Distributed Power Control for Time Varying Wireless Networks: Optimality and Convergence

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Abstract

This paper presents a new distributed power control algorithm for ad-hoc wireless networks in random channel environments. Previous work in this area has focused on distributed power control for ad-hoc networks with fixed channels. We show that the algorithms resulting from such formulations do not accurately capture the dynamics of a time-varying channel. The performance of the network, in terms of power consumption and generated interference, can be severely degraded when a power control algorithm designed for a deterministic channel is applied to a random channel. In particular, some well-known strong optimality results for such algorithms no longer hold. In order to address these problems we propose a new criterion for power optimality in ad-hoc wireless networks. We then show that the optimal power allocation for this new criterion can be found through an appropriate stochastic approximation algorithm. Ultimately, the iterations of the stochastic approximation algorithm can be decoupled to form an optimal fully distributed on-line power control algorithm for ad-hoc wireless networks with time-varying channels.

1 Introduction

Adaptive control of transmission power in wireless networks allows devices to setup and maintain wireless links with minimum power while satisfying constraints on quality of service (QoS). The benefits of power minimization are not just increased battery life. Effective interference mitigation can also increase overall network capacity by allowing higher frequency reuse.

Typically, power control and interference mitigation techniques are designed for wireless networks with cellular architectures. The benefit of such an architecture is that one can assume a centralized controller has knowledge of the channel states for all users in the system. In this paper we consider a fundamentally different architecture where there is no centralized controller to distribute power control commands or channel information. Hence, the model we consider here is that of an ad-hoc wireless network with purely distributed control (we will clarify the details of this definition in the next section).

Some of the earliest work on decentralized control for wireless networks was published by Foschini and Miljanic [3] in 1993. Their proposed control algorithm (now well known in the wireless community as simply the Foschini-Miljanic algorithm) provides for distributed on-line power control of ad-hoc networks with user-specific SIR requirements. Furthermore, their algorithm yields the minimum transmitter powers that satisfy the SIR requirements. This seminal work spawned a number of further publications [1, 9, 10] by various authors that extended the original algorithm to account for additional issues (i.e. link protection, user admission control, and so forth).

However, the original algorithm proposed by Foschini and Miljanic did require a significant assumption on the channel gains between nodes in the ad-hoc network. That is, the channel gains were modelled as constants. This is not to say that the authors believed the channel gains in a wireless network were deterministic quantities. Rather, they assumed the power adaptation interval was substantially longer than the fluctuation periods of the wireless channels between users. Hence a deterministic model provided a sufficient, yet non-trivial, level of abstraction.

In this paper we consider the same distributed power control problem proposed in [3], but we permit the links between network nodes to be time-varying stochastic processes. Within this setting we evaluate the performance of the original Foschini-Miljanic algorithm and show that it no longer satisfies the minimum power optimality conditions (optimality in this case is in terms of expected transmitter powers). Moreover, we also show that the SIR targets of the Foschini-Miljanic algorithm change dramatically in a random channel environment. In order to address these shortcomings we propose a new quality of service (QoS) criterion for SIR targets in wireless ad-hoc networks that has a form that is similar to the one proposed in [1]. We then find a power allocation that satisfies our new QoS criterion and minimizes expected power across all the transmitters simultaneously. Finally, we provide a fully distributed on-line power control algorithm that converges to the optimal power allocation for an ad-hoc network in a random channel environment.

The rest of this paper is organized as follows. In the next section we present a brief review of the formulation and results of [3]. In Section 3 we evaluate the performance of the Foschini-Miljanic algorithm in a random channel environment. In Section 4 we propose a new criteria for power optimality in wireless ad-hoc networks. Section 5 contains our presentation of a distributed stochastic approximation algorithm for power control. Numerical results are presented in Section 6 and we then conclude with a discussion of future research.

2 A Review of The Foschini-Miljanic Algorithm

In [3] the authors formulate the wireless network as a collection of radio links with each link corresponding to a transmitter and an intended receiver. Each transmitter is assumed to have a *fixed* channel gain to its intended receiver as well as fixed gains to all other receivers in the network. The quality of each link is determined by the signal to interference ratio (SIR) at the intended receiver. In a network with N interfering links we denote the SIR for the ith user as

$$R_i = \frac{G_{ii}P_i}{\eta_i + \sum_{i \neq j} G_{ij}P_j}, \tag{1}$$

where $G_{ij} > 0$ is the power gain from the transmitter of the jth link to the receiver of the ith link, P_i is the power of the ith transmitter, and η_i is the thermal noise power at the ith receiver.

Each link is assumed to have a minimum SIR requirement $\gamma_i > 0$ that represents the ith user's quality of service (QoS) requirements. This constraint can be represented in matrix form as

$$(\mathbf{I} - \mathbf{F})\mathbf{P} \geq \mathbf{u} \;\text{ with }\; \mathbf{P} > 0, \tag{2}$$

where $\mathbf{P} = (P_1, P_2, \ldots, P_n)^T$ is the column vector of transmitter powers,

$$\mathbf{u} = \left(\frac{\gamma_1 \eta_1}{G_{11}}, \frac{\gamma_2 \eta_2}{G_2 2}, \ldots, \frac{\gamma_N \eta_N}{G_N N} \right)^T, \tag{3}$$

is the column vector of noise powers scaled by the SIR constraints and channel gain, and $\mathbf{F}$ is a matrix with

$$F_{ij} = \begin{cases} 0, & \text{if } \ i = j \\ \frac{\gamma_i G_{ij}}{G_{ii}}, & \text{if } \ i \neq j \end{cases} \tag{4}$$

with $i, j \in \{1, 2, \ldots, N\}$.

2.1 Key Results for the Deterministic Channel

The matrix $\mathbf{F}$ has non-negative elements and, by assumption, is irreducible (i.e. we do not have multiple isolated networks). Let ρ_F be the Perron-Frobenius eigenvalue of $\mathbf{F}$. Then from the Perron-Frobenius theorem and standard matrix theory [8] we have the following equivalent statements

1. $\rho_F < 1$
2. There exists a vector $\mathbf{P} > 0$ (i.e. $P_i > 0$ for all i) such that $(\mathbf{I} - \mathbf{F})\mathbf{P} \geq \mathbf{u}$
3. $(\mathbf{I} - \mathbf{F})^{-1}$ exists and is positive componentwise.

Furthermore, if any of the above conditions holds we also have that $\mathbf{P}^* = (\mathbf{I} - \mathbf{F})^{-1}\mathbf{u}$ is the Pareto optimal solution to (2). That is, if $\mathbf{P}$ is any other solution to (2) then $\mathbf{P} \geq \mathbf{P}^*$ componentwise. Hence, if the SIR requirements for all users can be met simultaneously the best power allocation is $\mathbf{P}^*$, so as to minimize power consumption. We should note that previous publications on this power control algorithm and its variants refer to the above optimality criteria as Pareto optimal – even though the result is much stronger. Indeed, the above conditions guarantee *global* power optimality for every transmitter in the system. We remain consistent with previous publications and continue to use the phrase "Pareto optimality" to refer to the above conditions as well as some new conditions we propose in Section 4.

In [3] the authors show that the following iterative power control algorithm converges to $\mathbf{P}^*$ when $\rho_F < 1$, and diverges to infinity otherwise

$$\mathbf{P}(k+1) = \mathbf{F}\mathbf{P}(k) + \mathbf{u}, \tag{5}$$

for $k \in \{1, 2, 3, \ldots\}$. Furthermore, the above iterative algorithm can be simplified into the following distributed version. Let

$$P_i(k+1) = \frac{\gamma_i}{R_i(k)} P_i(k), \tag{6}$$

for each link $i \in \{1, 2, \ldots, N\}$. Hence, each link increases power when its SIR is below its target and decreases power when its SIR exceeds its target. It is easy to show that (5) and (6) are pathwise equivalent and hence the distributed version of the power control algorithm also converges to $\mathbf{P}^*$.

The fact that (6) converges to the optimal power allocation is one of the most remarkable results from [3]. While the full version of the power control algorithm (5) requires knowledge of all G_{ij}'s at each time k (and hence a centralized controller), the distributed version of the algorithm only requires each receiver to transmit its SIR to the appropriate transmitter. In this sense, the algorithm is fully distributed since each transmitter only requires information from its intended receiver and the intended receiver only collects observations rather than requesting any information from other transmitters. This suggests that ad-hoc wireless networks can provide robust and reliable QoS while utilizing strictly distributed link control algorithms. This result, along with several other enhancements to the basic algorithm [1], provides some theoretical assurance in the ability of power control algorithms to maintain link quality in ad-hoc wireless networks. However, it is not clear how this algorithm would perform when the channel is time-varying and random. We address this issue in the next section.

3 The Foschini-Miljanic Algorithm in a Random Channel Environment

We now consider the performance of (5) and (6) when the channel gains G_{ij} are allowed to vary with time. Let $\mathbf{G} = (\mathbf{G}(k) : k \geq 0)$ be a stationary ergodic sequence of random channel gain matrices. The elements of $G_{ij}(k)$ denote the channel gain from the jth transmitter to the ith receiver at time k. The sequence $\mathbf{G}$ takes values in a discrete or continuous set of N x N non-negative and irreducible matrices. Given this definition we create another sequence of random matrices $\mathbf{F} = (\mathbf{F}(\mathbf{k}) : k \geq 0)$ with elements

$$F_{ij}(k) = \begin{cases} 0, & \text{if } i = j \\ \frac{\gamma_i G_{ij}(k)}{G_{ii}(k)}, & \text{if } i \neq j \end{cases}, \tag{7}$$

and the random vector sequence $\mathbf{u} = (\mathbf{u}(\mathbf{k}) : k \geq 0)$ where

$$\mathbf{u}(\mathbf{k}) = \left(\frac{\gamma_1 \eta_1}{G_{11}(k)}, \frac{\gamma_2 \eta_2}{G_2 2(k)}, \ldots, \frac{\gamma_N \eta_N}{G_N N(k)} \right)^T. \tag{8}$$

For the sake of simplicity we assume the thermal noise power terms η_i remain time-invariant.

We now evaluate the properties of the "random version" of (5), which we define as

$$\mathbf{P}(\mathbf{k+1}) = \mathbf{F}(\mathbf{k})\mathbf{P}(\mathbf{k}) + \mathbf{u}(\mathbf{k}). \tag{9}$$

The distributed version of this algorithm is identical to (6)

$$P_i(k+1) = \frac{\gamma_i}{R_i(k)} P_i(k), \tag{10}$$

with $R_i(k)$ now given by

$$R_i(k) = \frac{G_{ii}(k) P_i(k)}{\eta_i + \sum_{i \neq j} G_{ij}(k) P_j(k)}. \tag{11}$$

Clearly the power vector $\mathbf{P}(\mathbf{k})$ will not converge to some deterministic constant as it did in (5). Rather, in a random channel environment a statement regarding stability requires the power vector to converge in distribution to a well defined random variable. In (5) the key convergence condition is to require that the Perron-Frobenius eigenvalue

$\rho_F < 1$. Since $\mathbf{F}$ is now a random matrix process the key convergence condition is that the Lyapunov exponent $\lambda_F < 0$, where λ_F is defined as

$$\lambda_F = \lim_{k\to\infty} \frac{1}{k} \log ||\mathbf{F}(\mathbf{1})\mathbf{F}(\mathbf{2}) \cdots \mathbf{F}(\mathbf{k})||. \tag{12}$$

See [6] for more details on Lyapunov exponents and the convergence properties of products of random matrices. We have the following lemma

Lemma 1: If the transmitter powers are updated according to (9) or (10), $\lambda_F < 0$, and

$$\mathrm{E}\left[\log(1 + ||\mathbf{u}(\mathbf{k})||)\right] < \infty \tag{13}$$

then the power vector $\mathbf{P}(\mathbf{k}) \Rightarrow \mathbf{P}(\infty)$. If $\lambda_F > 0$ then $\mathbf{P}(\mathbf{k}) \Rightarrow \infty$ as $k \to \infty$.

Proof: See [4].

Since the powers (and hence the SIRs) of all users are now random variables, clearly we cannot meet the original QoS constraint that required $R_i \geq \gamma_i$ for $i \in \{1, 2, \ldots, N\}$ with probability one. In a random environment an appropriate first step is to evaluate the expected value of R_i or the expected value of increasing functions of R_i. For the power control algorithm (9) in a random channel environment we have

Lemma 2: If the transmitter powers are updated according to (9) or (10) and $\lambda_F < 0$ then

$$\lim_{k\to\infty} E[\log R_i(k)] = \log \gamma_i \;\; \text{for all} \;\; i \in \{1, 2, \ldots, N\} \tag{14}$$

Proof: See [4].

Notice that in a random channel environment the "target QoS levels" have changed. Rather than aim for $R_i = \gamma_i$, the random version of the power update algorithm aims for $\mathrm{E}[\log R_i] = \log \gamma_i$. (Note that $\log E[R_i] \geq E[\log R_i]$.) Moreover, we no longer have a proof of optimality for (9).

At least intuitively, it is rather easy to see that the power updates (9) are unlikely to provide a minimum expected power solution in the case where the gain matrices $\mathbf{G}(\mathbf{k})$ are i.i.d. In that case, the power update applied at time $k+1$ is determined by the channel gains at time k, which provide *no information* about the gains at time $k+1$. Hence, the transmitter powers and resulting SIRs will hop erratically while trying to track useless channel information (see the Numerical Results Section for an example). The fundamental problem is that (9) is designed to only track transmitter power information. The power control algorithm does not sufficiently track any information on the channel statistics (nor should it, since the algorithm was designed for a deterministic channel). In the next section we propose new criteria for optimality in the random channel setting as well as a modified version of (9) that addresses the problems discussed above.

4 A Statistical Reformulation of the Quality of Service Criterion

When designing an adaptive power control algorithm for a random channel environment we essentially have two options. The first option is to develop an algorithm that attempts to predict channel behavior based on past observations and then uses those predictions to update the transmitter powers. The resulting sequence of transmitter powers will then be a random process that potentially satisfies some QoS constraint. The second option is to eschew a prediction algorithm for an appropriate deterministic power allocation. In this paper we will use the second option for a number of reasons. The most significant reason is that our design objective is a simple, robust, and *distributed* algorithm for controlling the transmitter powers in a random channel environment. Any power update algorithm that uses channel prediction will require each transmitter to learn something about the channels between other transmitters and receivers. This violates our constraints on the simplicity and distributed nature of the algorithm. Hence we assume the appropriate power control algorithm should aim for an optimal fixed power allocation.

In order to facilitate our construction of a power control algorithm we develop a slightly modified version of the original QoS definition used by (5). Consider the original SIR requirement

$$R_i(k) = \frac{G_{ii}(k)P_i(k)}{\eta_i + \sum_{i\neq j} G_{ij}(k)P_j(k)} \geq \gamma_i, \tag{15}$$

and re-write it as

$$G_{ii}(\infty)P_i(\infty) - \gamma_i\eta_i - \gamma_i \sum_{i\neq j} G_{ij}(\infty)P_j(\infty) \geq 0. \tag{16}$$

Further suppose we want this constraint to hold in expected value. If we re-write the set of constraints for all $i \in \{1, 2, \ldots, N\}$ in matrix form and assume a fixed power allocation $\bar{\mathbf{P}}$, we have

$$(\mathbf{I} - \bar{\mathbf{F}})\bar{\mathbf{P}} - \bar{\mathbf{u}} \geq 0 \tag{17}$$

where

$$\bar{F}_{ij} = \begin{cases} 0, & \text{for } i = j \\ \gamma_i \frac{\mathrm{E}[G_{ij}]}{\mathrm{E}[G_{ii}]} & \text{for } i \neq j \end{cases}, \tag{18}$$

and $\bar{\mathbf{u}} = \mathrm{E}[\mathbf{u}]$. If $\rho_{\bar{F}} < 1$ then from the arguments presented in [3] we know there exists a Pareto optimal vector $\bar{\mathbf{P}}^* = (I - \bar{\mathbf{F}})^{-1}\bar{\mathbf{u}}$, such that for any other vector $\mathbf{P}$ that satisfies (17) we have $\mathbf{P} \geq \bar{\mathbf{P}}^*$ componentwise.

This condition for Pareto-optimality presents a rather interesting conclusion, yet it also raises a number of significant theoretical and practical questions. The obvious conclusion is that a seemingly minor modification of the QoS conditions gives us optimality conditions that appear to be very similar to those needed for the deterministic channel case. It is tempting to simply state that expectations in the QoS constraints result in expectations in the optimality conditions. However, we should point out that the SIR constraints we have constructed are slightly different from what one might expect. Notice that the elements of the modified constraint (17) can also be written as

$$\frac{\mathrm{E}\left[G_{ii}(k)\bar{P}_i(k)\right]}{\mathrm{E}\left[\eta_i + \sum_{i\neq j} G_{ij}(k)\bar{P}_j(k)\right]} \geq \gamma_i. \tag{19}$$

That is, we have constrained the ratio of the expected values of the received signal power and interference powers – as opposed to the expected value of their ratio. This raises the question of which QoS constraint is the appropriate one to use? In this paper we have a number of reasons for examining the ratio of expected values constraint. The most significant reason is that this constraint form presents us with a tractable power control model and optimality conditions. Moreover, the constraint (19) implies that $\mathrm{E}[R_i] > \gamma_i$, and hence (19) is conservative relative to the requirement that $\mathrm{E}[R_i] > \gamma_i$. However, since the interference in our system is not Gaussian, the $\mathrm{E}[R_i]$ does not determine the probability distribution for the received signal at the intended receiver. Hence, it is not even clear that the expected value of the ratio constraint provides a better measure of system performance than (19). In many cases, this is a moot discussion since the two metrics are often quite close for many wireless channels of interest.

Another matter of interest is the stationary distribution of R_i under our proposed optimality criteria. This distribution will assist in channel capacity computations as well as computing bounds on outage probability. Define the probability of outage, for some outage SIR δ, to be

$$P\left(\frac{G_{ii}P_i}{\eta_i + \sum_{i\neq j} G_{ij}P_j} < \delta\right) = P\left(G_{ii}P_i < \delta\left(\eta_i + \sum_{i\neq j} G_{ij}P_j\right)\right). \tag{20}$$

Notice that if $P_i = \bar{P}_i$ in (20) then the computation of the outage probability can be quite easy. In fact, if the G_{ij}'s are Gaussian then the computation is trivial. Even if the channel gains are more complex (e.g. correlated exponential random variables) we can still perform a straightforward computation to bound (20). However, if the P_i's are random variables, as would be the case if we applied (9) as the power control algorithm, the computation of this probability would be substantially more difficult.

Finally, we also have a much more practical question to answer. How do we design an algorithm that converges to the optimal solution of (17)? More importantly, will this algorithm be *distributed*? The answer is not obvious. Notice that a distributed algorithm must iteratively solve an expected value matrix equation without observations of all the (random) elements of the matrix process $\mathbf{F}$. This issue is solved in the next section.

5 A Distributed Pareto-Optimal Algorithm in the Random Channel Environment

Our goal is to develop a distributed power control algorithm that converges to $\bar{\mathbf{P}}^*$. Recall that in the original algorithm (6) for deterministic channels, each receiver provided feedback of its SIR to the appropriate transmitter and this was all that was required for the distributed algorithm to converge. As we will see in this section, in the random channel environment we require slightly more information at each transmitter. In addition to the SIR at the intended receiver we assume each transmitter also has knowledge of the channel gain to the receiver. That is, the ith transmitter has knowledge of $R_i(k)$ and $G_{ii}(k)$ when selecting $P_i(k+1)$.

As a first step towards a distributed algorithm we first consider a centralized solution of

$$(\mathbf{I} - \bar{\mathbf{F}})\bar{\mathbf{P}}^* - \bar{\mathbf{u}} = 0, \tag{21}$$

(a centralized solution assumes knowledge of the $\mathbf{F}(\mathbf{k})$'s). If we write

$$g(\bar{\mathbf{P}}) = (I - \bar{\mathbf{F}})\bar{\mathbf{P}} - \bar{\mathbf{u}} \tag{22}$$

then we can view the solution $\bar{\mathbf{P}}^*$ of (21) as the zero of the function $g(\bar{\mathbf{P}})$. Of course, the centralized controller of the network might not have access to $\bar{\mathbf{F}}$ and $\bar{\mathbf{u}}$, just random observations of the matrix sequence $\mathbf{F}$ and vector sequence $\mathbf{u}$. Hence, finding a solution to our optimality equation is equivalent to estimating the zero of the function $g(\bar{\mathbf{P}})$ when we only have access to noisy estimates of $g(\bar{\mathbf{P}})$. Therefore, one possible iterative estimation procedure is a version of the Robbins-Monro stochastic approximation algorithm [7]. We will define our centralized power control algorithm as follows. Let $\hat{\mathbf{P}}(\mathbf{k})$ be our estimate for the solution to (21) at time k, then

$$\hat{\mathbf{P}}(\mathbf{k+1}) = \hat{\mathbf{P}}(\mathbf{k}) - a_k g\left(\hat{\mathbf{P}}(\mathbf{k})\right) + a_k \epsilon_k, \tag{23}$$

where a_k is the algorithm step-size satisfying $a_n \to 0$ and $\sum_{n=1}^{k} a_n \to \infty$. Define the matrix $\tilde{\mathbf{G}}(\mathbf{k})$ as

$$\begin{cases} \tilde{G}_{ij}(k) = G_{ii}(k) & \text{for } i = j \\ 0 & \text{otherwise,} \end{cases} \tag{24}$$

then ϵ_k is the noise term

$$\epsilon_k = g(\hat{\mathbf{P}}(\mathbf{k})) - \left(\tilde{\mathbf{G}}(\mathbf{k})\hat{\mathbf{P}}(\mathbf{k}) - \tilde{\mathbf{G}}(\mathbf{k})\mathbf{F}(\mathbf{k})\hat{\mathbf{P}}(\mathbf{k}) - \tilde{\mathbf{G}}(\mathbf{k})\mathbf{u}(\mathbf{k})\right). \tag{25}$$

The stochastic approximation algorithm (23) requires the following conditions in order to ensure convergence,

A1:

$$\lim_{k\to\infty} \frac{\sum_{n=1}^{k} \alpha_n \epsilon_n}{\sum_{n=1}^{k} \alpha_n} = 0 \tag{26}$$

where

$$\alpha_n = \begin{cases} a_1, & \text{if } n = 1 \\ a_n \prod_{n=2}^{k} 1/(1-a_n) & \text{otherwise.} \end{cases} \tag{27}$$

A2: $\sup\{\mathrm{Re}\,\rho : \rho \in \sigma(\bar{\mathbf{F}})\} < 1$.

Note that if we set $a_k = 1/k$ then in **A1** we have $\alpha_k = 1$ for all k. In [4] we show that if the power vector $\hat{\mathbf{P}}(\mathbf{k})$ is updated according to (23) and conditions **A1** and **A2** are satisfied then $\hat{\mathbf{P}}(\mathbf{k}) \to \bar{\mathbf{P}}^*$ as $k \to \infty$, and if $\rho_{\bar{F}} > 1$ then $\hat{\mathbf{P}}(\mathbf{k}) \to \infty$.

It is interesting to note that the convergence conditions for this algorithm do not require that all possible values of $\mathbf{F}$ have $\rho_F < 1$. Recall that in the deterministic channel case $\rho_F > 1$ corresponded to an unstable system with powers increasing to infinity. In the random channel case it is possible for the wireless network to operate in an unstable environment for some fraction of time and still remain stable on average. We will see the consequences of this result in the numerical results presented in the next section.

Finally, we can construct a distributed version of the full-information stochastic approximation algorithm (23). Let

$$\hat{P}_i(k+1) = \hat{P}_i(k) - a_n\left(G_{ii}(k)\hat{P}_i(k) - \frac{\gamma_i G_{ii}(k)\hat{P}_i(k)}{R_i(k)}\right), \tag{28}$$

and we have

Theorem 1: The components of $\hat{\mathbf{P}}(\mathbf{k})$ from the full-information algorithm (23) are equivalent (on each sample path) to the individual transmitter powers determined by the distributed algorithm (28). Hence, all of the convergence results and properties of the full information algorithm also apply to the distributed algorithm.

Proof: The proof follows immediately from element by element analysis of $\hat{\mathbf{P}}(\mathbf{k})$.

6 Numerical Analysis

Consider an ad-hoc network consisting of three mobile devices. In the first numerical example we will assume every link in the system is an independent exponential random variable where the expected value of the gain matrix

$$\mathrm{E}[G] = \begin{pmatrix} 1 & .0375 & .02 \\ .0375 & 1 & .04 \\ .02 & .04 & 1 \end{pmatrix}. \tag{29}$$

We assume that $\gamma_i = 5$ and $\eta_i = 1$ for each transmitter. For this setup we have $\rho_{\bar{F}} = 1/3$, so we should expect the power control algorithms to be fairly stable. The plot in Figure 1 shows the power of the first transmitter for the same channel sample path using (10) and (28) as the power control algorithms. Note that the power axis is on a *logarithmic scale* . Clearly the stochastic approximation algorithm provides better power stability as well as power consumption.

Fig 1: Transmit Power of the Standard Foschini−Miljanic Algorithm and the Stochastic Approximation Algorithm

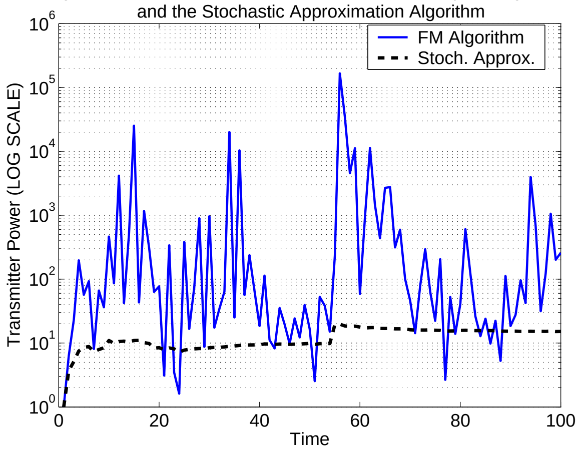
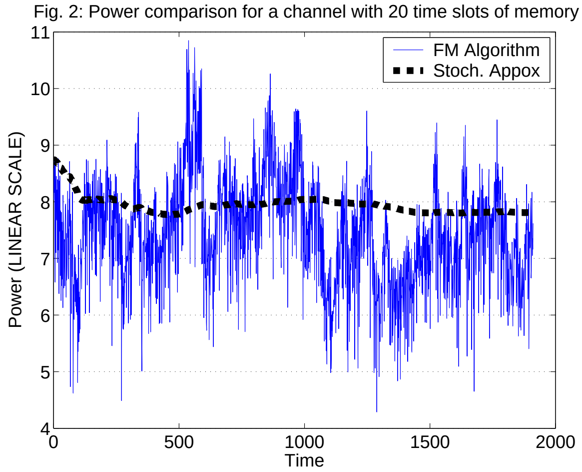


Fig. 2: Power comparison for a channel with 20 time slots of memory



Admittedly, independent exponential fading is probably the worst case channel for (10). Suppose we now allow the channel to have memory. We will model a channel gain matrix with memory of M timeslots as

$$\mathbf{G}(k+M) = \frac{1}{M}\mathbf{G_{iid}} + \frac{1}{M}\sum_{i=1}^{M-1}\mathbf{G}(\mathbf{k}+\mathbf{i}), \tag{30}$$

where $\mathbf{G_{iid}}$ is a matrix with independent exponential elements with mean (29). This type of memory model is highly favorable for (10) as the amount of "randomness" in each iteration of the channel decreases with $1/M$. The plot in Figure 2 shows the power comparison for a channel with $M = 20$, which is a substantial amount of memory. While (10) performs much better in this case, the stochastic approximation algorithm is still preferred.

7 Conclusion

We have presented an evaluation of the Foschini-Miljanic power control algorithm in a random channel environment. The analysis shows that their proposed algorithm does not meet its intended QoS requirements nor does it perform well in terms of power consumption. In order to address these issues we proposed a new criteria for power optimality in wireless ad-hoc networks. We then showed that the optimal power allocation could be discovered through a stochastic approximation algorithm. Moreover, the structure of this stochastic approximation algorithm yielded an optimal fully distributed on-line algorithm for controlling transmitter powers in an ad-hoc network.

In [4] we will present the proofs withheld from this short paper. We will also present our random channel extensions to a number of additional topics, including the active link protection and admission control protocols proposed in [1]. Finally, we also plan to address the issue raised in Section 4 of this paper regarding the choice of an appropriate QoS metric. Given the results presented in this paper it is still not clear whether the "expected value of the ratio" approach or the "ratio of expectations" metric provides the most appropriate measure of QoS.

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