Analysis of the leaky mode spectrum in strongly coupled cavity QED systems

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I analyze the transmission spectrum of a strongly coupled atom with a FP cavity. The transmission is done through the side, not through the cavity mirrors. I also analyze the spectrum of the leaky modes when the atom is excited by an external mechanism and produces photoluminescence.

I. INTRODUCTION

The system that we analyze is composed of a cavity mode with frequency ω_c and decay rate κ. The cavity is coupled to a dipole emitter with frequency ω_d with coupling constant g_c. The atom is also coupled to n leaky modes that span a frequency bandwidth $\Delta\Omega$ around the frequency ω_d. We take the coupling to each of the leaky modes to be equal to g_l

The Hamiltonian of the system is a modified Jaynes-Cummings Hamiltonian that includes the interaction of the atom with the leaky modes.

$$H = H_A + H_{FC} + H_{AFC} + H_{FO} + H_{AFO} \tag{1}$$

where

$$H_A = \frac{\hbar\omega_d}{2}\sigma_z; \tag{2}$$

$$H_{FC} = \hbar\omega_c(a_c^\dagger a_c + \frac{1}{2}) \tag{3}$$

$$H_{AFC} = i\hbar(g_c^\star a_c^\dagger \sigma_- - g_c\sigma_+ a_c) \tag{4}$$

$$H_{FO} = \sum_k \hbar\omega_k(a_k^\dagger a_k + \frac{1}{2}) \tag{5}$$

$$H_{AFC} = \sum_k i\hbar(g_l^\star a_k^\dagger \sigma_- - g_l\sigma_+ a_l) \tag{6}$$

Including a decay term for the cavity mode and considering, the Hamiltonian can be written in matrix form as follows:

$$\begin{pmatrix} \omega_d & -ig_c & -ig_l & .. & -ig_l \\ ig_c & \omega_c - i\kappa & 0 & .. & 0 \\ ig_l & 0 & \omega_1 & .. & 0 \\ : & : & : & & 0 \\ ig_l & 0 & 0 & .. & \omega_n \end{pmatrix}$$

Suppose the parameters of the system are such that the dipole can strongly couple to the cavity. There are two questions that we want to answer.

1. We design the experiment such that we first excite the dipole into the excited state, then immediately we place it into the cavity and we monitor the radiation that leaks out from the system into the leaky modes as shown in Fig.1. The question is, what is the spectrum of these leaky modes.

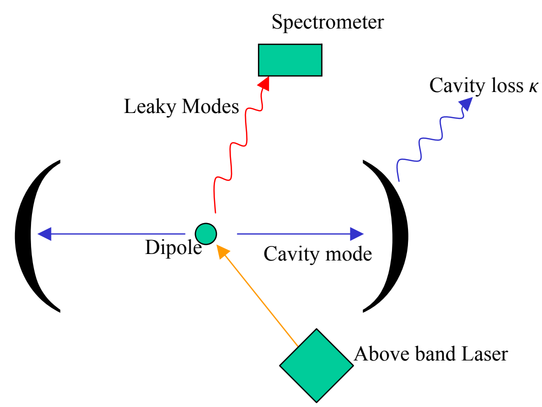

FIG. 1: Photoluminescence measurement - schematic representation. The dipole is strongly coupled to the cavity. The laser excites the dipole into its excited state (like the above band excitation of a quantum dot). Then the PL emitted in the leaky modes is collected and sent to a spectrometer. We want to know what the spectrum looks like.

2. A dipole in the excited state is placed into the cavity as shown if Fig.2. Then we scan (frequency) a laser beam through the dipole. The scanning is not done through the cavity mode but from the side of the cavity (think of a Fabry Perot cavity and that the scanning is done through the side of the PF). Then we monitor the laser beam after it interacted with the dipole-cavity system. The question is how would the absorption spectrum look like?

Both questions are answered using the same formalism. First we find the eigenvalues λ_m and eigenvectors $|V_m\rangle$ of the Hamiltonian. Then suppose we start the system in some specific state $|V_0\rangle$. The state of the system will time in time according to:

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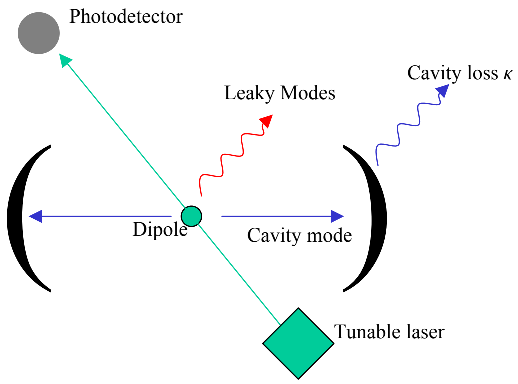

FIG. 2: Schematic representation of the measurement. The dipole is strongly coupled to the cavity. The laser frequency is scanned and the transmitted signal is monitored on the photodetector.

$$|\psi(t)\rangle = \sum_m a_m |V_m\rangle e^{-i\lambda_m t} \tag{7}$$

where $a_m = \langle V_0 | V_m \rangle$. The probability of finding the system in some specific state is given by projecting the state $|\psi(t)\rangle$ onto the eigenvectors of the unperturbed Hamiltonian.

The problem is solved numerically, and I use parameters similar to those in atomic physics experiments [1] $g_c = 34MHz$, $\kappa = 4MHz$. The parameter γ that represents the coupling of the dipole to free space is related to g_l and the number of leaky modes. I use $N = 98$ leaky modes that span a bandwidth of $BW = 185MHz$ around the dipole frequency $\omega_0 = 3\times10^8 MHz$ and $g_l = 0.5MHz$ that result in a free space decay rate $\gamma \approx 3.8MHz$, close to the atomic physics vale $\gamma = 2.6MHz$.

I consider the cavity to be resonant with the dipole.

II. PHOTOLUMINESCENCE OBSERVED IN THE LEAKY MODES

In this case we start with the dipole in the excited state which means that $V_0 = (\ 1\ \ 0\ \ ..\ \ 0\ )$. Then we let the system evolve for a period of time equal to $50\pi/BW$. The PL spectrum observed in the leaky modes can be found by plotting the power in each one of the leaky modes during the time evolution. Since a spectrometer would collect all the photons emitted in that time interval, we simply sum the energy time evolution in each one of the leaky modes. After running the Matlab code presented in at the end of this document we obtain the spectrum in Fig.3 for the energy distribution into the leaky modes.

We observe that the photoluminescence scattered in the leaky modes shows only the strong coupling peaks and no central peak corresponding to spontaneous emission from the atom.

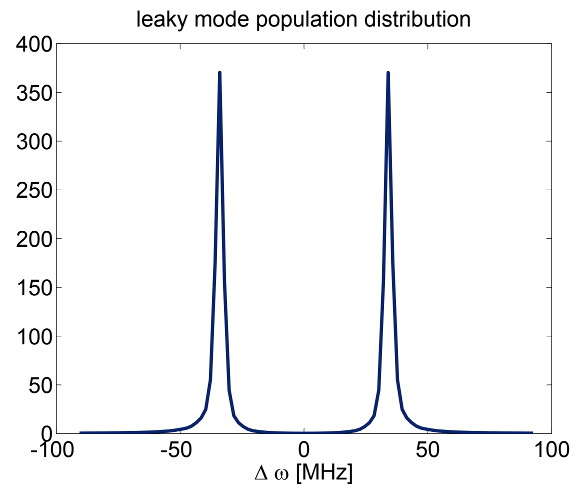

FIG. 3: Photoluminescence scattered in leaky modes

III. LATERAL TRANSMISSION THROUGH THE CAVITY

In this case we start with the dipole in the ground state and one photon into the leaky modes which means that $V_0 = (\ 0\ \ 0\ \ ..\ \ 1\ \ ..\ \ 0\ )$ (one takes any slot in the array except the first two). Then I integrate the field in the mode we start. I do this for every leaky mode and the spectrum is shown in if Fig.4

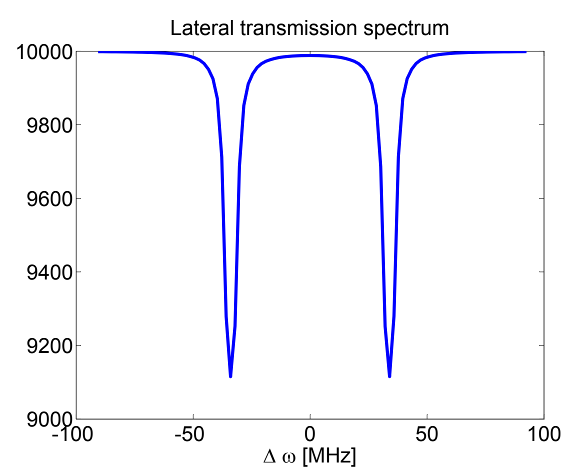

FIG. 4: Lateral transmission through the cavity

We observe that the transmission completely splits into two absorption peaks corresponding to the two strong-coupling polaritons.

IV. FREE SPACE QUANTUM DOT BEHAVIOR

To test if the code is correct I look at the free space quantum dot behavior. For this, we simply set the cavity coupling to zero and we get that in transmission there is only a single absorption peak with atom linewidth (Fig.6). In PL we also observe the peak corresponding to spontaneous emission (5).

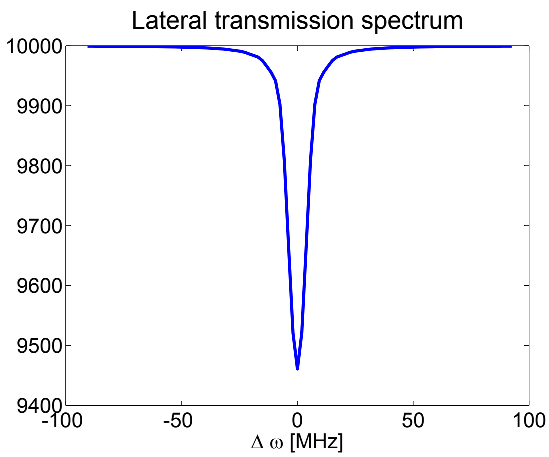

FIG. 5: Lateral transmission through the uncoupled dipole

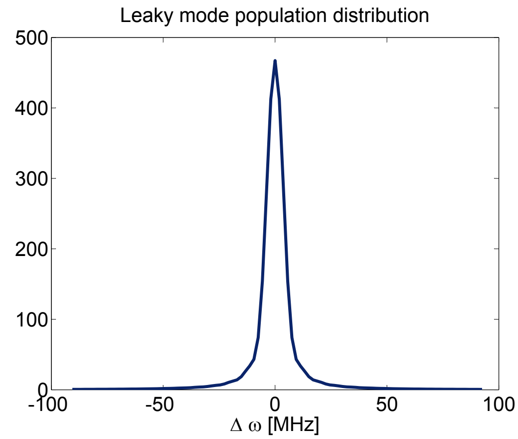

FIG. 6: Dipole spontaneous emission

V. CONCLUSION

In conclusion we show that, when observed from the lateral direction of a Fabry Perot cavity with a strongly coupled dipole, both the PL and the transmission show a split spectrum corresponding to the two polaritons of the strongly coupled system.

[1] K. M. Birnbaum, A. Boca, R. Miller, A. D. Boozer, T. E. Northup, and H. J. Kimble. Photon blockade in an optical cavity with one trapped atom. Nature, 436:87–90, 2005.

```
close all;
clear all;

%Here I define the coupling constants

gc=34; %coupling to the cavity
%gc=0;
gl=0.5; %coupling to each of the leaky modes
wc=3*10^(8); %cavity frequency
wd=wc; %dipole frequency
i=sqrt(-1);
n=100; %number of leaky modes plus 2
kappa=4; %cavity loss

bw=5*37; % frequancy interval spanned by leaky modes

%now I define the hamiltonian matrix
H=zeros(n);
w=[];

H(1,1)=wd;
H(2,2)=wc-i*kappa;
H(1,2)=-i*gc;
H(2,1)=i*gc;

for j=3:n
    w(j)=wc+(j-2-(n-2)/2)*bw/(n-2);
    H(j,j)=w(j);
    H(1,j)=-i*gl;
    H(j,1)=i*gl;
end


% now I find the eignevalues and eigenvectors of the Hamiltonian
[V,D]=eig(H);

% l(j) are the eigenvalues
for j=1:n
    l(j)=D(j,j);
end


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%% Compute Photoluminescence Spectrum%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% start with atom in excited state
v0=zeros(1,n);
v0(1)=1;

%a(j) are the initial coefficient of each of the eigenvectors in the
```

```
%function psi describing the state of the system

psi=[];
for j=1:n
    v=V(:,j);
    a(j)=v0*v;
    psi(:,j)=a(j)*V(:,j);
end

%these are the number of time iterations

    tpoints=10000
    t=linspace(0,50*pi/bw,tpoints);


% %each one of the components psi[j] evolves in time as exp(i*lambda*t);

% %first find the evolution of the QD, cavity and the leaky mode resonant
% with the cavity

st=zeros(1,tpoints);%stores qd time evolution
ct=zeros(1,tpoints);%stores cavity time evolution

vs=zeros(1,n);%eigenvector corresponding to the QD
vs(1)=1;
vc=zeros(1,n);%eigenvector corresponding to the cavity
vc(2)=1;

for j=1:n
  st=st+vs*psi(:,j)*exp(-i*l(j)*t);
  ct=ct+vc*psi(:,j)*exp(-i*l(j)*t);
end


mode_en=[];

% in this for loop I look at the PL leaked in eack of the leaky modes. The
% variable llt stores the evolution of the leaky mode field

for k=1:n-2;

    vl=zeros(1,n);
    vl(k+2)=1;
    llt=zeros(1,tpoints);
    for j=1:n
      llt=llt+vl*psi(:,j)*exp(-i*l(j)*t);
    end
     mode_en(k)=sum(abs(llt).^2)
end
```

```
 figure;
 plot(t,abs(st).^2); hold on;
 title('dot population');
 xlabel 't [\mu s]'

 figure;
 plot(t,abs(ct).^2,'r');
 title('cavity mode population');
 xlabel 't [\mu s]'

 figure;
 plot(w(3:n)-wc,mode_en,'g');
 title('leaky mode population distribution');
 xlabel '\Delta \omega [MHz]'

 % now let's plot the fft of each of these time evolutions

figure;
plot(abs(fftshift(fft((st)))));
title('ft of dot');

figure;
plot(abs(fftshift(fft((ct)))),'r');
title('ft of high q mode');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%% Compute transmissio Spectrum%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%I start with  the initial state in each of the leaky modes and I evolve it
%in time. Then I integrate (sum) the intensity in the leaky mode for the
%entire time evolution. This integral is proportional to the transmitted
%intensity.


trans=[]; %here I store the transmitted intensity for each leaky mode.

for k=1:n-2

    v0=zeros(1,n);
    v0(k+2)=1;

    for j=1:n
        v=V(:,j);
        a(j)=v0*v;
    end

    % % the components of the initial state
    psi=[];
    for j=1:n
        psi(:,j)=a(j)*V(:,j);
```

```
    end

    lt=zeros(1,tpoints);

    for j=1:n
      lt=lt+v0*psi(:,j)*exp(-i*l(j)*t);
    end

    trans(k)=sum(abs(lt).^2);
    k
end

figure
plot(w(3:n)-wc,trans)
title 'Lateral transmission spectrum'
xlabel '\Delta \omega [MHz]'
```