Optimal Power Control for CDMA Systems in the Wideband Limit

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Abstract — **Traditional methods for solving multiuser control problems in CDMA systems do not scale well as the number of users in the system increases. To address this problem we develop a wideband limit approximation for dynamic programming problems in CDMA systems. This approximation greatly simplifies the state space and evolution equations for the users in a CDMA cell, thereby simplifying the optimal control problem. This wideband convergence result holds for a number of different linear multiuser receiver structures including the MMSE and matched filter receivers.**

I. System Model

Consider a group of K wireless users transmitting data to a base station. We assume the variations in the wireless channel gain between each user and the base station can be described by a finite-state discrete-time Markov chain (DTMC). We will denote the state of the ith user's channel gain at time t as $z_i(t)$. Let $y_i(t)$ be the state of the ith user's data buffer at time t.

We define the state of user i at time t as $x_i(t) = (y_i(t), z_i(t)) \in \mathcal{X}$, where the set $\mathcal{X}$ consists of all combinations of buffer states and channel states for a single user. Using the state of each user we can define a state for the entire system as the empirical distribution $\Pi_K(t)$ over all K users in the cell at time t:

$$\Pi_K(j,t) = \frac{1}{K}\sum_{k=1}^{K} I_{[x_k(t)=j]}, \tag{1}$$

where $I_{[x_k(t)=j]} = 1$ if mobile k is in state j at time t and zero otherwise. Note that $\Pi_K(t)$ is a random vector for finite K.

In each time slot every mobile selects a power level $a_i(t) \in A$, where A is a finite set of transmission powers. We assume each mobile chooses a transmission power according to a power control policy $g(x_i(t), \Pi_K(t))$, which maps the state of the ith user and the empirical distribution $\Pi_K(t)$ into a probability distribution across A.

We assume the probability of mobile i successfully transmitting a packet of data at time t depends only on the selected control g and the empirical distribution $\Pi_K(t)$. Hence we can define a transition matrix for the ith user as $P(g, \Pi_K(t))$ such that $P_{jk}(g, \Pi_K(t)) = p(x_i(t+1) = k | x_i(t) = j, g, \Pi_K(t))$.

The receiver structure employed at the basestation is a linear multi-user receiver. As in [1], we assume a random spreading sequence model. Each user is assigned an N length spreading sequence with chips that are independently and identically distributed with mean zero and variance one.

The linear receiver provides a feature that is critical to our analysis. Namely, the wideband limiting SIR for the ith user can be computed directly [1] for many different receiver structures; provided we know the power control policy $g(x_i(t), \Pi_K(t))$, the state of the ith user $x_i(t)$, and the empirical distribution of all users $\Pi_K(t)$.

II. Wideband Optimal Control Problem

Define a cost vector $r(g)$, which determines the cost of choosing control g for each state in $\mathcal{X}$.

Proposition 1: As $K, N \to \infty$ with $\frac{K}{N} \to \alpha$ the empirical distribution $\Pi_K(t)$ obeys the non-linear matrix equation

$$\lim_{K\to\infty} \Pi_K(t+1) = \pi(t+1) = \pi(t)P(g, \pi(t)). \tag{2}$$

Furthermore, this non-linear matrix equation has a unique fixed point.

Let $R_i(g,t)$ be the random process describing the cost associated with the control for user i at time t, we write the value of the power control policy g as

$$V_K(g,i) = \lim_{T\to\infty} \frac{1}{T+1}\sum_{t=0}^{T} E_{x,\Pi_K}[R_i(g,t)], \tag{3}$$

where the expectation is taken with respect to the state $x_i(t)$ and empirical distribution $\Pi_K(t)$.

Proposition 2: As $K, N \to \infty$ with $\frac{K}{N} \to \alpha$ we have $V_K(g,i) \to \pi(\infty,g)r(g)^T$, where $\pi(\infty, g)$ is the unique fixed point of (2). This simplification allows us to solve the following non-linear optimization problem for the optimal multi-user control.

$$\min_g \pi(\infty,g)r(g)^T \tag{4}$$

subject to:

$$\begin{aligned} \pi(\infty,g) &= \pi(\infty,g)P(\pi(\infty,g)) \\ \pi(\infty,g)A &\leq b \\ \pi(\infty,g)\mathbf{1}^T &= 1, \end{aligned}$$

where $\pi(\infty,g)A \leq b$ represents a set of quality of service (QoS) constraints .

Proofs of these propositions can be found in [2]. We also show that these results hold for a general class of cross-layer controls with tight QoS constraints, rather than just power control. Traditional formulations of these multi-user cross-layer problems are typically intractable due to the curse of dimensionality. The low-dimensional wideband approximation provides an approximately optimal control for system sizes comparable to those proposed for the next generation CDMA networks.

References

[1] D. Tse and S. Hanly, "Linear Multiuser Receivers: Effective Interference, Effective Bandwidth and User Capacity", *IEEE Transactions on Information Theory*, v.45, No. 2, Mar. 1999.

[2] T. Holliday, A. Goldsmith, P. Glynn, "Wideband Approximations for Optimal Control of CDMA Systems", *In Preparation.*

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