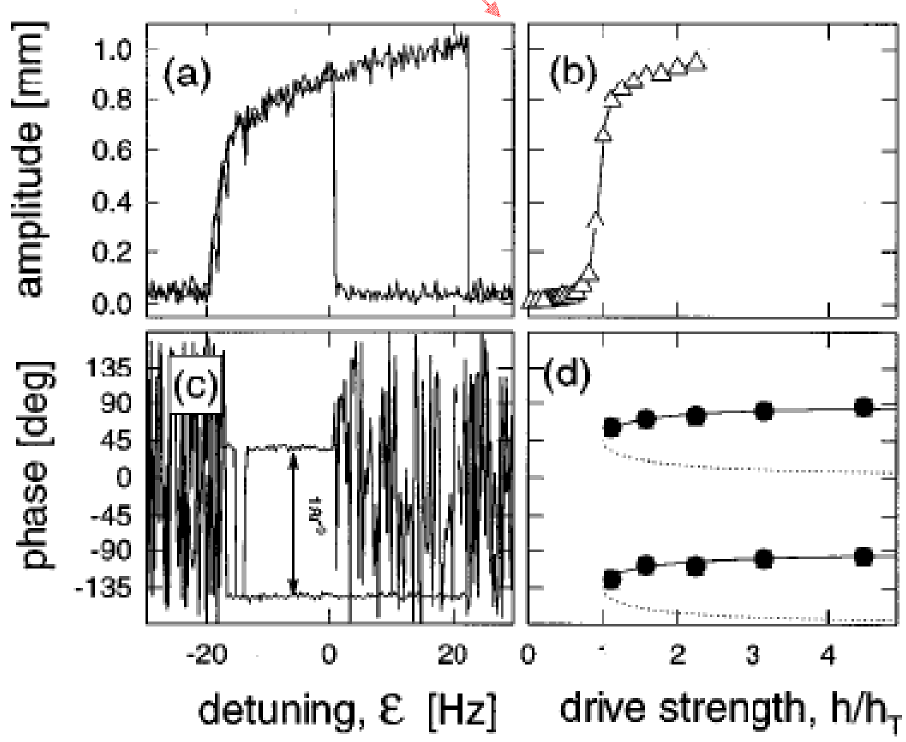
(Received 24 June 1998)

## RF-Driven Josephson Bifurcation Amplifier for Quantum Measurement

I. Siddiqi, R. Vijay, F. Pierre, C. M. Wilson, M. Metcalfe, C. Rigetti, L. Frunzio, and M. H. Devoret
*Departments of Applied Physics and Physics, Yale University, New Haven, Connecticut 06520-8284, USA*
(Received 11 February 2004; published 10 November 2004)



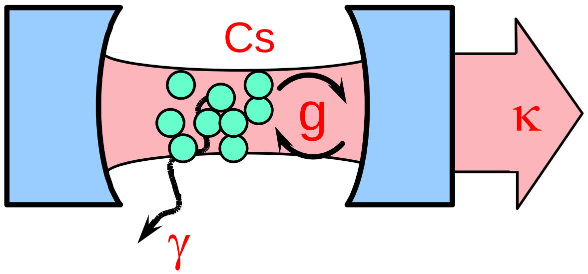
# Bifurcations, instabilities & fluctuations in cavity QED

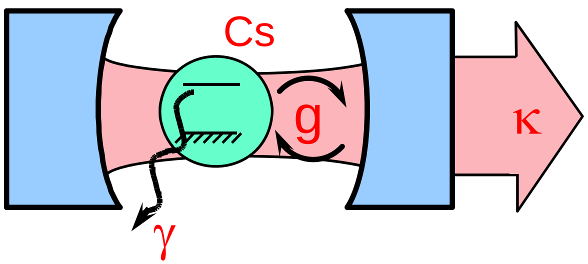
C. Savage and H. J. Carmichael IEEE J. Quantum Electron. **24**, 1495 (1988)

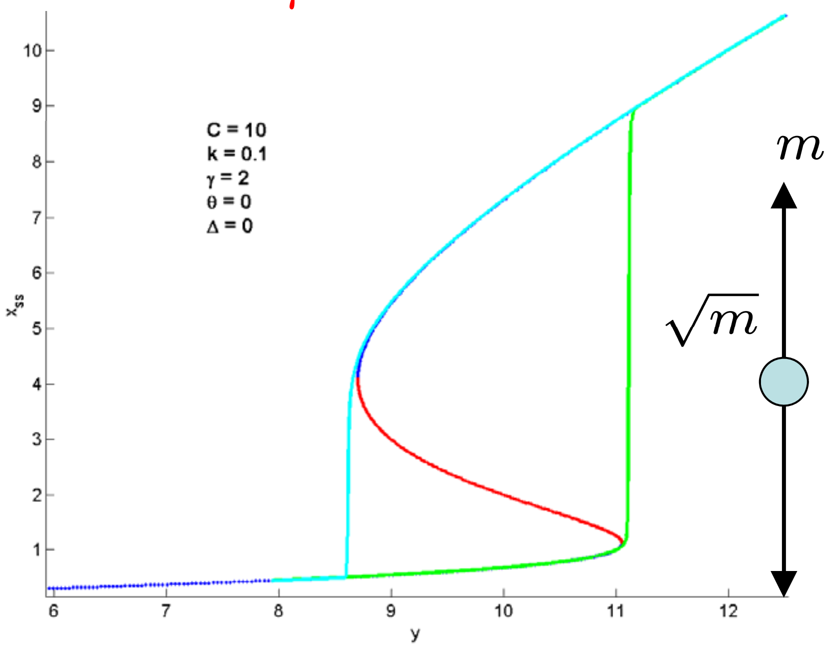
$$C = \frac{Ng^2}{2\kappa\gamma_\perp}$$

weak coupling

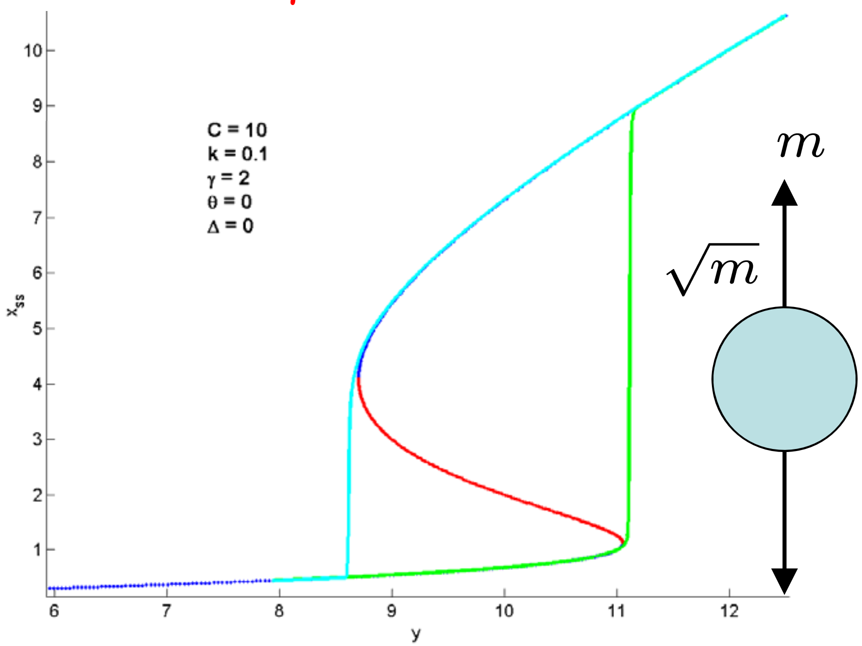
strong coupling




*semi-classical nonlinear dynamics*

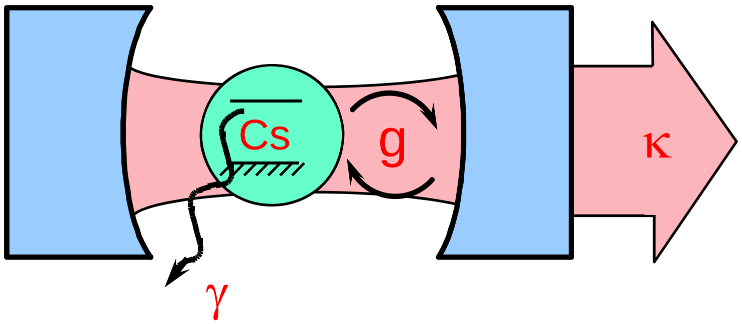



*un-localized in phase portrait*

# Cavity QED with strong coupling

H. J. Kimble, Physica Scripta **176**, 127 (1998)

One atom, strongly coupled




**Critical Photon Number**

$$m_0 \approx \frac{\gamma_\perp^2}{2g^2} < 1$$

1 photon can induce strong nonlinearity

**Critical Atom Number**

$$N_0 \approx \frac{2\kappa\gamma_\perp}{g^2} < 1$$

1 atom may be used as an optical switch

The Master Equation for the Jaynes-Cummings Hamiltonian:

$$\dot{\rho} = \frac{1}{i\hbar}[\,H\,,\,\rho\,] + 2\kappa\mathcal{D}[a]\rho + 2\gamma_\perp\mathcal{D}[\sigma]\rho$$

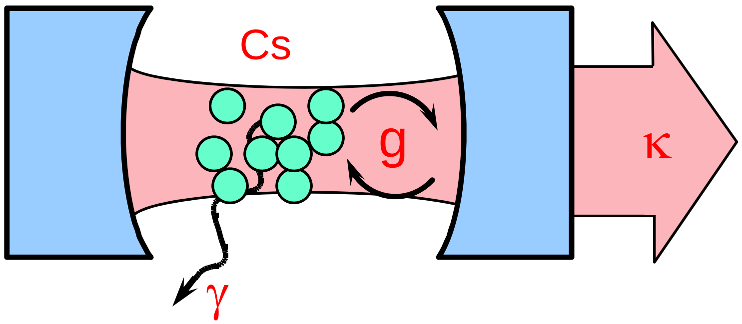
$$H = \hbar\Delta_c a^\dagger a + \hbar\Delta_a \sigma^\dagger\sigma + i\hbar\mathcal{E}(a^\dagger - a) + i\hbar g(a^\dagger\sigma - a\sigma^\dagger)$$

- Driven, coherent + dissipative linear evolution of the joint state
- Generation of entanglement between atom and cavity field
- Fundamental paradigm for study of open quantum systems

# Semiclassical Maxwell-Bloch Equations

L. A. Lugiato, in *Progress in Optics*, edited by E. Wolf (North-Holland, 1984), Vol. XXI

Many atoms, weakly coupled




Cooperativity Parameter

$$C = \frac{g_{eff}^2}{2\kappa\gamma_\perp} = \frac{Ng^2}{2\kappa\gamma_\perp}$$

Absorptive Bistability for C > 4

Maxwell-Bloch Equations:

$$\begin{aligned} \dot{x} &= \kappa(1+i\Theta)\; x + (g_{eff}/2)\; p + \mathcal{E} \\ \dot{p} &= \gamma_\perp(1+i\Delta)\; p + 2g_{eff} m x \\ \dot{m} &= -\gamma_\parallel(m+1) - g_{eff}(\; x^* p + p^* x \;) \end{aligned}$$

$$\begin{aligned} \sqrt{m_0}\; x &\leftrightarrow \langle a \rangle \\ p &\leftrightarrow 2\langle \sigma \rangle \\ m &\leftrightarrow \langle \sigma_z \rangle = \langle [\sigma^\dagger, \sigma] \rangle \end{aligned}$$

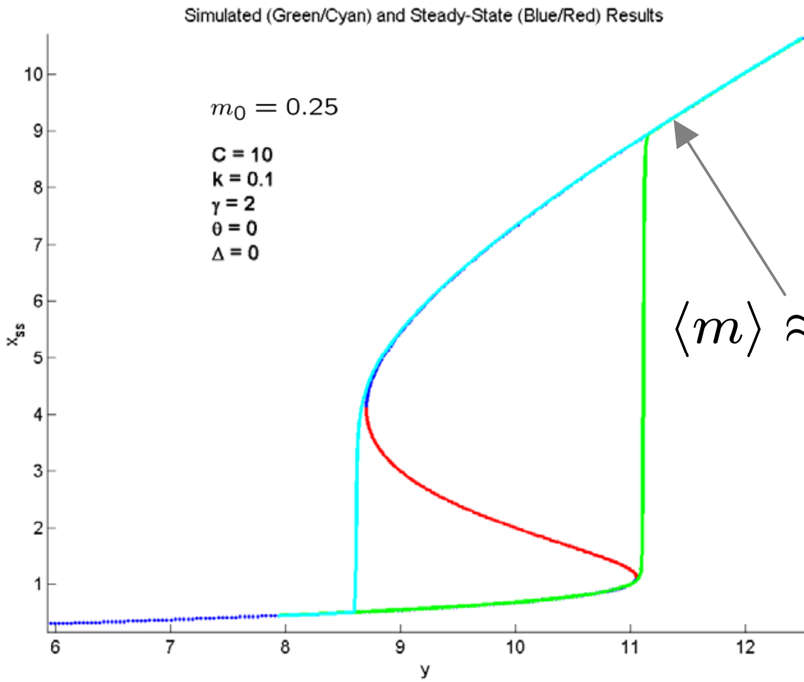
$$\langle a^\dagger \sigma \rangle \rightarrow \langle a^\dagger \rangle \langle \sigma \rangle$$

- Valid in weak-coupling (per atom) regime ⇒ $m_0$ is relatively large
- Mean-field approximation for cavity field; no atom-field entanglement
- Fluctuations so far incorporated additively, via local linearization

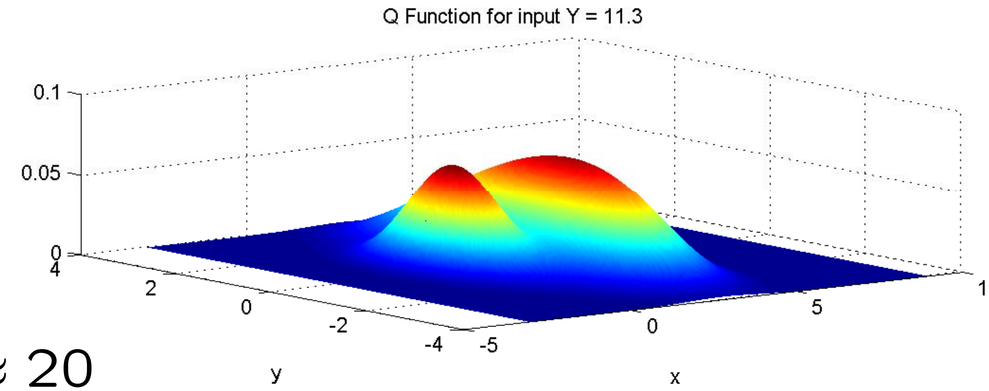
H. J. Carmichael, PRA **33**, 3262 (1986)

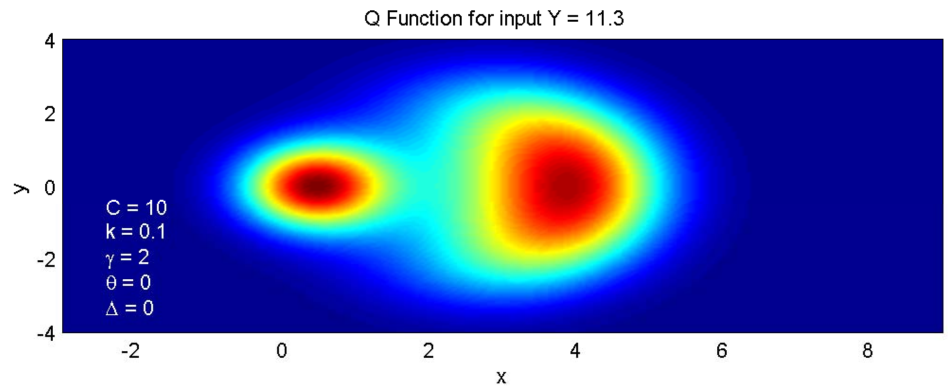
# Single-atom absorptive bistability, revisited

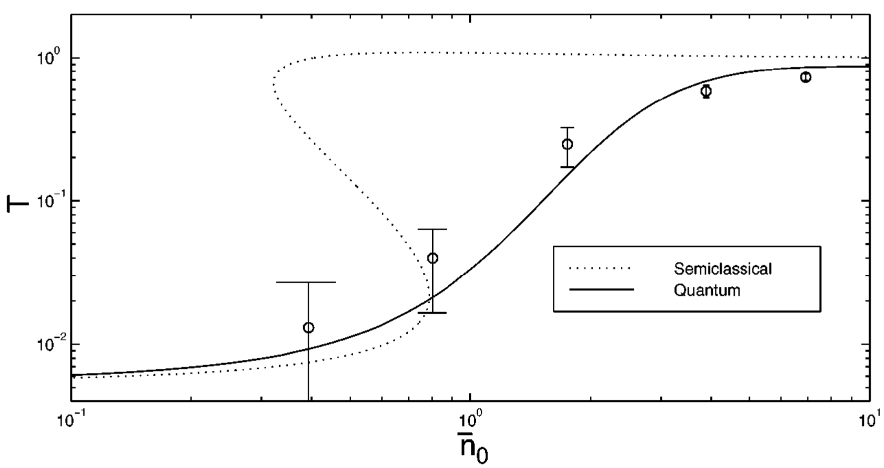
M. Armen and HM, PRA **73**, 063801 (2006)



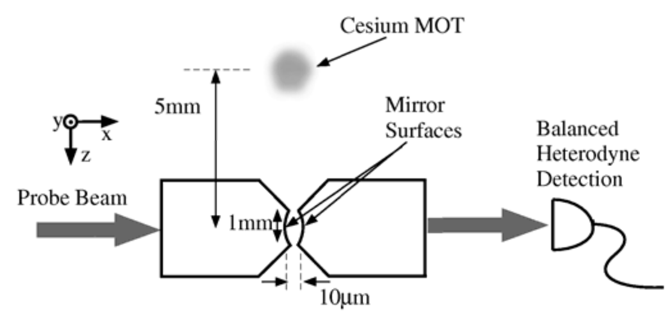

$\langle m \rangle \approx 20$



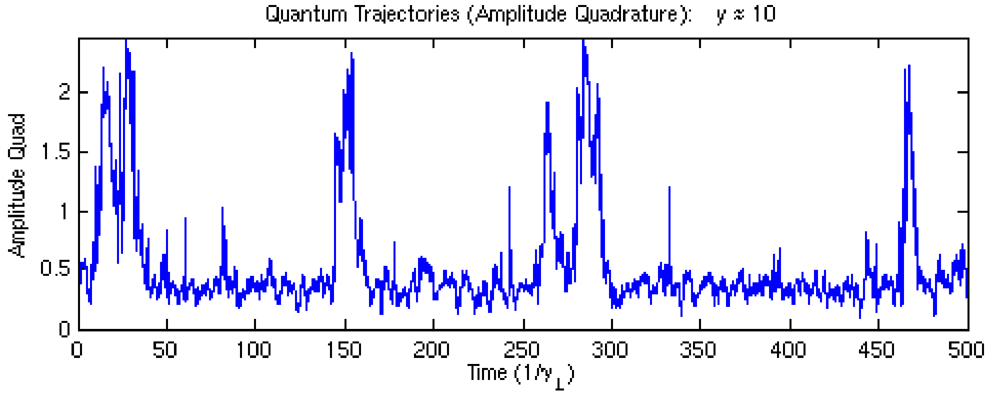



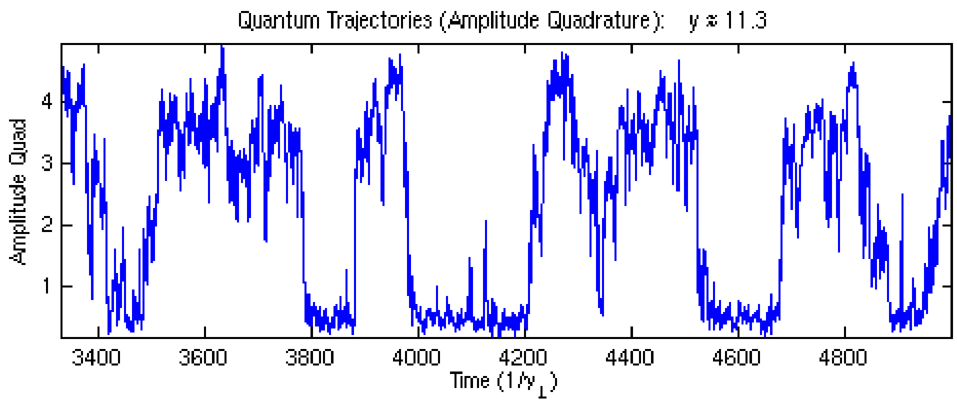



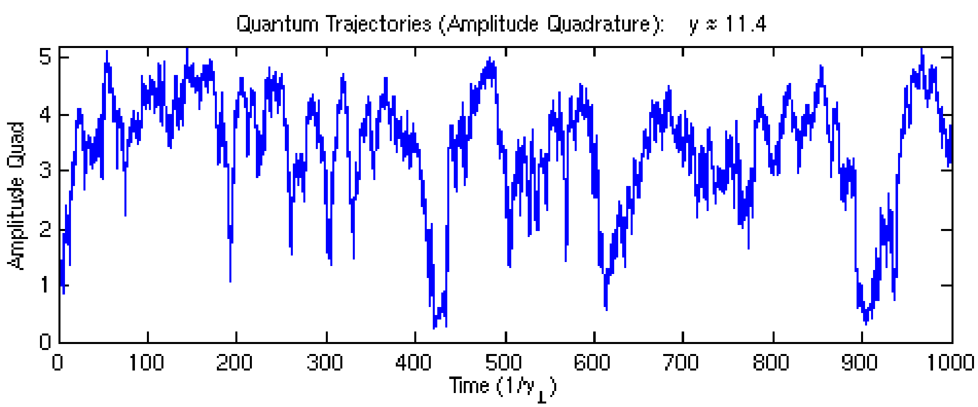

← *Kimble and co-workers ('98)*

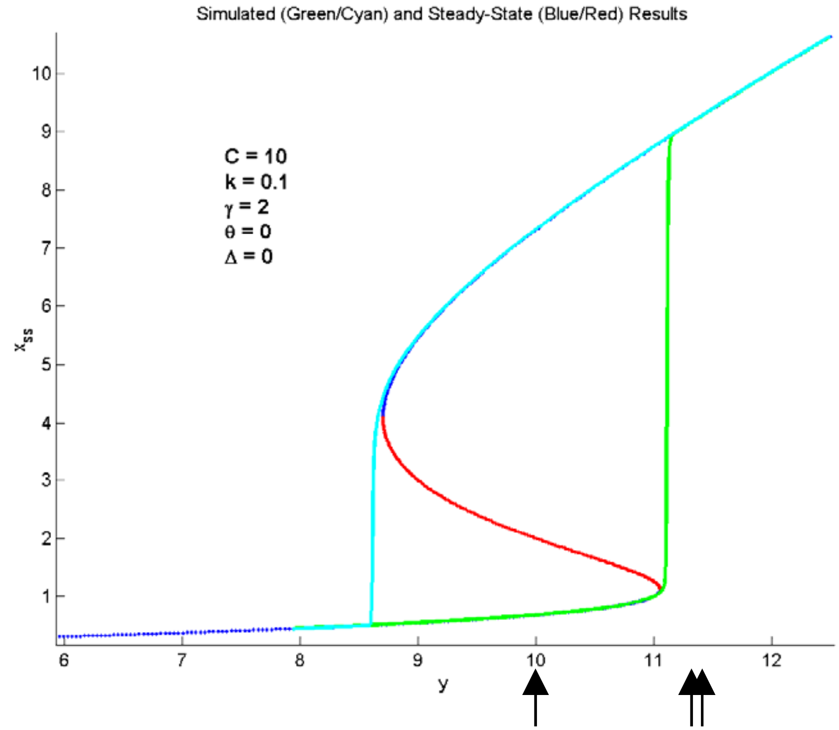


# Quantum trajectories for single-atom bistability



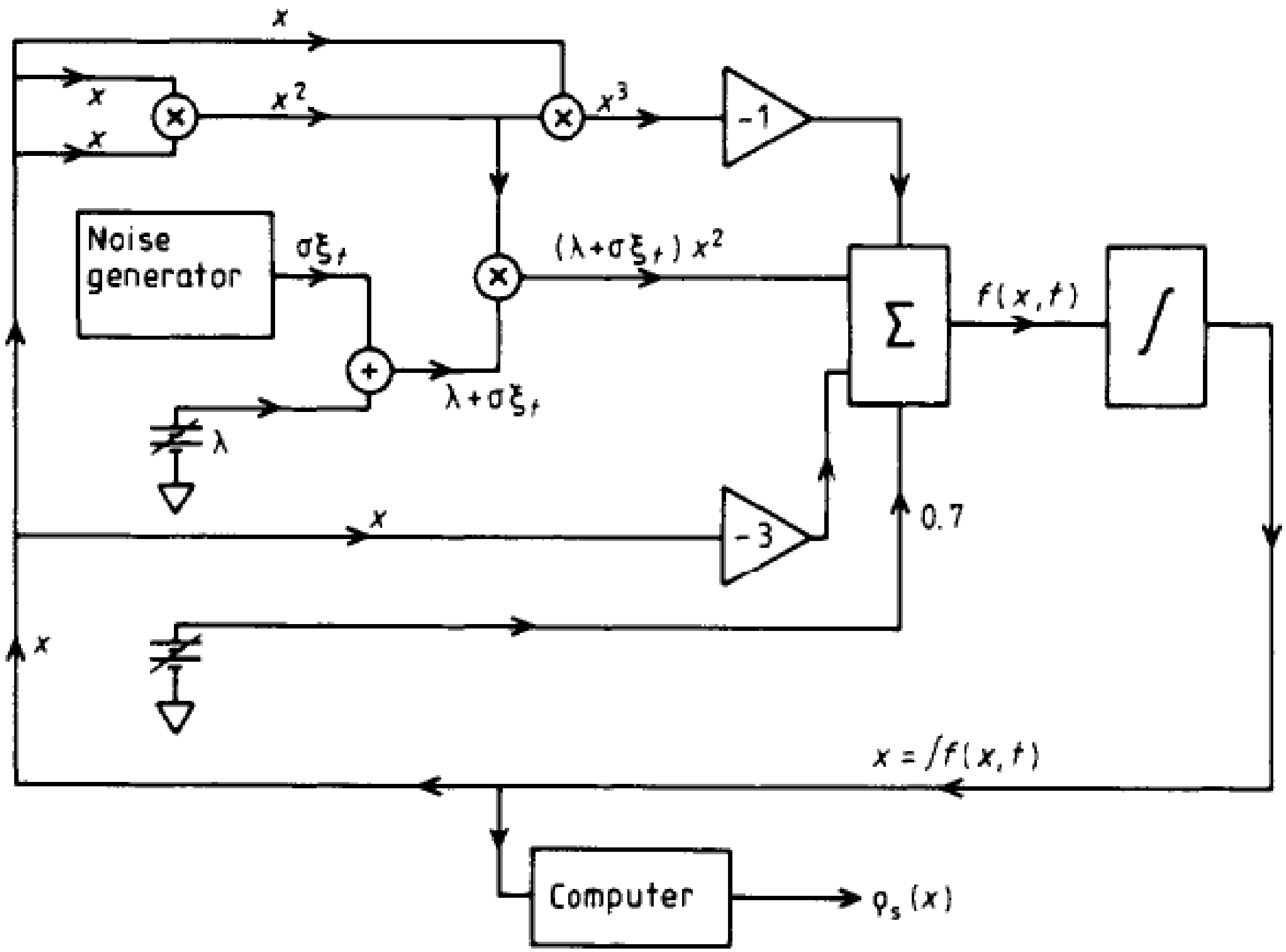




- relative stability of lower branch?
- persistence > upper turning point?

# Electrical circuit simulation of a cubic bistable system

S. D. Robinson, F. Moss and P.V. E. McClintock, J. Phys. A: Math. Gen. **18**, L89 (1985)

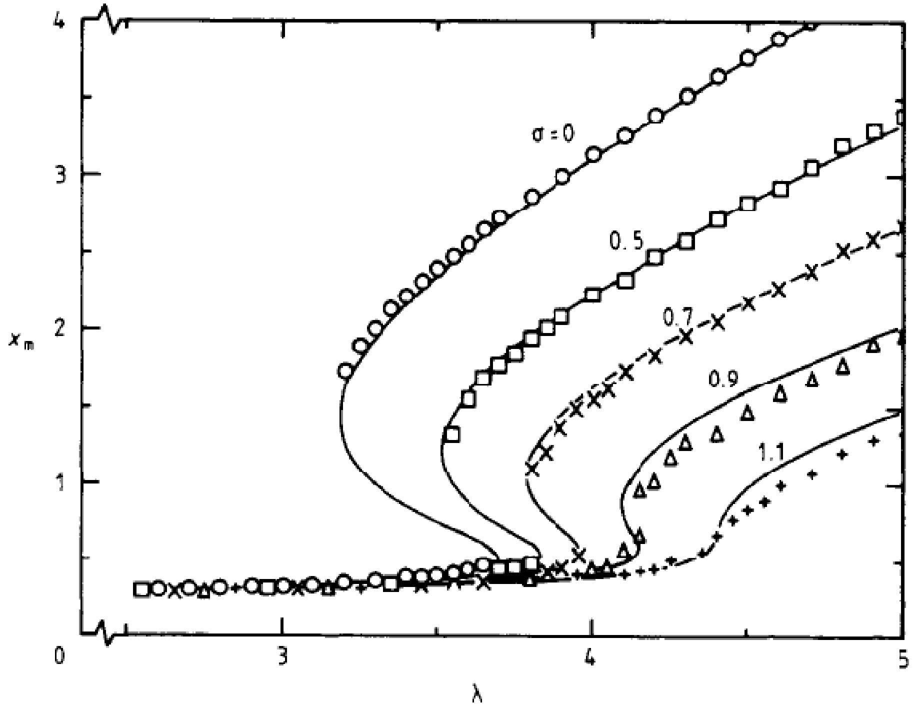


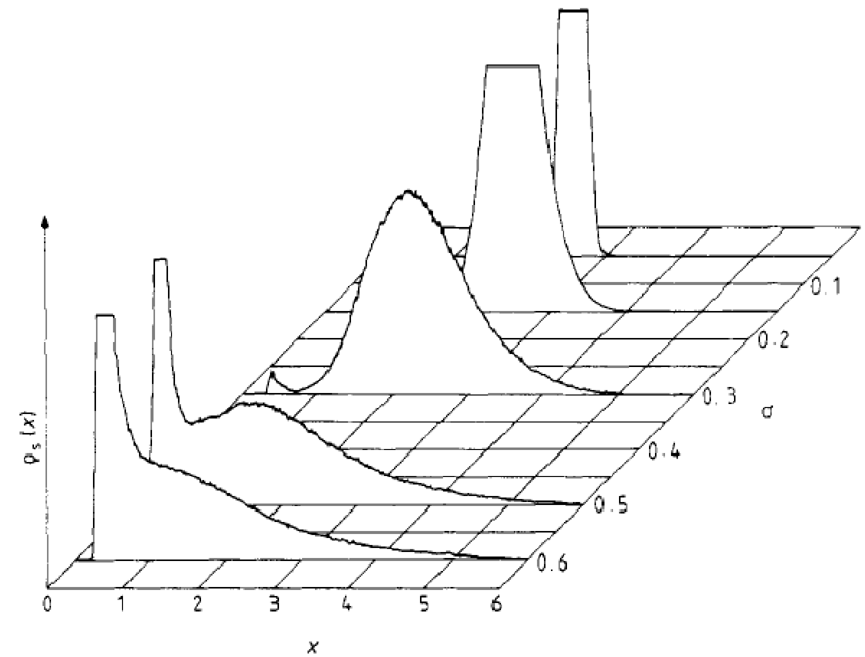

$$\dot{x} = -x^3 + \lambda_t x^2 - Qx + R = f(x,t) \quad (1)$$

$$\lambda_t = \lambda + \sigma\,\xi_t \quad (2)$$

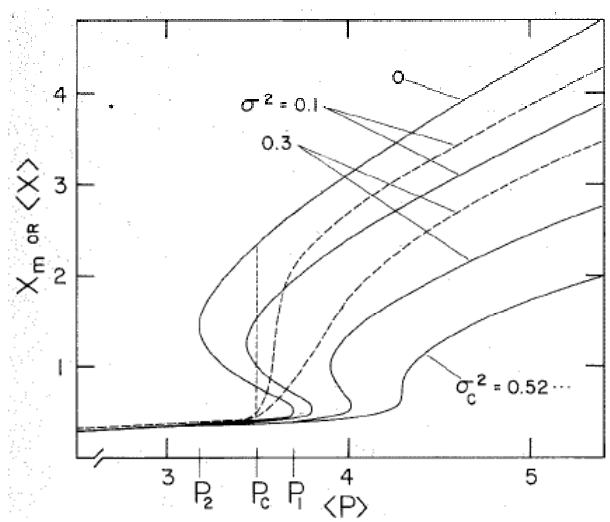
# “Postponement of critical onsets”

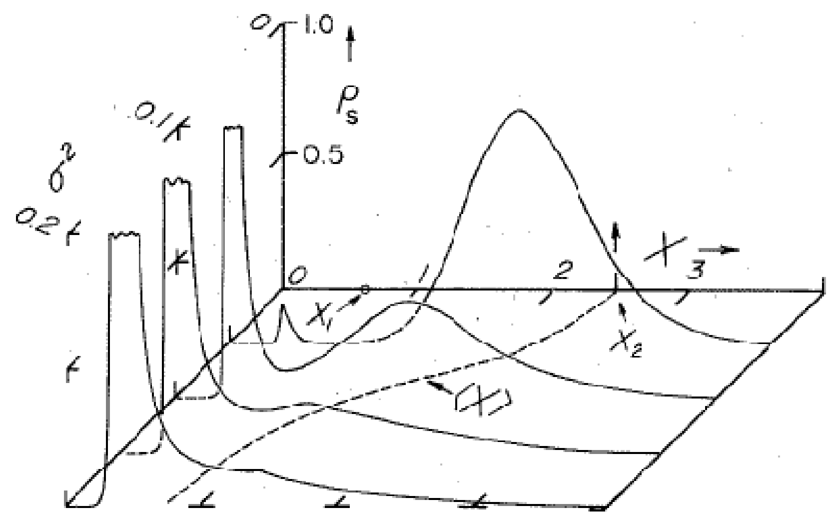
S. D. Robinson, F. Moss and P.V. E. McClintock, J. Phys. A: Math. Gen. **18**, L89 (1985)



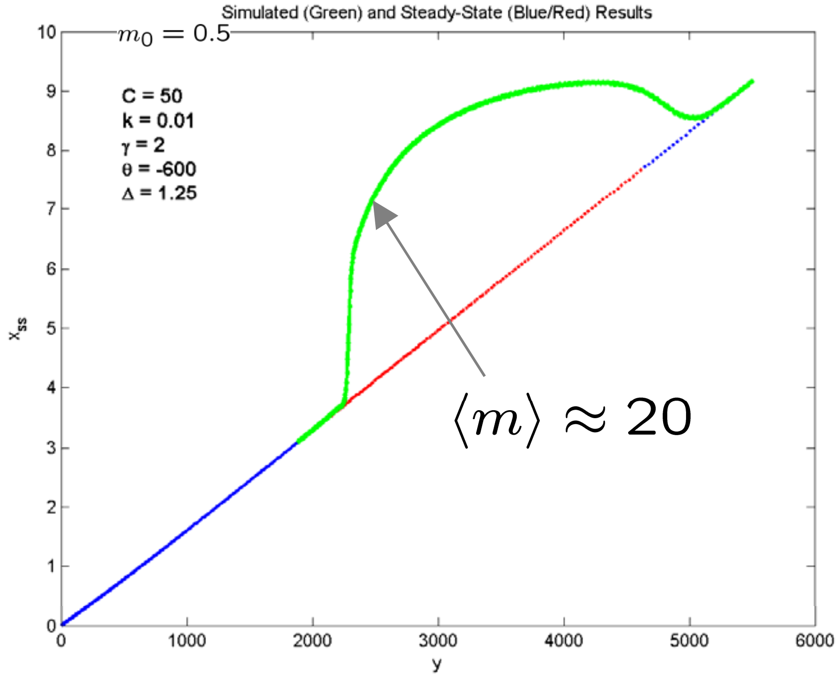

Theory by
G. V. Welland and
F. Moss:
Phys. Lett. **89A**,
273 (1982)

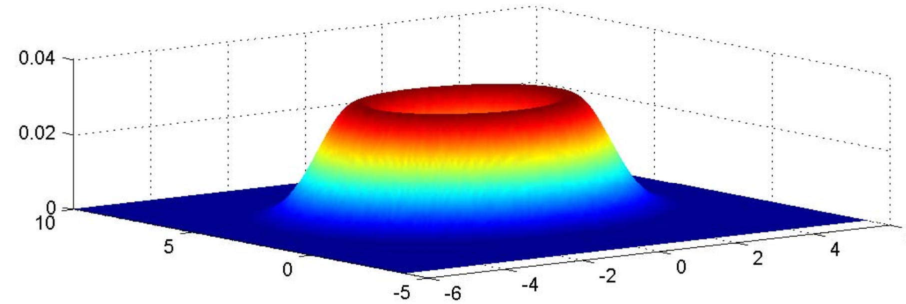


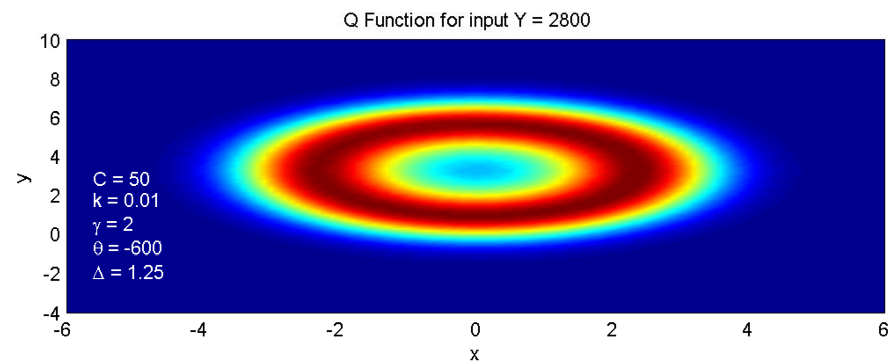



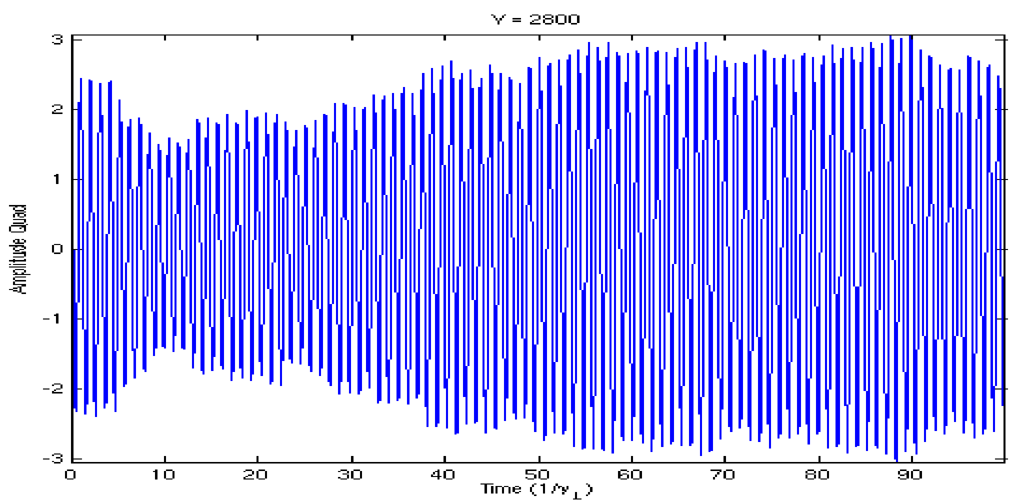
# Super-critical Hopf bifurcation: stable limit cycle

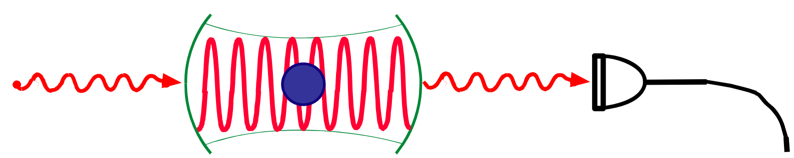
M. Armen and HM, PRA **73**, 063801 (2006)



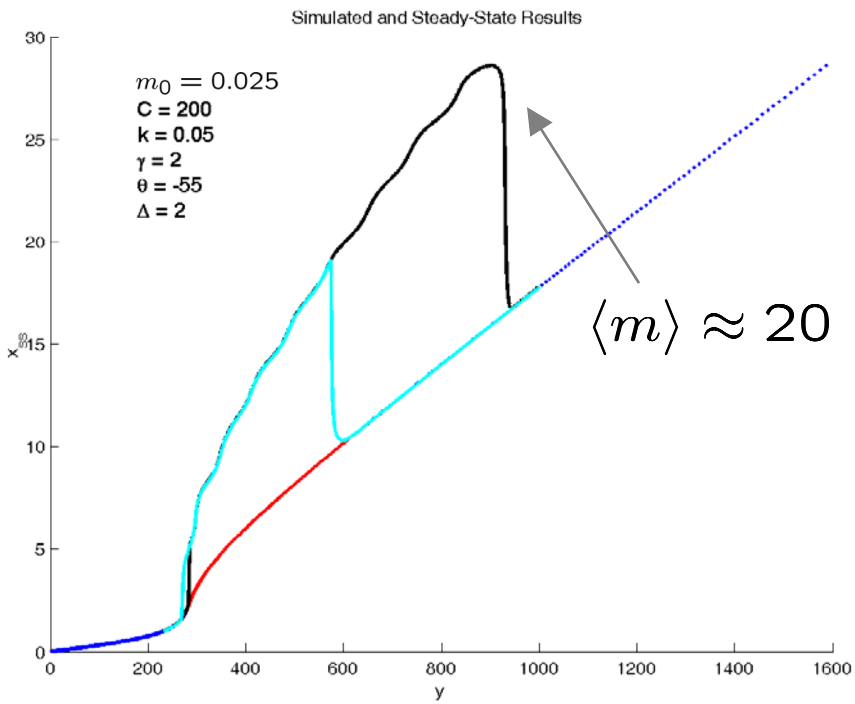


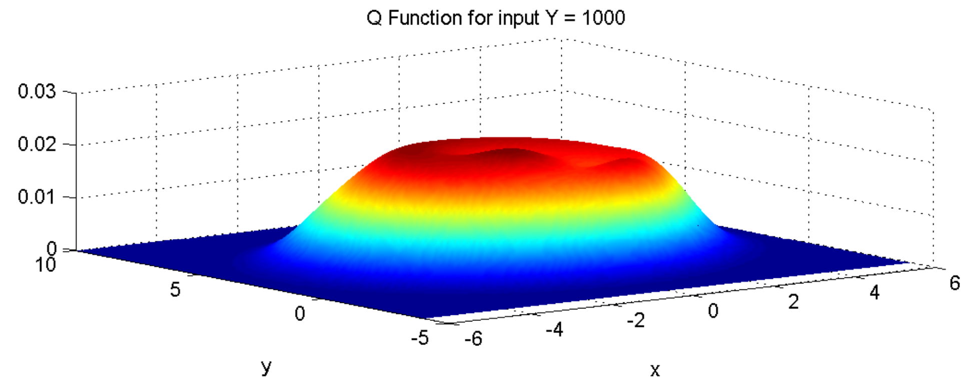



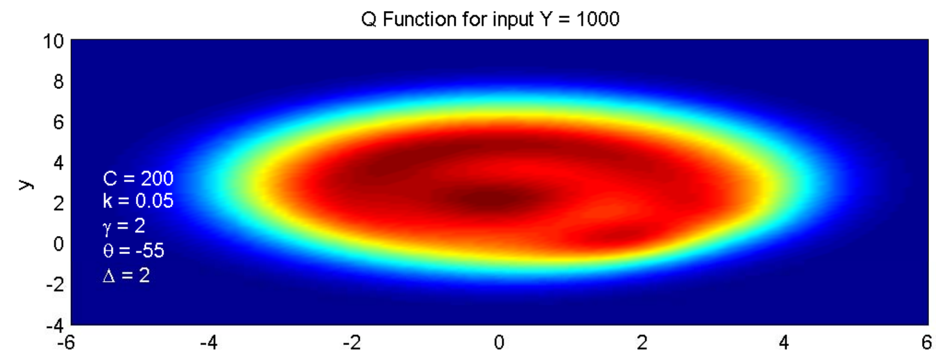



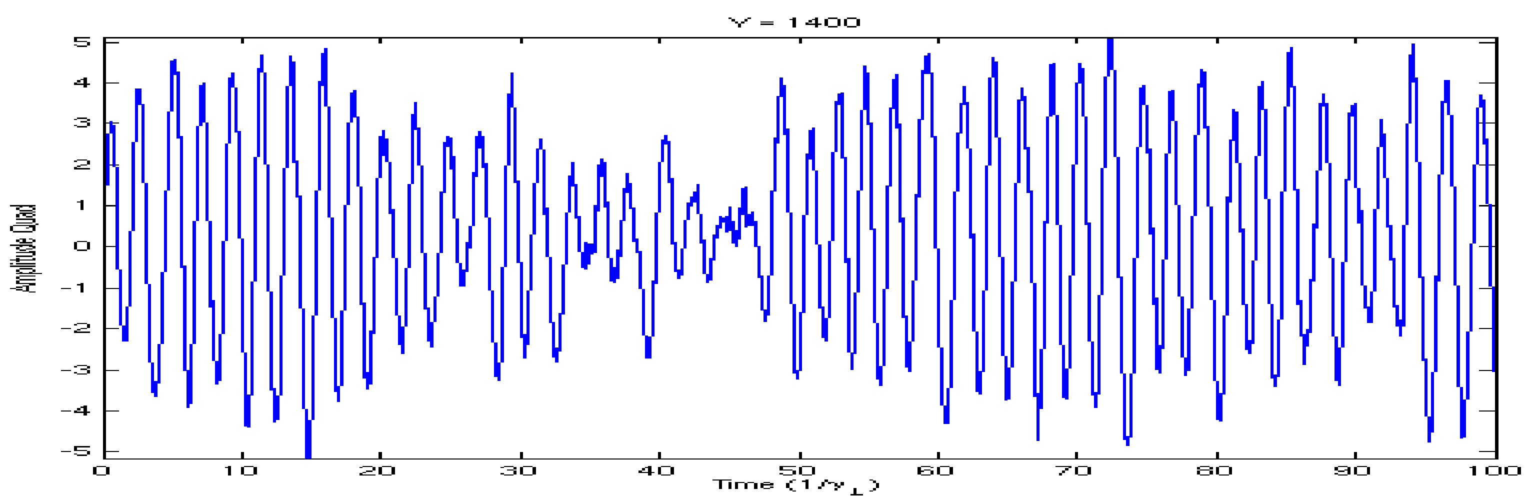
Oscillatory dynamics should be observable in homodyne/heterodyne measurements + filtering

# Sub-critical Hopf: fixed point plus limit cycle



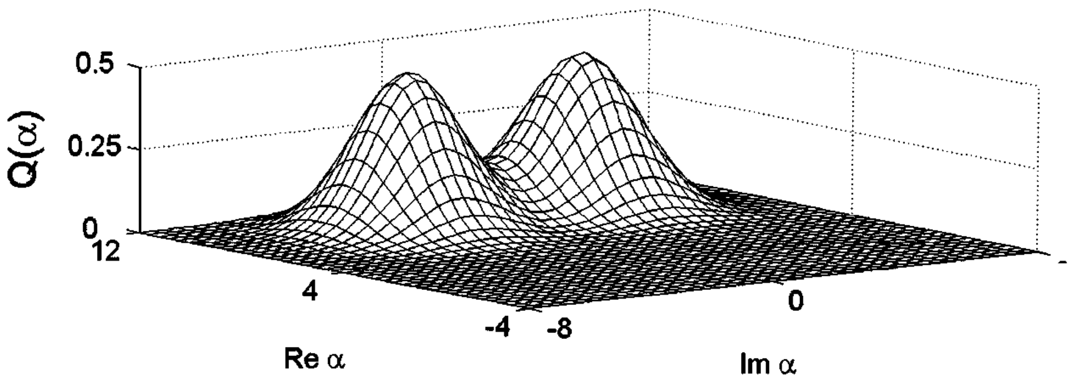



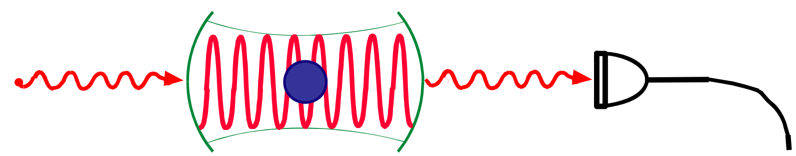



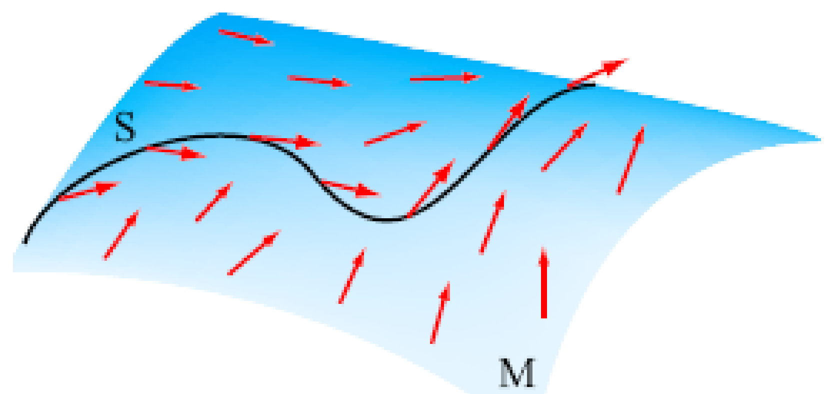



# Phase bistability and quantum filter projection

P. Alsing and H. J. Carmichael, Quantum Opt. **3**, 13 (1991)
Ramon van Handel and HM, J. Opt. B: Quantum Semiclass. Opt. **7**, S226 (2005)
H. Mabuchi, Phys. Rev. A **78**, 015801, (2008)

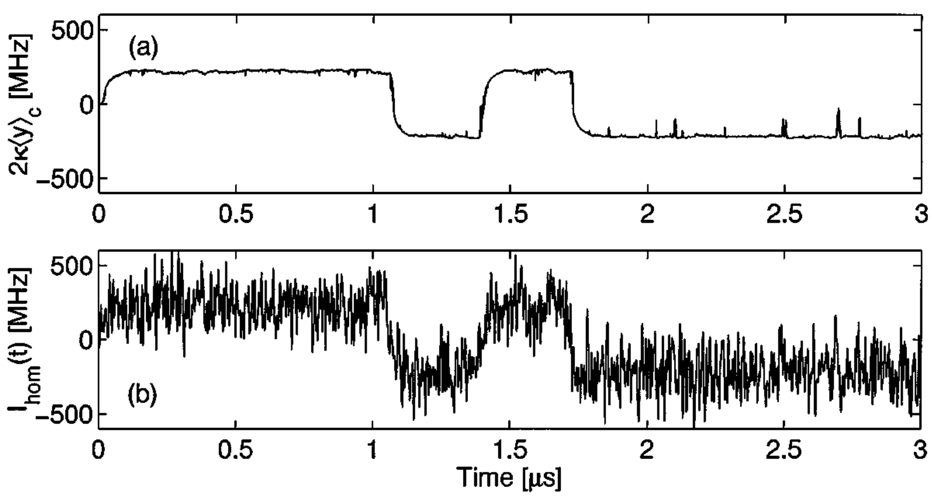





$$d\rho = \mathcal{L}\rho dt + i\sqrt{2\kappa\eta}\left\{a\rho - \rho a^\dagger - \mathrm{Tr}\left[\rho\left(a - a^\dagger\right)\right]\right\} dW_t$$

$$I_{\mathrm{hom}}(t) = 2\eta \mathrm{Tr}\left[(-ia + ia^\dagger)\rho\right] + \sqrt{2\kappa\eta} dW_t/dt$$



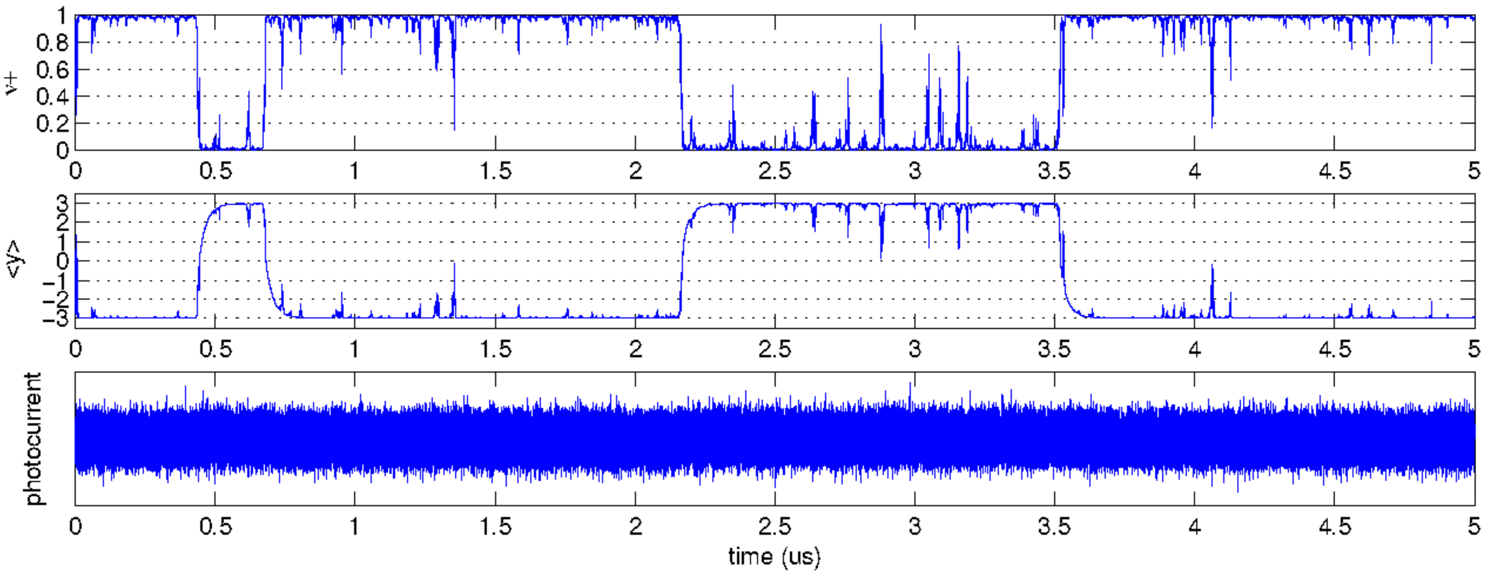

- parameterize sub-manifold of states
- intuition: two coupled “line segments”
- project stochastic equations of motion
- *e.g.*, stochastic Maxwell-Bloch Eqns.

# Bi-Gaussian approximate filter

Ramon van Handel and HM, J. Opt. B: Quantum Semiclass. Opt. **7**, S226 (2005)

Physical intuition motivates Gaussian *ansatz*;
restriction by geometric methods *(D. Brigo et al., M. H. Vellekoop and J. M. C. Clark, …)*

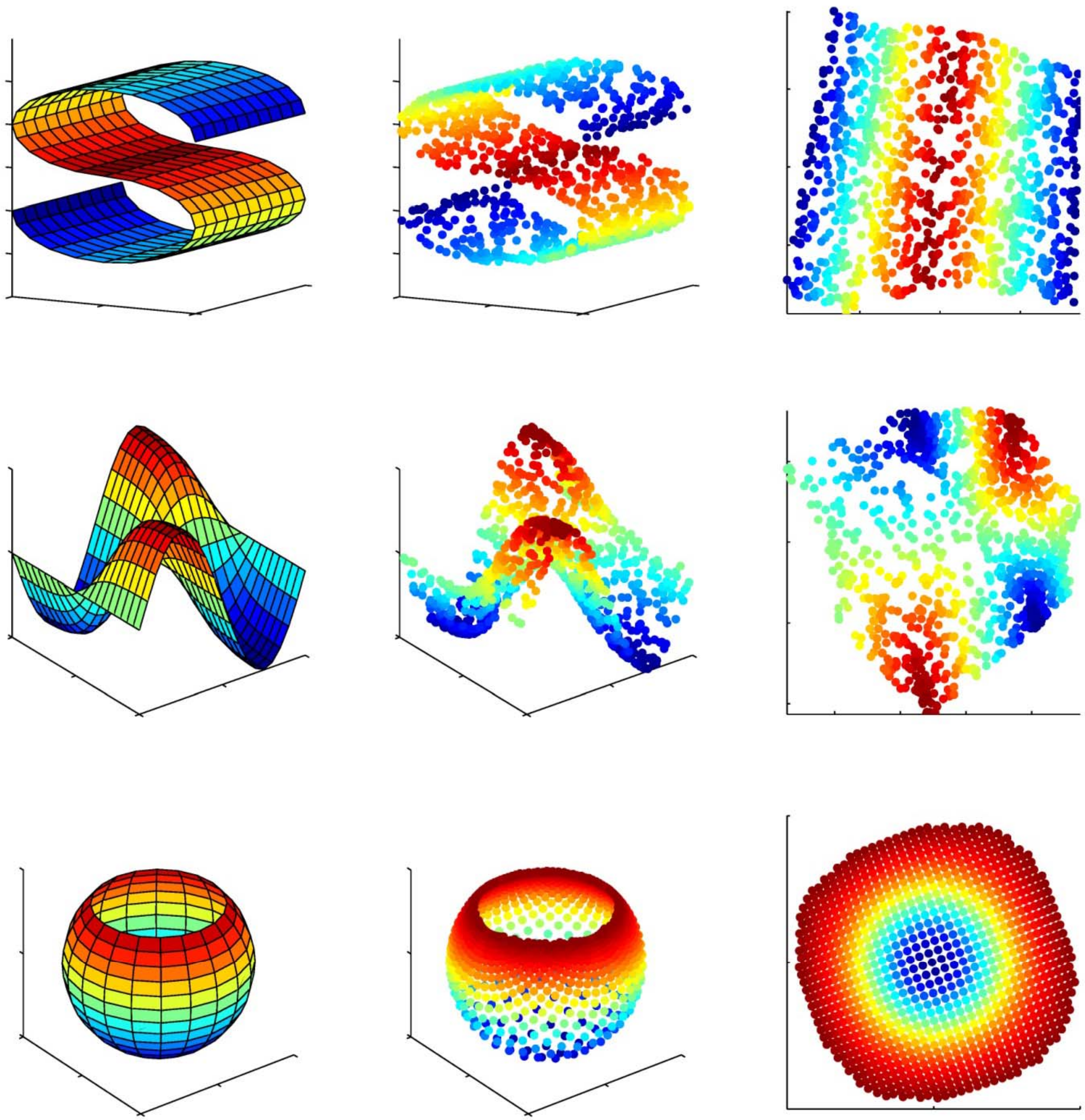
$$
\begin{aligned}
d\tilde{\nu}_t^+ &= -\gamma_\perp(\tilde{\nu}_t^+ - \tfrac{1}{2})dt + \sqrt{2\kappa\eta}\tilde{\nu}_t^+(1-\tilde{\nu}_t^+)(\mu_t^+ - \mu_t^-)(dY_t - \sqrt{2\kappa\eta}(\mu_t^+\tilde{\nu}_t^+ + \mu_t^-(1-\tilde{\nu}_t^+))dt) \\
\frac{d\mu_t^+}{dt} &= -g - \kappa\mu_t^+ + \frac{\gamma_\perp}{2}\frac{1-\tilde{\nu}_t^+}{\tilde{\nu}_t^+}(\mu_t^- - \mu_t^+) \\
\frac{d\mu_t^-}{dt} &= +g - \kappa\mu_t^- + \frac{\gamma_\perp}{2}\frac{\tilde{\nu}_t^+}{1-\tilde{\nu}_t^+}(\mu_t^+ - \mu_t^-)
\end{aligned}
$$




accomplishes $\sim 10^5 \to 1$ reduction, but relies on knowing sub-manifold

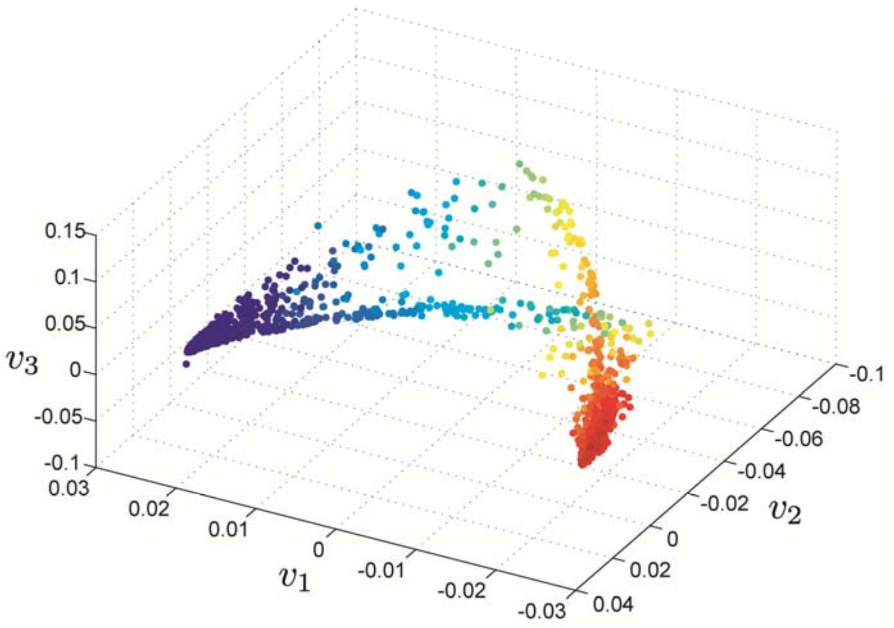
# Think Globally, Fit Locally: Unsupervised Learning of Low Dimensional Manifolds

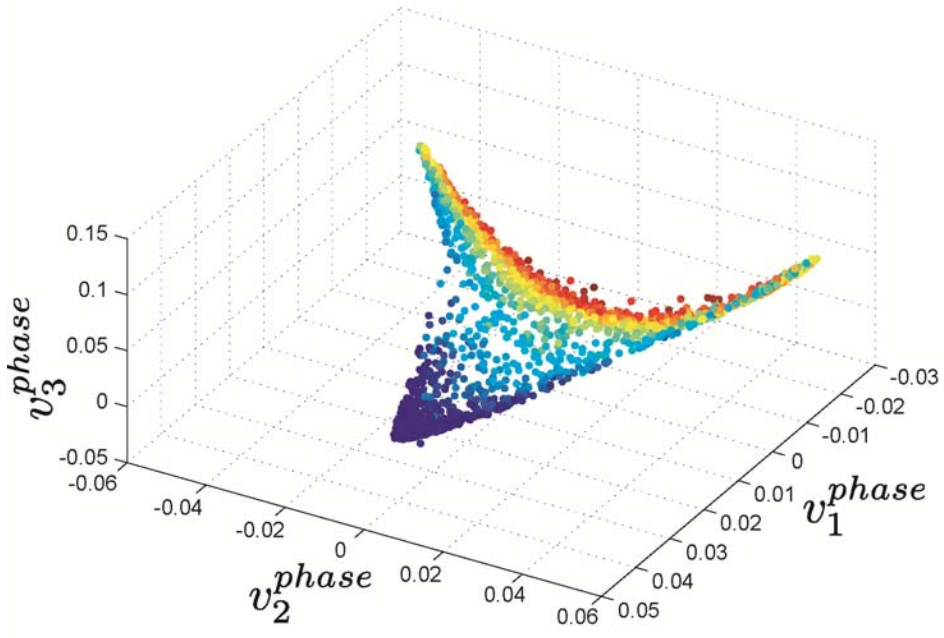
L. K. Saul and S. T. Roweis, Journal of Machine Learning Research **4**, 119 (2003)



# Embedding coordinates via local alignment (LTSA)

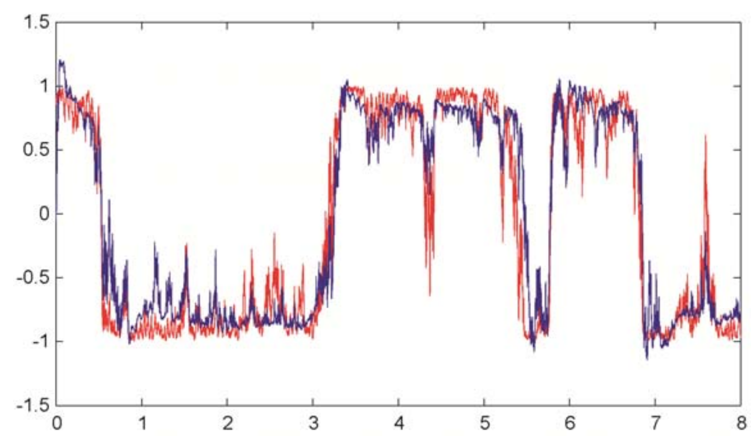
Z. Zhang and H. Zha, SIAM J. Sci. Comput. **26**, 313 (2004)




*"Basis pursuit" inference of dynamical equations*

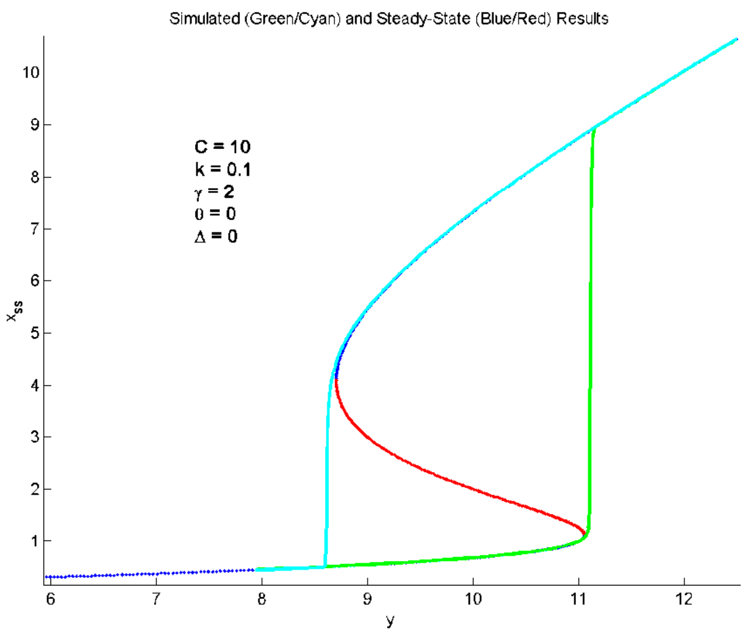
$$
\begin{aligned}
dv_1 &= \frac{-g_0}{2} v_4 dt - \kappa v_1 dt + \sqrt{8\kappa}(v_2 - v_1^2) dW \\
dv_2 &= -g_0(B_0 + B_1 v_1 v_4) dt - 2\kappa v_2 dt + \frac{\kappa}{2} dt \\
&\quad + \sqrt{8\kappa}(G_0 + G_1 v_1 + (G_2 - 1) v_1 v_2) dW \\
dv_4 &= -\gamma v_4 dt + \sqrt{8\kappa}(B_0 + (B_1 - 1) v_1 v_4) dW
\end{aligned}
$$

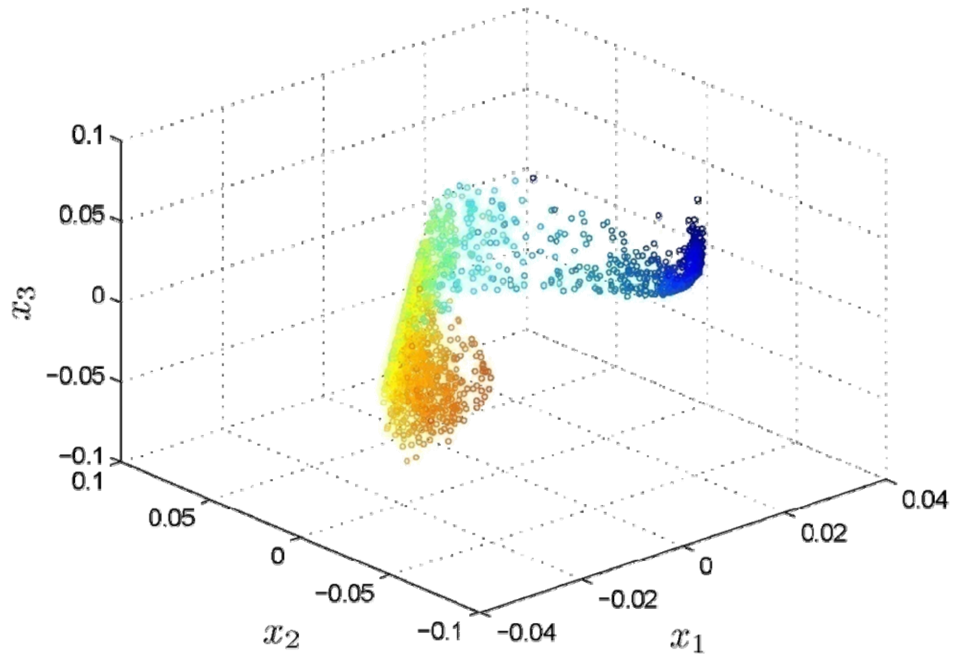


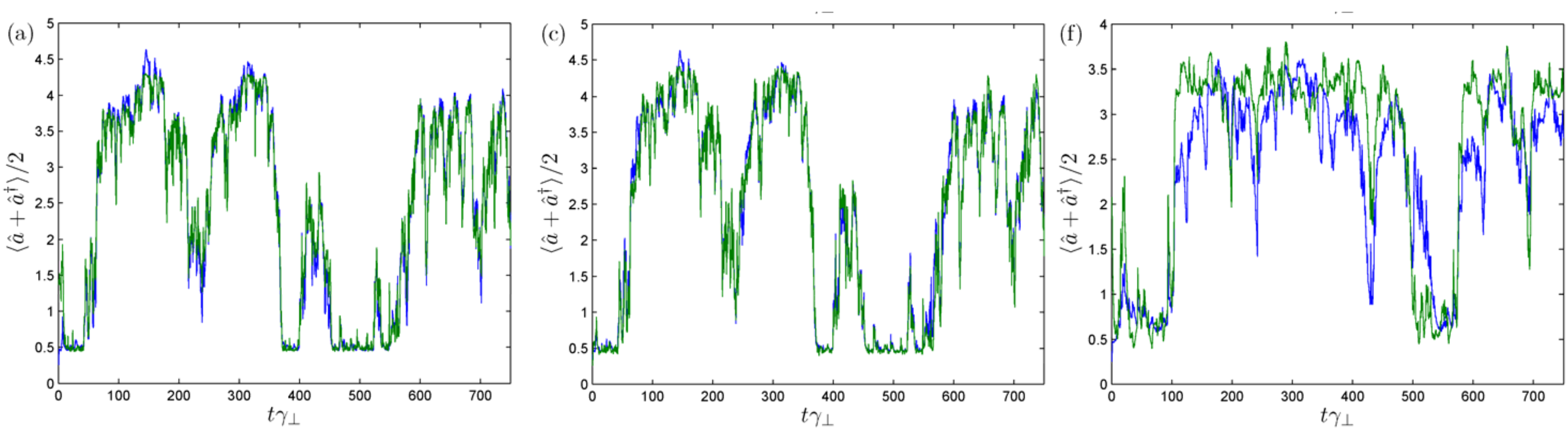
*Promising but some issues; projection preferable?*

# Nonlinear stochastic model reduction

A. B. Nielsen, A. S. Hopkins and HM, submitted



# Reduced filters for real-time measurement feedback

A. B. Nielsen, A. S. Hopkins and HM, submitted