

CONFIDENCE INTERVAL ESTIMATION FOR THE DIFFERENCE BETWEEN TWO STOCHASTIC PROCESSES USING STANDARDIZED TIME SERIES; Bor-Chung Chen and Robert G. Sargent, Working Paper #84-004, October 1984, Department of Industrial Engineering and Operations Research, Syracuse University, Syracuse, N. Y. 13210.

This paper describes confidence interval estimation for the difference between the means of two independent strictly stationary phi-mixing stochastic processes. This estimation is based on the asymptotic properties of the standardized time series of observations from the two stochastic processes. Interval estimators for the cases of equal sample sizes with unknown variances, unknown common variance, and unequal sample sizes with unknown variances are developed. Two applications of these intervals are validating stationary discrete event simulation models and comparing two alternative policies or system designs via simulation.

AN ILLUSTRATION OF THE SAMPLE SPACE DEFINITION AND VARIANCE REDUCTION, Barry L. Nelson, Department of Industrial and Systems Engineering, The Ohio State University, 1971 Neil Avenue, Columbus, Ohio 43210.

A simulation experiment performed to estimate the expected number of customers served per day in a bank is used to illustrate a new definition of simulation experiments, a new framework for variance reduction, and several variance reduction techniques (VRTs). Particular emphasis is placed on techniques that might be suitable for automated variance reduction procedures.

### EDITOR'S CORNER

REGENERATIVE PROCESSES AND THE  $\phi$ -MIXING PROPERTY, by Peter W. Glynn, Department of Industrial Engineering, University of Wisconsin, Madison, Wisconsin 53706.

The development of limit theory for steady-state output analysis algorithms is often simplified by assuming that the output process is either  $\phi$ -mixing or regenerative. The purpose of this note is to show that, contrary to the folklore, stationary positive recurrent regenerative processes are not necessarily  $\phi$ -mixing.

Let  $X = \{X_n : n \geq 0\}$  be a Markov chain on state space  $E = \{0, 1, \dots\}$ , with transition matrix

$$\begin{aligned} P_{i,i+1} &= p \quad , \quad i \geq 0 \\ P_{i,i-1} &= q \quad , \quad i \geq 1 \\ P_{0,0} &= q \end{aligned}$$

where  $p > q = 1-p > 1/2$ . If  $P\{X_0 = j\} = (1-p/q)^{-1} (p/q)^j$ , then  $X$  is a stationary Markov chain which is a positive recurrent regenerative sequence with respect to hitting times of state 0 (for instance).

In order that  $X$  be  $\phi$ -mixing (see BILLINGSLEY (1968)), there must exist constants  $\phi(n)$  decreasing to zero such that

$$(1) \sup \{ |P(E_2 | E_1) - P(E_2)| : E_1 \in \mathcal{F}_0^k, E_2 \in \mathcal{F}_{k+n}^\infty, k \geq 0 \} \leq \phi(n)$$

where  $\mathcal{F}_1^j$  consists of all events generated by  $X_1, \dots, X_j$  ( $i \leq j$ ). The left-hand side of (1) clearly dominates

$$\begin{aligned} & \sup \{ |P\{X_n = 0 | X_0 = j\} - P\{X_n = 0\}| : j \geq 0 \} \\ & \geq |P\{X_n = 0 | X_0 = n+1\} - P\{X_n = 0\}| \\ & \geq P\{X_n = 0\} = (1-p/q)^{-1}; \end{aligned}$$

The second inequality is a consequence of the fact that a birth-death chain cannot move more than  $n$  states in  $n$  steps. It follows that the left-hand side of (1) does not converge to zero as  $n$  tends to infinity. Hence,  $X$  is not  $\phi$ -mixing.

Stationary positive recurrent regenerative sequences do satisfy a somewhat weaker mixing hypothesis, namely that of strong mixing:

$$\sup \{ |P(E_1 \cap E_2) - P(E_1)P(E_2)| : E_1 \in \mathcal{F}_0^k, E_2 \in \mathcal{F}_{k+n}^\infty, k \geq 0 \} \leq \alpha(n)$$

where the  $\alpha(n)$ 's converge to zero (see GLYNN (1982)). It turns out that strong mixing is sufficient to prove a large variety of limit theorems that are relevant to simulation; see, for example, HALL and HEYDE (1980).

$$\sup \{ |P(E_1) | P(E_2 | E_1) - P(E_2) | : E_1 \in \mathcal{F}_0^k, E_2 \in \mathcal{F}_{k+n}^\infty, k \geq 0 \} \leq \alpha(n)$$

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#### REFERENCES

- [1] BILLINGSLEY, P. (1968). *Convergence of Probability Measures*. John Wiley, New York.
- [2] GLYNN, P. (1982). Some new results in regenerative process theory. Technical Report 16, Department of Operations Research, Stanford University, Stanford, CA.
- [3] HALL, P. and HEYDE, C. C. (1980). *Martingale Limit Theory and its Applications*. Academic Press, New York.

(Please address comments to the author. Ed.)

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- [ ] Nabil R. Adam, "Achieving A Confidence Interval for Parameters Estimated by Simulation," *Management Science* 29, No 7, July 1983, pp. 856-866.
- [ ] Carl A. Mauro and Dennis E. Smith, "Factor Screening in Simulation: Evaluation of Two Strategies based on Random Balancing Sampling," *Management Science* 30, No. 2, February 1984, pp. 209-221.
- [ ] David Goldsman and Lee Schruben, "Asymptotic Properties of some Confidence Interval Estimators for Simulation Output," *Management Science* 30, No. 10, October 1984, pp. 1217-1225.
- [ ] James R. Wilson and A. Alan B. Pritsker, "Experimental Evaluation of Variance Reduction Techniques for Queueing Simulation using Generalized Concomitant Variables," *Management Science* 30, No. 12, December 1984, pp. 1459-1472.