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ON THE VALUATION OF PAYOFFS FROM A GEOMETRIC RANDOM WALK OF OIL PRICES

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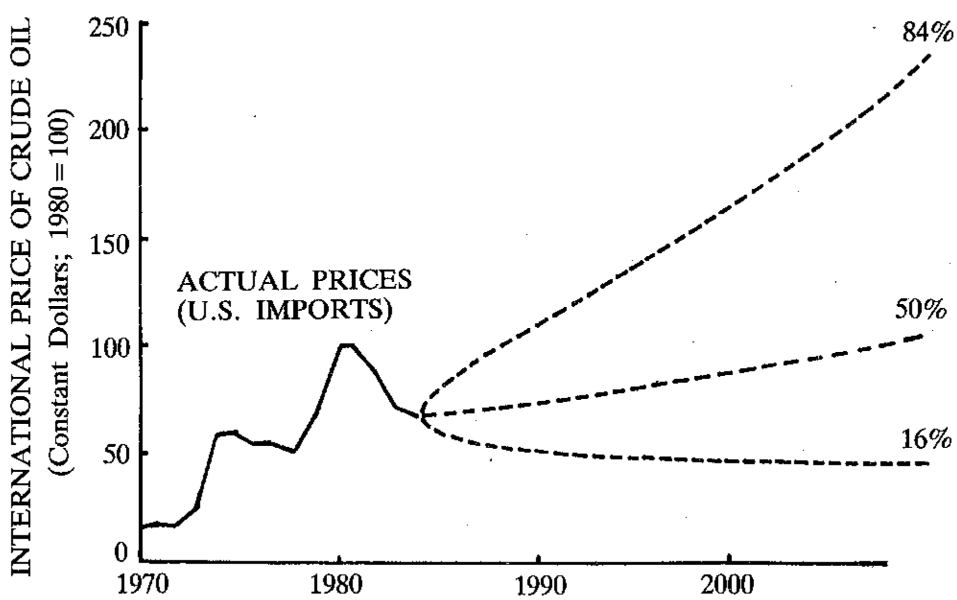
INTRODUCTION

Consider a risk-averse decision maker. How much should that individual be willing to pay for an investment whose returns are generated by a geometric random walk process? The following propositions may be readily proved, but do not seem to be well known.

A geometric random walk process is one for which the *logarithm* of the process may be described by the Brownian motion with drift. From one time interval to another, the ratio change is independent. Geometric random walks are frequently used to model stock market behaviour, and they have also been proposed to describe the history of crude oil prices (see Figure 1) [1]. The three lines correspond respectively to the 16th, 50th and 84th percentile of the

Figure 1



price distribution estimated from 1968–84 statistics. Hereafter, for vividness, we will refer to the random variable as the price of crude oil.

FIVE PROPOSITIONS

Without loss of generality, units of measurement may be chosen so that the initial level of the crude oil price is unity. Suppose that time is measured in annual intervals. Let μ be the expected value of the annual change in the logarithm of the price, and let σ^2 be the variance of the annual change in this logarithm. The following five propositions may then be demonstrated:

1. A geometric random walk process implies that at any given point in time, the price of oil will be lognormally distributed. At year $t \geq 0$, the mean and variance of this distribution will be, respectively:

$$\mu(t) = \exp(\mu t + .5\sigma^2 t);$$
$$\sigma^2(t) = \exp(2\mu t + \sigma^2 t)[\exp(t\sigma^2) - 1].$$

2. At year $t \geq 0$, let $m(t)$ denote the *median* of the price distribution. This may be calculated by using the expression $m(t) = \exp(\mu t)$. The distribution of price is skewed to the right, and the median lies below the mean. That is, whenever $t > 0$,

$$m(t)/\mu(t) = \exp(-.5t\sigma^2) < 1.0$$

In this special case, the mode (the most likely price) is identical with the median. There is an equal probability that the price of oil will exceed $\exp(2\mu t)$, and that it will lie below $\exp(.5\mu t)$.

3. If an investor's attitudes toward future oil price risks are described by a *logarithmic* utility function, the lognormal oil price distribution has a money value equal to its median (and, therefore, its mode). Note that this valuation is lower than the EMV (expected money value) by an amount depending upon the variance of the year-to-year price changes.

4. Suppose that the physical output from an oil investment declines at the annual rate of d over an infinite planning horizon. The decision maker discounts future utilities at the annual rate r. With a logarithmic utility function, the present value certainty equivalent of this prospect is then:

$$1/[r + d - \mu]$$

5. Let the decision maker's attitudes toward risk in year t be described by an *isoelastic* utility function $u(x)$, where x denotes the price of one unit of oil in that year. Such a utility function is characterized by a, an exponent which determines, the elasticity of instantaneous marginal utility with respect to the price:

$$u(x) = x^a \text{ for } 0 < a \leq 1$$
$$-x^a \text{ for } a < 0$$

Again, let r be the discount rate and d the decline rate of physical output from the investment. Assuming that the following denominator is positive, the present value certainty equivalent is:

$$1/[r + d - \mu - .5\, a\, \sigma^2]$$

If the exponent $a = 1$, this describes a decision maker who is *not* risk averse. Such an individual is sometimes termed an EMU-er, one who maximizes the expected money value of the outcome. The preceding expression may then be interpreted as the discounted mean of the oil price in each future year.

EXTENSIONS AND SUGGESTIONS FOR ADDITIONAL RESEARCH

There are more general stochastic models for the price process, e.g. ones that have jumps at randomly determined intervals. These will require additional analysis. Suggestions will be welcomed from the readers of this note.

REFERENCE

1. Manne, A.S. and Schrattenholzer, L., "International Energy Workshop: Oil Price Projections", *The Energy Journal*, 7(1): 109–114, 1986.