J. Fluid Mech. (2017), *vol.* 831, R3, doi:10.1017/jfm.2017.701

JFM RAPIDS
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An unambiguous definition of the Froude number for lee waves in the deep ocean

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(Received 24 July 2017; revised 23 August 2017; accepted 26 September 2017)

There is a long-standing debate in the literature of stratified flows over topography concerning the correct dimensionless number to refer to as a Froude number. Common definitions using external quantities of the flow include $U/(ND)$, $U/(Nh_0)$, and Uk/N, where U and N are, respectively, scales for the background velocity and buoyancy frequency, D is the depth, and h_0 and k^{-1} are, respectively, height and width scales of the topography. It is also possible to define an internal Froude number $Fr_\delta = u_0/\sqrt{g'\delta}$, where u_0, g', and δ are, respectively, the characteristic velocity, reduced gravity, and vertical length scale of the perturbation above the topography. For the case of hydrostatic lee waves in a deep ocean, both $U/(ND)$ and Uk/N are insignificantly small, rendering the dimensionless number Nh_0/U the only relevant dynamical parameter. However, although it appears to be an inverse Froude number, such an interpretation is incorrect. By non-dimensionalizing the stratified Euler equations describing the flow of an infinitely deep fluid over topography, we show that Nh_0/U is in fact the square of the internal Froude number because it can identically be written in terms of the inner variables, $Fr_\delta^2 = Nh_0/U = u_0^2/(g'\delta)$. Our scaling also identifies Nh_0/U as the ratio of the vertical velocity scale within the lee wave to the group velocity of the lee wave, which we term the vertical Froude number, $Fr_{vert} = Nh_0/U = w_0/c_g$. To encapsulate such behaviour, we suggest referring to Nh_0/U as the lee-wave Froude number, Fr_{lee}.

Key words: internal waves, stratified flows, topographic effects

1. Introduction

In its most generally accepted use, the Froude number Fr represents a ratio of the speed with which two processes, namely advection and wave propagation, carry information of a disturbance throughout a system. Locally, the Froude number also represents the partitioning between kinetic and potential energy of the flow and

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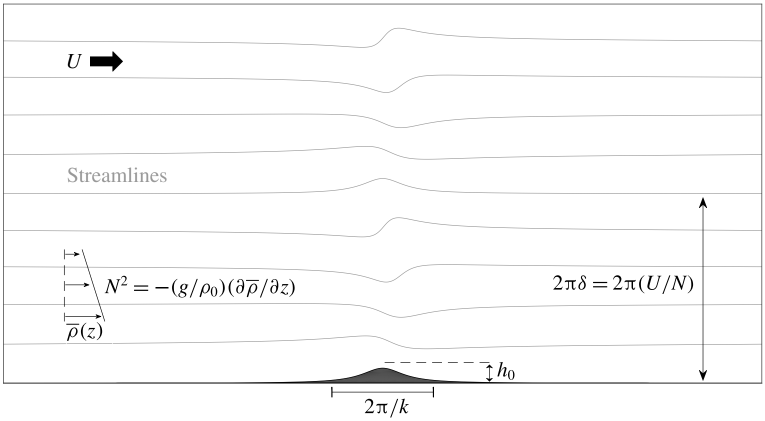

FIGURE 1. Stratified flow over a two-dimensional hill with height h_0, width $2\pi/k$, and upstream flow U and stratification N. Streamlines represent a solution of Long's equation over a Witch of Agnesi with $\epsilon = Uk/N = 0.01$ and $J = Nh_0/U = 0.5$ obtained with the iterative method of Laprise & Peltier (1989*b*).

identifies the flow as sub- or supercritical. For flow over an isolated sill, the critical value of $Fr = 1$ can only occur at the sill crest and indicates that the flow here is under hydraulic control, meaning that the energetics of the flow are equally partitioned and cannot support any greater volume flux above the obstacle (Armi 1986; Farmer & Armi 1986). In the simple case of inviscid open channel flow in water much shallower than the horizontal scale of the topography, the Froude number is given by $Fr = v/\sqrt{gd}$, where v is the local depth-averaged velocity, d is the local depth, and g is the acceleration due to gravity. Upon extension to two flowing layers with different densities, Armi (1986) defines a Froude number for each of $i = 1, 2$ layers as $Fr_i = u_i/\sqrt{g'\delta_i}$, where u_i and δ_i are, respectively, the flow speed and thickness of layer i, $g' = g(\Delta\rho/\rho_0)$ is the reduced gravity, and $\Delta\rho$ is the density difference between the two layers. Armi uses these layer Froude numbers to characterize the criticality of the flow by computing the composite Froude number, G, which, along with the Boussinesq approximation and subcritical bulk flow ($v/\sqrt{gd} \ll 1$, with $v = (u_1\delta_1 + u_2\delta_2)/d$ and $d = \delta_1 + \delta_2$), is defined as $G^2 = Fr_1^2 + Fr_2^2$.

In the case of continuously stratified flow over an isolated ridge, the relevant parameters are the ridge height h_0, its width $2\pi/k$, the depth D, and velocity and stratification scales for the unperturbed flow, U and $N^2 = -(g/\rho_0)\bar{\rho}_z$ (see figure 1). Using these parameters, one can form three independent dimensionless numbers that resemble a Froude number, namely $U/(ND)$, $U/(Nh_0)$, and Uk/N. However, not all of these numbers represent a ratio of advection to wave propagation speed or relate to the criticality of the flow, and thus referring to all of them as Froude numbers robs the concept of its intuitive dynamical significance. In his seminal text on stratified flow over topography, Baines (1995) proposed that, as a solution to this 'Froude for everything syndrome,' one should only refer to the most obvious extension of open channel flow as a Froude number. That is, $Fr = U/(ND)$, where ND is a scale for the first-mode (fastest) internal gravity wave speed (which is given exactly by ND/π from linear theory with uniform N).

For relatively shallow flows over topography, such as tidal flows over a sill in an inlet (Farmer & Smith 1980), $U/(ND)$ is indeed a dynamically significant Froude number. However, for lee waves generated by abyssal hills at the bottom of the ocean, $Fr = U/(ND)$ is very small and offers little information about the system. For example, in the Drake Passage region of the Antarctic Circumpolar Current (ACC), where abyssal lee waves are predicted to be dynamically important, $U \approx 0.1$ m s^{-1}, $N \approx 10^{-3}$ rad s^{-1}, and $D \approx 4000$ m, giving $U/(ND) \approx 0.025$ (Nikurashin & Ferrari 2010). In these systems, the dynamics are captured elegantly by considering the case of an infinitely deep ocean, implying $U/(ND) = 0$, for which solutions describe a stationary wave above the topography with energy propagating upwards at an angle downstream to infinity (Long 1953).

In such deep systems, the relevant dimensionless number is Nh_0/U, which has various names in the literature. Miles (1969) refers to it as the Russell number, *Ru*, after the fluid mechanician John Scott Russell, who described the reduction in drag on shipping vessels when propelled faster than the shallow water wave speed $\sqrt{gd}$. Aguilar & Sutherland (2006) refer to it as the Long number, *Lo*, in honour of Robert Long's pioneering work on the lee-wave problem (Long 1953). Nikurashin & Ferrari (2010) refer to it as a steepness parameter, ϵ, after showing that, in the hydrostatic limit, Nh_0/U is identical to the ratio of the topographic slope to the slope of the internal wave phase lines, a parameter that the internal tide literature commonly refers to as ϵ (Garrett & Kunze 2007). Finally, in much of the literature, Nh_0/U is simply referred to either as an inverse Froude number, Fr^{-1} (Drazin 1961; Kitabayashi 1977; Durran 1986; Kimura & Manins 1988; Smolarkiewicz & Rotunno 1989; Crook, Clark & Moncrieff 1990; Scinocca & Peltier 1994; Legg & Klymak 2008; Eckermann *et al.* 2010; Winters & Armi 2012), or a vertical Froude number, Fr_z (Peltier & Clark 1983; Clark & Peltier 1984; Laprise & Peltier 1989*a*,*c*; Afanasyev & Peltier 1998; Welch *et al.* 2001; Klymak, Legg & Pinkel 2010). However, none of these papers offer either mathematical or physical justification for their association of Nh_0/U with the Froude number.

Winters & Armi (2012) consider a continuous extension of the layer Froude number, $Fr_\delta = u_0/\sqrt{g'\delta}$, where u_0, g', and δ are, respectively, the perturbation velocity over the topography, the reduced gravity arising from the perturbation density, and the characteristic vertical scale of the perturbation, and find that Fr_δ is held at a constant critical state for flows in which $Nh_0/U > O(1)$. Note, however, that the internal Froude number defined by Fr_δ does not represent a ratio of advection to wave speed, since $\sqrt{g'\delta}$ is not the speed of propagation of an internal gravity wave.

By non-dimensionalizing the equations describing infinitely deep stratified flow over topography, the purpose of this paper is to show that, regardless of its name, Nh_0/U is in fact identical to both the square of the internal Froude number Fr_δ and a ratio of the vertical velocity within the lee wave to the group speed of the wave itself, w_0/c_g, which we refer to as Fr_{vert}. The results suggest that it is indeed appropriate to define the lee-wave Froude number as $Fr_{lee} = Nh_0/U$.

Additionally, we discuss how our scaling breaks down as Nh_0/U approaches 1, in which case the height of the topography approaches the vertical scale of the lee wave, U/N, and the flow becomes hydraulically controlled, or blocked, with the internal Froude number held constant at $Fr_\delta = 1$, indicating a saturation of the internal wave field energetics (Winters & Armi 2012). Thereafter, Nh_0/U informs instead the magnitude of nonlinear dynamics such as upstream blocking (Smith 1989) or downslope windstorms (Laprise & Peltier 1989*a*).

2. Non-dimensional equations

In this section we develop a scaling for internal (perturbation) quantities of the flow field in a lee wave based on the characteristic external (background) quantities.

As discussed in the Introduction, the steady, two-dimensional (in the horizontal x, and vertical z directions) lee wave formed by the flow of a stratified, inviscid fluid over an isolated hill of height h_0 and width $2\pi/k$ with an infinite depth is characterized by the external dimensional quantities U, k, N, and h_0 (See figure 1). Rotation is neglected for simplicity, and it is assumed that the depth of the domain, D, is irrelevant, i.e. $U/(ND) \ll 1$.

Following Winters & Armi (2012), we separate the flow into its external and internal quantities such that $\boldsymbol{u}_{total} = U\boldsymbol{e}_x + \boldsymbol{u}'$, $\rho_{total} = \overline{\rho}(z^*) + \rho'$, and $p_{total} = \rho_0\overline{p}(z^*) + \rho_0 p'$, where it is assumed that physical quantities are dimensional, dimensional coordinates are indicated with the *, and $\boldsymbol{e}_x$ is a unit vector in the x direction. Under these definitions, the governing dimensional equations after employing the Boussinesq approximation are given by

$$U\frac{\partial u'}{\partial x^*} + \boldsymbol{u}' \cdot \nabla^* u' = -\frac{\partial p'}{\partial x^*}, \tag{2.1}$$

$$U\frac{\partial w'}{\partial x^*} + \boldsymbol{u}' \cdot \nabla^* w' = -\frac{\partial p'}{\partial z^*} - \frac{\rho'}{\rho_0}g, \tag{2.2}$$

$$U\frac{\partial \rho'}{\partial x^*} + \boldsymbol{u}' \cdot \nabla^* \rho' = \frac{\rho_0 N^2}{g}w', \tag{2.3}$$

where $\nabla^* = \boldsymbol{e}_x \partial/\partial x^* + \boldsymbol{e}_z \partial/\partial z^*$, $N^2 = -g/\rho_0\ \partial\overline{\rho}/\partial z^*$, subject to continuity $\nabla^* \cdot \boldsymbol{u}' = 0$ and the dimensional kinematic boundary condition at $z^* = h'(x^*)$

$$U\frac{\partial h'}{\partial x^*} + u'\frac{\partial h'}{\partial x^*} = w', \tag{2.4}$$

where h' is the topography as a deviation from a flat bottom.

The equations are non-dimensionalized using the inner variable scales defined by

$$u' = u_0 u, \tag{2.5}$$

$$w' = w_0 w, \tag{2.6}$$

$$\rho' = R\rho, \tag{2.7}$$

$$p' = Pp, \tag{2.8}$$

$$x^* = k^{-1}x, \tag{2.9}$$

$$z^* = \delta z, \tag{2.10}$$

where non-primed inner variables and x and z are assumed to be dimensionless. Using these scales, non-dimensionalizing the kinematic bottom boundary condition (2.4), and requiring a balance between the linear terms to leading order implies $w_0 = kh_0U$, which gives the non-dimensional kinematic bottom boundary condition

$$\frac{\partial h}{\partial x} + \left(\frac{u_0}{U}\right) u\frac{\partial h}{\partial x} = w. \tag{2.11}$$

The vertical scale of the flow as indicated by δ is not the same as the hill height h_0, since δ must be finite as $h_0 \to 0$ (the linear limit). The vertical scale is thus dictated by continuity, which requires $ku_0 = w_0/\delta$. Combined with the scaling for the kinematic bottom boundary condition, this implies $\delta = w_0/(ku_0) = kh_0U/(ku_0) = Uh_0/u_0$.

Non-dimensionalizing the x-momentum equation (2.1) gives

$$ku_0 U \frac{\partial u}{\partial x} + ku_0^2 \boldsymbol{u} \cdot \boldsymbol{\nabla} u = -kP \frac{\partial p}{\partial x}. \tag{2.12}$$

If we require a leading-order balance between the pressure gradient and the linear momentum advection term, we must have $P = u_0 U$, which gives

$$\frac{\partial u}{\partial x} + \left(\frac{u_0}{U}\right) \boldsymbol{u} \cdot \boldsymbol{\nabla} u = -\frac{\partial p}{\partial x}. \tag{2.13}$$

In a similar manner, the non-dimensional density transport equation (2.3) is given by

$$kUR \frac{\partial \rho}{\partial x} + ku_0 R \boldsymbol{u} \cdot \boldsymbol{\nabla} \rho = \frac{k\rho_0 h_0 N^2 U}{g} w, \tag{2.14}$$

which implies

$$R = \frac{\rho_0 N^2 h_0}{g} \tag{2.15}$$

for the linear terms to balance. The non-dimensional density transport equation is then given by

$$\frac{\partial \rho}{\partial x} + \left(\frac{u_0}{U}\right) \boldsymbol{u} \cdot \boldsymbol{\nabla} \rho = w. \tag{2.16}$$

Finally, the non-dimensional vertical momentum equation is given by

$$k^2 h_0 U^2 \frac{\partial w}{\partial x} + k^2 h_0 U u_0 \boldsymbol{u} \cdot \boldsymbol{\nabla} w = -\frac{P}{\delta} \frac{\partial p}{\partial z} - \frac{gR}{\rho_0} \rho. \tag{2.17}$$

If we require a vertical hydrostatic balance to leading order, then we must have

$$\frac{P}{\delta} = \frac{gR}{\rho_0} = N^2 h_0, \tag{2.18}$$

and

$$P = \delta N^2 h_0 = \frac{N^2 h_0^2 U}{u_0}, \tag{2.19}$$

which gives

$$\epsilon^2 \left[\frac{\partial w}{\partial x} + \left(\frac{u_0}{U}\right) \boldsymbol{u} \cdot \boldsymbol{\nabla} w\right] = -\frac{\partial p}{\partial z} - \rho, \tag{2.20}$$

where

$$\epsilon = \frac{Uk}{N} \tag{2.21}$$

is the non-hydrostatic parameter and represents a ratio of the frequency with which the flow over the hill excites a wave, Uk, to the frequency of the buoyant response, N. A propagating wave is only possible if the excitation frequency is smaller than the buoyancy frequency ($\epsilon < 1$). Otherwise, the perturbation by the flow over a hill occurs faster than the buoyancy time scale, giving evanescent behaviour. Within the propagating regime, one can also think of ϵ as a ratio of the vertical wavelength of the wave (U/N) to the width of the hill (k^{-1}). For wavelengths much smaller than the hill length ($\epsilon \ll 1$), the wave is approximately hydrostatic and its group velocity is vertically oriented with magnitude $c_g = \epsilon U$ (Gill 1982).

Returning to the pressure, since the non-dimensional vertical momentum equation (2.20) requires $P = N^2 h_0^2 U/u_0$ and the non-dimensional horizontal momentum equation (2.13) requires $P = u_0 U$, this implies that $u_0 = N h_0$ and thus

$$\frac{u_0}{U} = \frac{N h_0}{U} \equiv J, \tag{2.22}$$

as deduced in Baines (1995), equation (5.2.2). In terms of J, the governing non-dimensional equations are given by

$$\frac{\partial u}{\partial x} + J\boldsymbol{u} \boldsymbol{\cdot} \boldsymbol{\nabla} u = -\frac{\partial p}{\partial x}, \tag{2.23}$$

$$\epsilon^2 \left(\frac{\partial w}{\partial x} + J\boldsymbol{u} \boldsymbol{\cdot} \boldsymbol{\nabla} w \right) = -\frac{\partial p}{\partial z} - \rho, \tag{2.24}$$

$$\frac{\partial \rho}{\partial x} + J\boldsymbol{u} \boldsymbol{\cdot} \boldsymbol{\nabla} \rho = w, \tag{2.25}$$

subject to $\boldsymbol{\nabla} \boldsymbol{\cdot} \boldsymbol{u} = 0$ and the kinematic bottom boundary condition

$$(1 + Ju) \frac{\partial h}{\partial x} = w. \tag{2.26}$$

These non-dimensional equations imply that the problem is uniquely characterized by ϵ and J, and the relevant inner scales (non-dimensionalized by N and U) are given by

$$\frac{u_0}{U} = J, \tag{2.27}$$

$$\frac{w_0}{U} = \epsilon J, \tag{2.28}$$

$$\frac{gR}{\rho_0 U N} = J, \tag{2.29}$$

$$\frac{P}{U^2} = J, \tag{2.30}$$

$$\frac{\delta N}{U} = 1. \tag{2.31}$$

An interesting property of this scaling is that the vertical scale δ, which defines the vertical wavelength of the lee wave $2\pi\delta$, is determined exclusively from the upstream quantities of the flow, U and N, as we expect from linear theory. To test this scaling, we plot the horizontal perturbation velocity normalized by JU for the case shown in figure 1 in figure 2. This figure shows that the maximum perturbation horizontal velocity is $O(JU)$ and that the vertical wavelength of the lee wave is $\lambda = 2\pi\delta$.

If we now borrow from Winters & Armi (2012) and use the inner scales to define the internal Froude number as $Fr_\delta = u_0/\sqrt{g'\delta}$, where $g'\delta = g(R/\rho_0)\delta = JU^2$, this gives $Fr_\delta = J^{1/2}$. Therefore, although it appears to represent an inverse Froude number when expressed in terms of external variables, this scaling shows that it is in fact appropriate to refer to $J = Nh_0/U$ as the square of an internal Froude number because of the relationship between the internal and external variables.

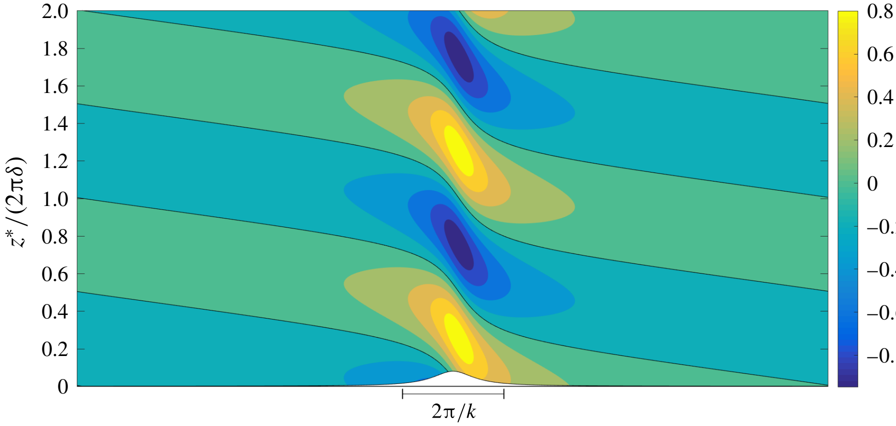

FIGURE 2. Dimensionless horizontal perturbation velocity, $u^*/(JU) = u/J$, corresponding to the streamlines plotted in figure 1. The aspect ratio is greatly exaggerated to display vertical structure. Contours of $u = 0$ are indicated to identify the 'layers' of the perturbation and to emphasize that they have a characteristic thickness of $\pi\delta$.

3. Discussion

Our scaling has shown that $J = Nh_0/U$ can be interpreted as the square of the internal Froude number, Fr_δ, as defined by Winters & Armi (2012). However, this interpretation alone is not sufficient to unambiguously define J as a Froude number because it is not readily apparent that Fr_δ is a ratio of advection to wave speed. Nonetheless, for the special case of the hydrostatic lee wave, our scaling identifies J as just such a ratio. Recall that in the hydrostatic limit the group velocity of the wave is oriented vertically with magnitude $c_g = \epsilon U$ (Gill 1982). Comparing this wave speed to the scale for the vertical perturbation velocity, $w_0 = \epsilon JU$, we form the ratio $w_0/c_g = J$. Thus, with our scaling J emerges as a Froude number in its most generally accepted sense, that is, as a ratio of an advection speed to a wave speed. In this case, it is the ratio of vertical advection speed within the lee wave to the nearly vertical propagation speed of the lee wave. Such an interpretation justifies reference of J as the 'vertical' Froude number, Fr_z or Fr_{vert}, following Laprise & Peltier (1989*a*,*c*).

This identification of J as a Froude number derives from a balance of the linear terms in the Euler equations, and is therefore mathematically unjustifiable for lee waves with significant perturbation velocities. Indeed, from the literature it is clear that the character of the lee wave changes as J approaches $O(1)$. A precondition on the wave solution to the flow is that it remains stable to both convective and shear instabilities (Long 1953; Miles 1961). Asymptotic and fully nonlinear solutions for the streamlines of the flow using Long's model (Long 1953) show both vertical streamlines (convective instability) and Richardson numbers smaller than 0.25 (shear instability) developing when $J = O(1)$ for flow over various ridge shapes (e.g. Miles 1969; Laprise & Peltier 1989*b*). Furthermore, laboratory experiments and numerical simulations demonstrate that in the $J > 1$ regime, the wave field saturates and J reflects instead the strength of non-wave dynamics such as upstream blocking and downslope windstorms (Laprise & Peltier 1989*a*,*c*; Baines 1995). For this reason, J is often interpreted as a nonlinearity parameter rather than a Froude number (Miles 1969; Baines 1995; Aguilar & Sutherland 2006; Eckermann *et al.* 2010; Klymak *et al.* 2010; Nikurashin & Ferrari 2010).

The upper limit on the valid identification of J as a Froude number is evident when using Fr_δ to devise a physical interpretation of J. As discussed in the Introduction, Fr_δ is the continuous extension of the layer Froude number as defined by Armi (1986), and represents the partitioning of energy within the lee wave between perturbation kinetic energy, u_0^2, and perturbation potential energy, $g'\delta$. The scaling $u_0^2 = JU = N^2h_0^2$ indicates that the perturbation kinetic energy results from the conversion of the potential energy of a displaced isopycnal in the wave, $N^2h_0^2 = g(R/\rho_0)h_0 = g'h_0$, into kinetic energy. This conversion takes place within the vertical scale of the lee wave, $\delta = U/N$, and thus sets a limit on the maximum possible isopycnal displacement. In this sense, we can think of δ as a maximum wave-making capacity; the largest that u_0 can become is when $h_0 = \delta$, at which point $u_{max} = N\delta = U$ and the perturbation energetics are equally partitioned between kinetic and potential energy. As h_0 grows beyond this height, the wave component of the flow will no longer change, as it has saturated its energetic capacity. Indeed, in their recent study for flow past a half-cylinder when $J > 1$, Winters & Armi (2012) show that in this regime, Fr_δ is held constant at unity, the energetics of the wave are equally partitioned, and, in analogy to hydraulic control of a two-layer exchange flow over a sill, the flow of the lowest unblocked layer exhibits a transition from subcritical flow upstream to a supercritical jet downstream followed by a dissipative hydraulic jump. Note that this scaling argument for saturation also confirms that it is inaccurate to identify J as Fr_{vert} when $J > O(1)$ because we can no longer expect w_0 to scale with J in this supercritical regime.

It is thus clear that the relationships we have identified between J, Fr_{vert}, and Fr_δ hold only up to $J = Nh_0/U = O(1)$. Below this limit, waves accommodate the disturbance of the hill adiabatically, and carry it away from the site of generation. As J approaches $O(1)$, the kinetic component of the wave energetics grows to the same magnitude as the potential component. All of this is consistent with the dynamical significance of a Froude number, and our scaling thus unambiguously identifies J with the Froude number for lee waves in the deep ocean. It is therefore appropriate to simply refer to J as the lee-wave Froude number, $Fr_{lee} = Nh_0/U$, noting that Fr_{lee} informs the degree of blocking or other nonlinear processes once $Fr_{lee} = O(1)$.

Acknowledgement

We gratefully acknowledge the support of ONR Grant N00014-16-1-2256 (scientific officer: Dr T. Paluszkiewicz).

References

Afanasyev, Y. D. & Peltier, W. R. 1998 The three-dimensionalization of stratified flow over two-dimensional topography. *J. Atmos. Sci.* **55** (1), 19–39.

Aguilar, D. A. & Sutherland, B. R. 2006 Internal wave generation from rough topography. *Phys. Fluids* **18** (6), 1–9.

Armi, L. 1986 The hydraulics of two flowing layers with different densities. *J. Fluid Mech.* **163**, 27–58.

Baines, P. G. 1995 *Topographic Effects in Stratified Flows*. Cambridge University Press.

Clark, T. L. & Peltier, W. R. 1984 Critical level reflection and the resonant growth of nonlinear mountain waves. *J. Atmos. Sci.* **41** (21), 3122–3134.

Crook, N. A., Clark, T. L. & Moncrieff, M. W. 1990 The Denver cyclone. Part I: generation in low Froude number flow. *J. Atmos. Sci.* **47** (23), 2725–2742.

Drazin, P. G. 1961 On the steady flow of a fluid of variable density past an obstacle. *Tellus* **13** (2), 239–251.

DURRAN, D. R. 1986 Another look at downslope windstorms. Part I: the development of analogs to supercritical flow in an infinitely deep, continuously stratified fluid. *J. Atmos. Sci.* **43** (21), 2527–2543.

ECKERMANN, S. D., LINDEMAN, J., BROUTMAN, D., MA, J. & BOYBEYI, Z. 2010 Momentum fluxes of gravity waves generated by variable Froude number flow over three-dimensional obstacles. *J. Atmos. Sci.* **67** (7), 2260–2278.

FARMER, D. M. & ARMI, L. 1986 Maximal two-layer exchange over a sill and through the combination of a sill and contraction with barotropic flow. *J. Fluid Mech.* **164**, 53–76.

FARMER, D. M. & SMITH, J. D. 1980 Tidal interaction of stratified flow with a sill in Knight Inlet. *Deep-Sea Res.* **27**, 239–254.

GARRETT, C. & KUNZE, E. 2007 Internal tide generation in the deep ocean. *Annu. Rev. Fluid Mech.* **39**, 57–87.

GILL, A. E. 1982 *Atmosphere-Ocean Dynamics*. Academic Press.

KIMURA, F. & MANINS, P. 1988 Blocking in periodic valleys. *Boundary-Layer Meteorol.* **44** (1), 137–169.

KITABAYASHI, K. 1977 Wind tunnel and field studies of stagnant flow upstream of a ridge. *J. Met. Soc. Japan* **55** (2), 193–204.

KLYMAK, J. M., LEGG, S. M. & PINKEL, R. 2010 High-mode stationary waves in stratified flow over large obstacles. *J. Fluid Mech.* **644**, 321–336.

LAPRISE, R. & PELTIER, W. R. 1989*a* The linear stability of nonlinear mountain waves: implications for the understanding of severe downslope windstorms. *J. Atmos. Sci.* **46** (4), 545–564.

LAPRISE, R. & PELTIER, W. R. 1989*b* On the structural characteristics of steady finite-amplitude mountain waves over bell-shaped topography. *J. Atmos. Sci.* **46** (4), 586–595.

LAPRISE, R. & PELTIER, W. R. 1989*c* The structure and energetics of transient eddies in a numerical simulation of breaking mountain waves. *J. Atmos. Sci.* **46** (4), 565–585.

LEGG, S. & KLYMAK, J. 2008 Internal hydraulic jumps and overturning generated by tidal flow over a tall steep ridge. *J. Phys. Oceanogr.* **38**, 1949–1964.

LONG, R. R. 1953 Some aspects of the flow of stratified fluids I. A theoretical investigation. *Tellus* **V**, 42–58.

MILES, J. W. 1961 On the stability of heterogeneous shear flows. *J. Fluid Mech.* **10** (04), 496–508.

MILES, J. W. 1969 Waves and wave drag in stratified flows. In *Applied Mechanics. International Union of Theoretical and Applied Mechanics* (ed. W. G. Vincenti & M. Hetnyi), pp. 50–75. Springer.

NIKURASHIN, M. & FERRARI, R. 2010 Radiation and dissipation of internal waves generated by geostrophic motions impinging on small-scale topography: theory. *J. Phys. Oceanogr.* **40** (5), 1055–1074.

PELTIER, W. R. & CLARK, T. L. 1983 Nonlinear mountain waves in two and three spatial dimensions. *Q. J. R. Meteorol. Soc.* **109** (461), 527–548.

SCINOCCA, J. F. & PELTIER, W. R. 1994 Finite-amplitude wave-activity diagnostics for Long's stationary solution. *J. Atmos. Sci.* **51** (4), 613–622.

SMITH, R. B. 1989 Mountain-induced stagnation points in hydrostatic flow. *Tellus* A **41** (3), 270–274.

SMOLARKIEWICZ, P. K. & ROTUNNO, R. 1989 Low Froude number flow past three-dimensional obstacles. Part I: baroclinically generated Lee vortices. *J. Atmos. Sci.* **46** (8), 1154–1164.

WELCH, W. T., SMOLARKIEWICZ, P., ROTUNNO, R. & BOVILLE, B. A. 2001 The large-scale effects of flow over periodic mesoscale topography. *J. Atmos. Sci.* **58** (12), 1477–1492.

WINTERS, K. B. & ARMI, L. 2012 Hydraulic control of continuously stratified flow over an obstacle. *J. Fluid Mech.* **700**, 502–513.