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Disruption Risk and Optimal Sourcing in Multitier Supply Networks

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Abstract. We study sourcing in a supply chain with three levels: a manufacturer, tier 1 suppliers, and tier 2 suppliers prone to disruption from, e.g., natural disasters such as earthquakes or floods. The manufacturer may not directly dictate which tier 2 suppliers are used but may influence the sourcing decisions of tier 1 suppliers via contract parameters. The manufacturer's optimal strategy depends critically on the degree of overlap in the supply chain: if tier 1 suppliers share tier 2 suppliers, resulting in a "diamond-shaped" supply chain, the manufacturer relies less on direct mitigation (procuring excess inventory and multisourcing in tier 1) and more on indirect mitigation (inducing tier 1 suppliers to mitigate disruption risk). We also show that while the manufacturer always prefers less overlap, tier 1 suppliers may prefer a more overlapped supply chain and hence may strategically choose to form a diamond-shaped supply chain. This preference conflict worsens as the manufacturer's profit margin increases, as disruptions become more severe, and as unreliable tier 2 suppliers become more heterogeneous in their probability of disruption; however, penalty contracts—in which the manufacturer penalizes tier 1 suppliers for a failure to deliver ordered units—alleviate this coordination problem.

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Keywords: disruption risk • multisourcing • supply chains • multiple tiers

1. Introduction

In the wake of the March 11, 2011, Tōhoku earthquake off the eastern coast of Japan, many companies faced massive disruptions in their supply chains. Automotive manufacturers—particularly Toyota—were especially affected, as a number of key suppliers were located in the impacted region and were disabled as a result of a combination of earthquake damage, tsunami damage, and radiation exposure resulting from the meltdown of the Fukushima Daiichi nuclear power plant. In addition, some automotive suppliers that were not directly damaged were forced to cease production because of power shortages that followed in the weeks after the disaster. The consequences for manufacturers were severe: Toyota faced immediate shortages on over 400 parts, and production capacity was reduced for six months following the disaster (Tabuchi 2011).

Prior to the disaster, Toyota had primarily concerned itself with its tier 1 (immediate) suppliers and had left the management of higher tiers in the supply chain to its direct partners. In most cases, Toyota did not even know who its tier 2 suppliers were, much less their risk profiles or how tier 1 suppliers managed that risk via, for instance, sourcing decisions or holding safety stocks. As part of the recovery process, Toyota created a systematic map of the upper levels of its supply chain

and the disruption management strategies of its tier 1 suppliers for the first time. During this investigation, a fundamental problem emerged (Greimel 2012, Masui and Nishi 2012): much to the company's surprise, Toyota discovered that while it might have attempted to mitigate disruption risk by multisourcing with tier 1 suppliers, many tier 1 suppliers shared some or all of their tier 2 suppliers, leading to a significant degree of correlation in risk for supply coming from tier 1. This problem manifested itself with a wide variety of tier 2 suppliers, ranging from large firms with specific expertise, such as Renesas, an automotive semiconductor manufacturer, to smaller firms with generic capabilities, such as Fujikura, a rubber manufacturer (Masui and Nishi 2012). The substantial overlap in the upper levels of the network, which Toyota internally referred to as a "diamond-shaped" supply chain, clearly called for the firm to rethink its disruption management and sourcing decisions.

The findings of Toyota and other major manufacturers quickly led firms to realize that a more comprehensive disruption management strategy, extending beyond a firm's immediate suppliers and into the entire supply network, is necessary to build a robust enterprise (Greimel 2012, Masui and Nishi 2012, Tucker 2012). Motivated by this example and numerous recent

Figure 1. Supply Chain Overview

instances of disruption caused by natural disasters (e.g., flooding in Thailand, as described in Soble 2011), financial distress of suppliers (e.g., in the automotive supply chain, as described in Hoffman 2012), or industrial accidents (e.g., at apparel suppliers in Bangladesh, as described in Greenhouse 2012), in this paper we consider the issue of optimal sourcing when the disruption risk originates in the upper levels of a multitier supply chain. Specifically, we examine a stylized model in which a downstream manufacturer (such as Toyota) sources identical critical components from tier 1 suppliers. Those tier 1 suppliers, in turn, source subassemblies or raw materials from tier 2 suppliers. We assume that disruption may occur in tier 2, which in turn will cause a shortage impacting tier 1 and ultimately the manufacturer; see Figure 1.

Following the classic supply chain disruption approach (Tomlin 2006, Aydin et al. 2010), we assume that tier 2 suppliers have heterogeneous cost and disruption risk profiles, and hence the choice of tier 2 suppliers is critical in determining the risk profile of the supply chain. However, unlike the existing disruption risk literature, in our model the manufacturer cannot directly dictate that risk profile by choosing tier 2 suppliers. Instead, the manufacturer acts as a sequential leader, first deciding contract terms with its tier 1 suppliers; those tier 1 suppliers then choose sourcing quantities from one or more tier 2 suppliers to maximize their own expected profits, given the contract terms offered by the manufacturer. Consequently, the manufacturer's contract decisions with its tier 1 suppliers will induce some sourcing arrangement from tier 1 to tier 2, ultimately determining both the *probability of disruption* and the *correlation of disruption risk* for the tier 1 suppliers. Prompted by Toyota's findings after the Tōhoku earthquake, we pay particular attention to the impact of the supply chain configuration—the number of tier 2 suppliers and the manner in which they are connected to tier 1 suppliers—on the manufacturer's optimal risk management strategy, focusing on three key questions. First (in Sections 3–6), how should a manufacturer manage disruption risk when both the probabilities and the correlations in the disruption profile of its immediate suppliers are endogenously determined by the contracts that the manufacturer offers? How does the manufacturer's optimal sourcing strategy depend on the configuration of the supply network? Second (in Section 7), if tier 1 suppliers were capable of choosing the supply chain configuration,

would they choose the optimal configuration from the manufacturer's point of view? And third (in Section 8), if not, does a simple mechanism exist to align those preferences and coordinate the supply chain?

2. Literature Review

The existing disruption management literature has extensively explored mitigation strategies employable by a firm—primarily safety stocks and supplier diversification—to manage the risk of random supply at its own facilities or from its immediate tier 1 suppliers. Examples include Anupindi and Akella (1993), Yano and Lee (1995), Elmaghraby (2000), Minner (2003), Tomlin (2006), Chopra et al. (2007), Dada et al. (2007), Federgruen and Yang (2008), Tomlin (2009), Yang et al. (2009), Aydin et al. (2010), Tomlin and Wang (2011), Hu and Kostamis (2014), Wadecki et al. (2012), and many others. As we consider the issue of endogenous correlation in the supply network, our paper is particularly related to models with correlated risk and endogenous risk.

From the former category (correlated risk), Babich et al. (2007) find that a monopolistic firm might prefer positively correlated supplier defaults, as this increases price competition among its suppliers, while Tang and Kouvelis (2011) show that firms that compete in a common consumer market and share a supplier base will earn lower profits when supplier defaults are highly correlated. Our model differs from these in that, instead of focusing on the tension between competition and diversification in a firm's tier 1 supplier base, we introduce supply correlation between tier 1 suppliers through their common suppliers in tier 2. In this case, the firm can use diversification as a strategy to induce its tier 1 suppliers to source from different tier 2 suppliers.

From the latter category (endogenous risk), Swinney and Netessine (2009) consider how to contract with a supplier when that supplier's (financial) default risk depends on the contract price. Wang et al. (2010) explore how a firm can invest to improve the reliability of its suppliers and how this can be used alongside dual sourcing as a two-pronged strategy against yield uncertainty. Babich (2010) considers how financial subsidies can be used to mitigate the risk of supplier default. Hwang et al. (2014) explore how contracts and delegation can be employed to induce an immediate supplier to improve its reliability. In contrast to these models, in which risk is determined by firm actions

or financial subsidies, we consider how the choice of tier 1 and tier 2 suppliers impacts the shape, and subsequently the risk, of the supply network. Bimpikis et al. (2013) also consider how the endogenous choice of risky tier 2 suppliers impacts the supply chain; however, they focus on how nonconvexities of the production function affect supply chain risk, whereas we focus on how supplier asymmetries and access to different sets of tier 2 suppliers can influence the optimal sourcing strategy of the downstream manufacturer. To summarize, our model is, to the best of our knowledge, among the first to explore disruption risk management in supply chains with more than two tiers and the first to consider how supplier selection and contracting impacts the shape and disruption risk level of the supply chain.

3. Model

We analyze a supply chain consisting of three tiers: tier 0, tier 1, and tier 2. Tier 0 contains a single firm, the “manufacturer,” which assembles finished goods and sells to a consumer market. The manufacturer sources a critical component from tier 1, which consists of two identical suppliers that provide fully substitutable products. Each tier 1 supplier, in turn, sources a critical intermediate component from tier 2. Using an example from our introduction, the manufacturer in this supply chain might be Toyota, the tier 1 suppliers might be manufacturers of automotive air conditioning systems, and the tier 2 suppliers might be semiconductor manufacturers that provide the electronics used in the air conditioning units.

3.1. Tier 2

Disruption in the supply chain originates in tier 2. Following Tomlin (2006) and others, we assume there are two “classes” of tier 2 suppliers: reliable suppliers (denoted by the subscript r), which always deliver the requested quantity, and unreliable suppliers (denoted by u) that are prone to disruption but are less costly than reliable suppliers. This may be the case if, for example, some suppliers are located in an earthquake-prone region (e.g., Japan) that is close to the downstream manufacturing facilities, while other suppliers are located in more geologically stable but distant regions, requiring greater transportation costs (e.g., North America, Europe). We assume that reliable suppliers have infinite capacity, while unreliable suppliers have random capacity that follows a two-point disruption distribution: with probability $1 - \lambda_j$, there is no disruption and effective capacity is infinite, whereas with probability λ_j , a disruption occurs, and production capacity is reduced to some finite $K \geq 0$ (Yano and Lee 1995, Aydin et al. 2010). There are at most two unreliable suppliers, labeled $j \in \{1, 2\}$, and these suppliers may be heterogeneous in their disruption probability.

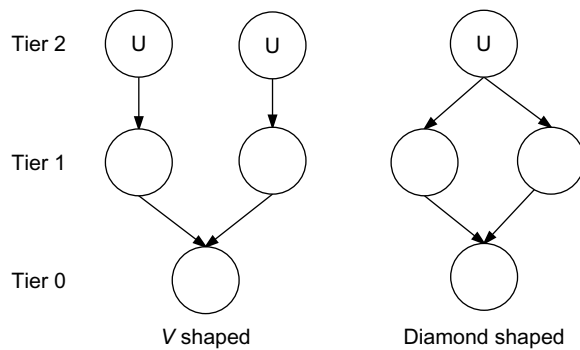
Without loss of generality, we assume that $\lambda_1 \leq \lambda_2$, i.e., unreliable supplier 1 is less likely to experience a disruption than is supplier 2. Disruptions at the unreliable suppliers are independent events.

The tier 2 sourcing cost is exogenous (e.g., because suppliers at this level tend to offer more commoditized goods, with prices determined by the market), and the per-unit sourcing cost from a reliable supplier is c_r , while the sourcing cost from an unreliable supplier is c_u , where $c_r > c_u$. Note that we assume that unreliable tier 2 suppliers, while potentially heterogeneous in their disruption probability, have identical sourcing costs. We make this assumption for two reasons. First, it is reflective of our motivating example of Toyota’s supply chain. In this case, the “unreliable” suppliers are, for the most part, domestic Japanese suppliers who may face slightly different levels of risk based on their location within Japan (e.g., suppliers located in the coastal area of the Tōhoku region may face a different chance of disruption than suppliers in the interior of the Kansai region) but, because of intense competition in the domestic marketplace and similar transportation costs, sell at similar prices to tier 1. It also includes, as a special case, identical (but independent) unreliable tier 2 suppliers. Second, this assumption enables tractable analysis of our problem, as simultaneous heterogeneity in cost and disruption risk increases the complexity of the model substantially; in the supplemental appendix, we numerically analyze the general case of heterogeneous risk and cost, and we find that our key insights continue to hold. Aside from their reliability in delivering production orders and their sourcing cost, the tier 2 suppliers are otherwise identical, i.e., they offer products of identical quality that are fully substitutable, and all tier 2 suppliers have sufficient capacity to fulfill any downstream order if they are not disrupted.

3.2. Tier 1

The two suppliers in tier 1, labeled $i \in \{A, B\}$, after receiving a contract from the manufacturer (described below), select which tier 2 suppliers to source from with the goal of maximizing their expected profit. If a tier 1 supplier selects a tier 2 supplier that disrupts, then all parts from that supplier in excess of the disrupted capacity K are lost, and the tier 1 supplier potentially faces a shortage of components, which may in turn lead to a failure to deliver product to the manufacturer. We assume that both tier 1 suppliers have access to one reliable and one unreliable tier 2 supplier. Tier 1 suppliers may choose to single source from either supplier type or to dual source, with an arbitrary quantity split between the suppliers.

Motivated by Toyota’s discovery of substantial overlap in the upper levels of its supply chain, we assume that there are two possible supply chain configurations,

Figure 2. The Two Possible Supply Chain Configurations

Notes. Only unreliable (U) tier 2 suppliers are pictured. Both tier 1 suppliers also have access to a perfectly reliable tier 2 supplier.

as shown in Figure 2: either unreliable tier 2 suppliers are independent, which we refer to as a V-shaped configuration, or the unreliable tier 2 supplier (either supplier 1 or 2) is shared by the tier 1 suppliers, which we refer to as a diamond-shaped configuration.¹ In the V-shaped configuration, the risk of disruption from the unreliable tier 2 suppliers is assumed to be statistically independent. In the diamond-shaped configuration, if the common unreliable tier 2 supplier disrupts, both tier 1 suppliers are impacted by reduced capacity at the disrupted supplier. There is no possibility of disruption originating in tiers 0 and 1.

For the initial part of our analysis, the supply chain configuration is assumed to be exogenously specified. In practice, this is frequently the case: given the high cost and long-term nature of establishing a list of approved suppliers, tier 1 suppliers operate over the short term with an effectively fixed set of tier 2 suppliers, around which the manufacturer optimizes its day-to-day procurement decisions. In addition, tier 2 suppliers are often chosen for reasons outside of our model, such as capability or quality, implying a key problem of how to optimally manage disruption risk given a fixed supply chain configuration. The degree of concentration of the tier 2 supplier base may also determine whether the supply chain is likely to end up with shared or independent tier 2 suppliers; if there are only two tier 2 suppliers in the world that manufacture the component, one reliable and one unreliable, then the tier 2 supply base is very “concentrated,” and the tier 1 suppliers necessarily share those two tier 2 suppliers, resulting in a diamond-shaped configuration. This may be the case for highly specialized tier 2 suppliers, such as semiconductor manufacturers like Renesas. Conversely, this may be driven by industry trends outside of our model: for example, Sasaki (2013) describes how consolidation in the Japanese automotive supply chain stemming from cost pressure throughout the 1990s and 2000s has generally lead to a much more concentrated supply base than in the past. By contrast, if there are

many tier 2 suppliers, and the likelihood that the tier 1 suppliers select identical tier 2 suppliers is negligible, then the tier 2 supply base is very “diffuse,” resulting in a V-shaped configuration; this can occur particularly with tier 2 suppliers that offer commoditized products with generic capabilities, such as rubber component manufacturers like Fujikura. While understanding the manufacturer’s optimal strategy under a fixed supply chain configuration is important, an equally interesting question is how the supply chain configuration is formed in the first place, i.e., as tier 1 suppliers select tier 2 suppliers. Hence, once we have determined the optimal actions of the manufacturer and tier 1 suppliers given a fixed supply chain configuration, we consider the longer-term strategic issue of how the supply chain configuration is formed in Section 7.

3.3. The Manufacturer

The manufacturer faces deterministic market demand D (Zimmer 2002, Babich et al. 2012) and makes fixed per-unit revenue of π for every unit it sells up to D .² We assume that $\pi \geq c_u$, so that it is possible for the supply chain to profitably sell the product; all manufacturer costs outside of procurement from tier 1 suppliers are normalized to zero. The sequence of events is as follows. In the first period, the manufacturer offers its tier 1 suppliers A and B a price and quantity contract of the form (p_i, Q_i) , $i \in \{A, B\}$. Such contracts are commonly observed in practice and in the literature, and they stipulate that the manufacturer pays the supplier p_i for every unit delivered up to the specified amount Q_i . The manufacturer makes no payment for ordered units that are not delivered. We assume that the choice of tier 2 suppliers is not directly contractible. Even manufacturers as powerful as Toyota are typically unable to unilaterally dictate supplier choice; after the 2011 earthquake, approximately 50% of Toyota’s tier 1 suppliers refused to even share the identities of their tier 2 suppliers, let alone allow Toyota to unilaterally dictate those suppliers, citing competitive reasons (Masui and Nishi 2012). In addition, while other tools may be available to influence the choice of tier 2 suppliers by tier 1 suppliers, such as manufacturer involvement in the development of the tier 2 supply base, these tools are longer-term mechanisms, leaving procurement contracts as the most obvious immediate short- and medium-term tool available to influence the sourcing decisions of tier 1 suppliers.

After receiving the manufacturer’s contract, each tier 1 supplier then places orders q_u^i and q_r^i with its tier 2 unreliable and reliable suppliers, to maximize expected profit. We call the pair (q_u^i, q_r^i) the *sourcing strategy* of tier 1 supplier i . In the second period, the tier 2 suppliers either disrupt or fully fulfill their deliveries to the tier 1 suppliers. After collecting all deliveries from tier 2 suppliers, tier 1 suppliers then attempt

to fulfill the order submitted by the manufacturer. Any excess inventory delivered by tier 2 suppliers to tier 1 suppliers has zero value. In turn, after collecting deliveries from tier 1 suppliers, the manufacturer attempts to fulfill consumer demand D , and if supply is less than demand, lost sales occur. Any excess components delivered by tier 1 suppliers to the manufacturer have zero value. The manufacturer seeks to maximize its expected profit from the consumer market at the end of the second period, and its decision variables are the price and quantity in the contracts it gives to its tier 1 suppliers, (p_A, Q_A) and (p_B, Q_B) ; hence, we define the manufacturer's *sourcing strategy* to be $\{(p_A, Q_A), (p_B, Q_B)\}$, the pair of contract offers to its two tier 1 suppliers. We assume that the disruption probabilities, sourcing costs, and supply chain configuration are known to all parties in the supply chain: there is no private information in the model.

4. Optimal Sourcing in a V-Shaped Supply Chain

4.1. Tier 1's Sourcing Decision

We first examine the manufacturer's optimal sourcing strategy when facing a supply chain with independent tier 2 suppliers. This resembles the type of V-shaped supply network that Toyota (mistakenly) believed it possessed prior to the Tōhoku earthquake and, as such, will serve as a benchmark for understanding how increased overlap in tier 2 impacts the manufacturer's sourcing strategy. We begin our analysis by characterizing the optimal sourcing strategy for each tier 1 supplier $i \in \{A, B\}$ with access to unreliable tier 2 supplier $j \in \{1, 2\}$, in response to a contractual offer (p_i, Q_i) from the manufacturer. Given a sourcing strategy (q_u^i, q_r^i) , the expected profit of supplier i is

$$\begin{aligned} \Pi^i(q_u^i, q_r^i) &= \lambda_j [p_i \min\{q_r^i + \min\{K, q_u^i\}, Q_i\} - c_r q_r^i - c_u \min\{K, q_u^i\}] \\ &\quad + (1 - \lambda_j) [p_i \min\{q_u^i + q_r^i, Q_i\} - c_u q_u^i - c_r q_r^i]. \end{aligned} \quad (1)$$

Let the superscript asterisk denote the optimal sourcing strategy. Optimizing the tier 1 supplier's expected profit over the quantity sourced from each tier 2 supplier leads to the following result.

Theorem 1. *In a V-shaped supply chain, the optimal sourcing strategy of tier 1 supplier $i \in \{A, B\}$ with access to unreliable tier 2 supplier $j \in \{1, 2\}$ is*

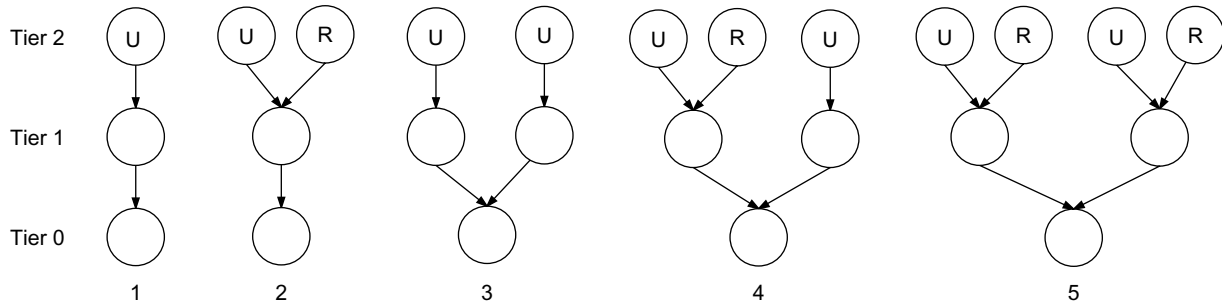
$$(q_u^i, q_r^i)^* = \begin{cases} (Q_i, 0) & \text{if } c_u \leq p_i < c_u + \frac{c_r - c_u}{\lambda_j}, \\ (K, Q_i - K) & \text{if } p_i \geq c_u + \frac{c_r - c_u}{\lambda_j}. \end{cases}$$

Proof. All proofs appear in the appendix. \square

Observe that it is never optimal for the supplier to single source from the reliable tier 2 supplier. This is because the supplier may completely eliminate disruption risk by sourcing only the disrupted output K from the unreliable supplier and sourcing the remainder, $Q_i - K$, from the reliable supplier, achieving the same outcome (at a lower cost) as sourcing everything from the reliable supplier. Moreover, sourcing inventory in excess of the manufacturer's contracted quantity Q_i is never optimal, because disruptions are manifested as random supplier capacity (as opposed to random yield; see Hwang et al. 2014), combined with the presence of a perfectly reliable tier 2 supplier. This implies that a tier 1 supplier effectively chooses from two possible sourcing strategies: sourcing from the unreliable supplier or sourcing from both suppliers. The former strategy is also known as passive acceptance (i.e., paying a low cost and merely accepting the risk of disruption), and the latter strategy is known as dual sourcing (Tomlin 2006). Intuitively, for low contract prices p_i , the supplier follows a passive acceptance strategy and single sources from the unreliable supplier, while for high prices, the supplier employs dual sourcing to completely eliminate the risk of disruption.³ The price necessary to induce the supplier to dual source is greater than the marginal cost of the reliable supplier, an agency cost arising from the moral hazard feature inherent in a decentralized supply chain.

4.2. The Manufacturer's Optimal Sourcing Strategy

Having derived the optimal sourcing strategy of a tier 1 supplier in response to a contract (p_i, Q_i) from the manufacturer, we may now determine the optimal contracts offered by the manufacturer to the two tier 1 suppliers. Without loss of generality, we assume that tier 1 supplier A has access to unreliable tier 2 supplier 1 (the lower-risk supplier), while tier 1 supplier B has access to unreliable tier 2 supplier 2 (the higher-risk supplier). In what follows, we use the term "supply chain structure" to refer to an induced set of sourcing quantities between tiers 0, 1, and 2, given a particular supply chain configuration (either V-shaped or diamond-shaped). In theory, the manufacturer could induce four sourcing strategies at each tier 1 supplier (no participation, single sourcing from either tier 2 supplier, or dual sourcing from both tier 2 suppliers), implying a total of 16 possible induced structures. However, from Theorem 1, the manufacturer can only feasibly induce three strategies for each of its tier 1 suppliers: no participation, single sourcing from the unreliable supplier (by offering a low price), or dual sourcing (by offering a high price). This implies that, via its contractual offers to tier 1 suppliers, the manufacturer is capable of generating a total of five unique supply chain structures (excluding the case of no production and all symmetric cases). We denote these as structures 1–5, and we summarize them in Figure 3.

Figure 3. The Five Distinct Structures That the Manufacturer Can Induce in a V-Shaped Supply Chain

Note. R, reliable; U, unreliable.

To determine the manufacturer's optimal sourcing strategy, we must consider the performance of each of these possible induced structures. In any structure, the manufacturer's expected profit as a function of its sourcing strategy $\{(p_A, Q_A), (p_B, Q_B)\}$ is

$$\begin{aligned} \Pi^M(\{(p_A, Q_A), (p_B, Q_B)\}) &= f_1(p_A, p_B) [\pi \min\{Q_A + Q_B, D\} - p_A Q_A - p_B Q_B] \\ &\quad + f_2(p_A, p_B) [\pi \min\{\min\{K, Q_A\} + Q_B, D\} \\ &\quad \quad - p_A \min\{K, Q_A\} - p_B Q_B] \\ &\quad + f_3(p_A, p_B) [\pi \min\{Q_A + \min\{K, Q_B\}, D\} \\ &\quad \quad - p_A Q_A - p_B \min\{K, Q_B\}] \\ &\quad + f_4(p_A, p_B) [\pi \min\{\min\{K, Q_A\} + \min\{K, Q_B\}, D\} \\ &\quad \quad - p_A \min\{K, Q_A\} - p_B \min\{K, Q_B\}]. \quad (2) \end{aligned}$$

In the above equation, f_1 , f_2 , f_3 , and f_4 denote the probability that the full orders are fulfilled by both tier 1 suppliers, only supplier B, only supplier A, or neither tier 1 supplier, respectively. Note that this is similar in form to the tier 1 supplier's profit function in Equation (1), except that (2) allows for the disruption of both immediate suppliers, and the probability terms in (2) are endogenously determined by the manufacturer's offered prices.

Each structure in Figure 3 corresponds to a particular set of prices offered to tier 1 suppliers, and this in turn determines the disruption probabilities f_1 – f_4 . For instance, structure 4 may be induced by offering supplier A a high price and supplier B a low price, and the resultant probabilities are $f_1 = 1 - \lambda_2$, $f_2 = 0$, $f_3 = \lambda_2$, and $f_4 = 0$, because only supplier B may fail to deliver the manufacturer's order, as supplier A dual sources and achieves perfect reliability. To determine the manufacturer's optimal strategy, we must find the optimal sourcing quantities in each structure given the prices and probabilities associated with the structures and then compare the optimal expected profit across the five structures. For the sake of brevity, we relegate the derivation of these profit expressions to Lemma 1 in the appendix, and we focus our subsequent discussion on the outcome of this analysis: the

determination of the manufacturer's optimal sourcing strategy and induced supply chain structure. We begin by considering the case of minor disruptions.

Theorem 2. In a V-shaped supply chain, if disruptions are minor ($K \geq D/2$), the manufacturer's optimal sourcing strategy is $\{(p_A, Q_A), (p_B, Q_B)\}^* = \{(c_u, \theta D), (c_u, (1 - \theta)D)\}$ for any $\theta \in [(D - K)/D, K/D]$.

The theorem shows if that the disrupted capacity is high (i.e., if sufficient disrupted capacity exists across the entire unreliable tier 2 supply base for the manufacturer to cover all of its demand), the manufacturer should induce structure 3 in Figure 3: single sourcing from the unreliable tier 2 suppliers by both tier 1 suppliers. As a result, tier 1 suppliers do nothing to mitigate disruption risk in the supply chain, and all risk mitigation is pursued directly by the manufacturer, who is able to completely eliminate risk via its own sourcing actions. We thus refer to this approach as a *manufacturer dual sourcing* (DS) strategy—the manufacturer mitigates risk by dual sourcing, while the tier 1 suppliers employ passive acceptance, by single sourcing from a risky tier 2 supplier. The only restriction on the manufacturer's sourcing strategy is that no more than K units should be sourced from each tier 1 supplier, to ensure that tier 2 suppliers are always able to deliver in the event of a disruption; this determines the feasible range of θ , the division of quantities between tier 1 suppliers, which the manufacturer can employ.

Next, we consider the case when disruptions are severe, meaning disrupted capacity is small. We first define the following cost threshold for the reliable supplier:

$$\bar{c}_r \equiv c_u + c_u(1 - \lambda_1)\lambda_2 \frac{D - 2K}{D - K}. \quad (3)$$

Observe that this cost threshold varies depending on the severity and likelihood of disruptions, and it lies between c_u and $2c_u$ if $K < D/2$. As the following theorem shows, the manufacturer's optimal sourcing strategy depends critically on this threshold.

Theorem 3. Define $\bar{\pi}_L \equiv ((c_r - c_u)/\lambda_1\lambda_2)((D - K)/(D - 2K)) + c_u$ and $\bar{\pi}_H \equiv ((c_r - c_u)/\lambda_1\lambda_2^2)((D - K)/(D - 2K)) - c_u((1 - \lambda_1 - \lambda_2)/\lambda_1\lambda_2)$. In a V-shaped supply chain, if disruptions are severe ($K < D/2$), we have the following:

(i) If $c_r \leq \bar{c}_r$, the manufacturer's optimal sourcing strategy is

$$\{(p_A, Q_A), (p_B, Q_B)\}^* = \begin{cases} \{(c_u, D - K), (c_u, K)\} & \text{if } c_u \leq \pi < \bar{\pi}_L, \\ \left\{ (c_u, K), \left(c_u + \frac{c_r - c_u}{\lambda_2}, D - K \right) \right\} & \text{if } \pi \geq \bar{\pi}_L. \end{cases}$$

(ii) If $c_r > \bar{c}_r$, the manufacturer's optimal sourcing strategy is

$$\{(p_A, Q_A), (p_B, Q_B)\}^* = \begin{cases} \{(c_u, D - K), (c_u, K)\} & \text{if } c_u \leq \pi < \frac{c_u}{\lambda_1}, \\ \{(c_u, D - K), (c_u, D - K)\} & \text{if } \frac{c_u}{\lambda_1} \leq \pi < \bar{\pi}_H, \\ \left\{ (c_u, K), \left(c_u + \frac{c_r - c_u}{\lambda_2}, D - K \right) \right\} & \text{if } \pi \geq \bar{\pi}_H. \end{cases}$$

The theorem shows that for small unit revenues (π), regardless of the cost of the reliable tier 2 suppliers, the manufacturer's approach is similar to the case when the magnitude of disruptions is small: the optimal strategy is to induce both tier 1 suppliers to single source from their unreliable tier 2 suppliers. The manufacturer prefers to source the majority of its order from tier 1 supplier A, who has access to the less risky tier 2 supplier, and places a smaller order with supplier B, who has access to the riskier tier 2 supplier. Hence, manufacturer dual sourcing (DS) is optimal for low unit revenues. As the unit revenue increases, the cost of reliable tier 2 suppliers begins to play a role. If they are inexpensive ($c_r \leq \bar{c}_r$), the manufacturer moves from DS to inducing one tier 1 supplier to dual source by paying a high price, while paying the other tier 1 supplier a low price to induce single sourcing. We call this a *manufacturer dual sourcing + supplier mitigation* (DS + SM) strategy, since it combines dual sourcing in tier 1 with induced supplier mitigation via dual sourcing in tier 2. Note that this strategy completely eliminates disruption risk for the manufacturer, but the manufacturer must pay a higher price to achieve this outcome.

If, on the other hand, reliable tier 2 suppliers are expensive ($c_r > \bar{c}_r$), at intermediate unit revenues, the manufacturer continues to pay tier 1 suppliers a low price but increases the quantity sourced from each tier 1 supplier to $D - K$, meaning the total amount of inventory procured exceeds demand. Tomlin (2006) refers to this strategy of excess procurement—in essence, holding safety stock—as “inventory mitigation.” Hence, when reliable tier 2 suppliers are costly, the optimal sourcing strategy switches from DS to *manufacturer*

Table 1. Optimal Sourcing Strategies in a V-Shaped Supply Chain for Severe Disruptions

Sourcing strategy	Use when ...	Quantity sourced	Tier 2 suppliers used
Manufacturer dual sourcing (DS)	π low	D	2 U
DS + Inventory mitigation (DS + IM)	π medium + c_r high	$2(D - K)$	2 U
DS + Supplier mitigation (DS + SM)	π high	D	2 U and 1 R

Note. R, reliable; U, unreliable.

dual sourcing + inventory mitigation (DS + IM), while still inducing tier 1 suppliers to follow a passive acceptance strategy by paying a low unit price. This strategy results in the manufacturer having more inventory than is necessary to satisfy demand if no disruption occurs; however, because the reliable tier 2 suppliers are expensive, the excess inventory cost is preferred to paying a high marginal procurement cost for reliable supply. Manufacturer efforts alone, however, cannot completely eliminate the risk of disruption, and even with the DS + IM strategy, there is still a chance that the manufacturer will have insufficient inventory to satisfy demand (i.e., if both tier 2 unreliable suppliers disrupt). Hence, as the unit revenue continues to increase, the manufacturer eventually switches to a DS + SM strategy, which eliminates disruption risk by engaging tier 1 suppliers in mitigation efforts. The optimal strategies when disruptions are severe are summarized in Table 1.

Theorem 3 illustrates that when disruptions are severe, the manufacturer's optimal strategy builds risk mitigation from the bottom of the supply chain up. Put differently, it is optimal to begin with manufacturer dual sourcing and, as the unit revenue increases, add manufacturer inventory mitigation (if reliable tier 2 suppliers are costly) and finally supplier mitigation. Consequently, for small and moderate unit revenues, tier 1 is not involved with disruption risk mitigation: the manufacturer bears all such responsibility. For large unit revenues, however, it is optimal to engage tier 1 in disruption risk mitigation efforts, by paying a higher price and inducing supplier mitigation. Comparing Theorems 2 and 3, we see that the manufacturer's optimal sourcing strategy is qualitatively different for minor and severe disruptions. For minor disruptions, the manufacturer bears all mitigation responsibility, following a strategy of pure manufacturer mitigation implemented via dual sourcing. For severe disruptions, the manufacturer may also employ inventory mitigation (at moderate unit revenues if reliable suppliers are costly) or induce supplier mitigation (at high unit revenues), sharing the burden of disruption risk mitigation with its suppliers. In addition, note

that the manufacturer always selects tier 1 supplier B, who has access to the riskier tier 2 supplier, to engage in supplier mitigation. This is because the price necessary to induce supplier B to dual source is lower than the price needed to induce supplier A to dual source, as indicated in Theorem 1. In other words, because supplier B faces a “riskier” unreliable supplier option, the agency costs incurred by the manufacturer are lower when inducing sourcing from the reliable supplier, and hence the manufacturer always prefers to engage in supplier mitigation with supplier B. This fact will have significant consequences in the supply chain game analyzed in Section 7.

After the 2011 Tōhoku earthquake, a key determination of Toyota executives was that they had placed too much emphasis on direct mitigation efforts and too little on supplier mitigation. Given that our model shows inducing supplier mitigation is indeed optimal for high unit revenues and severe disruptions, a natural question is, precisely how much does the manufacturer suffer when using the *wrong* strategy? Our final result in this section answers this question, by comparing manufacturer profit under direct dual sourcing (with or without inventory mitigation) to the profit under the optimal sourcing strategy (using supplier mitigation) for large unit revenues.

Corollary 1. *Let $\Pi^M(X)$ be the manufacturer’s expected profit under sourcing strategy $X \in \{DS, DS+IM, DS+SM\}$, and define $L(X) \equiv 1 - \Pi^M(X)/\Pi^M(DS+SM)$ to be the percentage loss in expected profit of strategy X relative to a DS + SM strategy. In a V-shaped supply chain and under severe disruptions, we have the following:*

- (i) *The percentage profit loss from employing manufacturer dual sourcing is $L(DS) \leq \lambda_1(1 - 2K/D)$.*
- (ii) *The percentage profit loss from employing manufacturer dual sourcing and inventory mitigation is $L(DS+IM) \leq \lambda_1\lambda_2(1 - 2K/D)$.*

For example, if the probability of disruption in tier 2 is 10%, the result shows that using an “incorrect” strategy that ignores inducements to tier 1 suppliers and solely uses manufacturer dual sourcing results in at most a 10% loss in profit compared with the optimal strategy. By adding inventory mitigation, the manufacturer’s profit is substantially improved, achieving a loss of at most 1% compared with the optimal profit. Note that the loss under either strategy depends similarly (i.e., linearly) on the *severity* of the disruption, as measured by the demand fill rate achievable when all unreliable tier 2 suppliers disrupt (i.e., $2K/D$). As one would expect, the performance of an incorrect strategy becomes increasingly poor as disruptions become very likely or very severe (i.e., high λ or low K), and all strategies perform equally well when disruptions are highly unlikely or not impactful (i.e., λ close to zero or K close to $D/2$). Importantly, however, the result

shows that in a V-shaped supply chain, manufacturer performance is substantially worse under a stand-alone dual sourcing strategy than under a dual sourcing and inventory mitigation strategy. Thus, focusing on dual sourcing alone in tier 1, as Toyota and many other lean manufacturers did prior to the Tōhoku earthquake, is potentially quite costly. This implies that, if the manufacturer cannot engage suppliers in disruption mitigation efforts when such efforts are optimal, a strategy that performs well is to employ manufacturer dual sourcing and inventory mitigation. While this cannot achieve the optimal profit earned under supplier mitigation, it results in an order of magnitude smaller loss than using dual sourcing alone. Indeed, Toyota has begun to implement inventory mitigation in instances where suppliers either refuse to engage in mitigation or find mitigation impractical (Greimel 2012, Masui and Nishi 2012).

5. Optimal Sourcing in a Diamond-Shaped Supply Chain

5.1. Tier 1’s Sourcing Decision

We now consider the case of a shared unreliable tier 2 supplier and the resulting “diamond”-shaped supply chain. Observe that whenever the tier 1 suppliers A and B are induced to source from their common unreliable tier 2 supplier, they potentially compete for limited capacity, i.e., when they respectively order q_u^A and q_u^B units, and the disrupted output is $K < q_u^A + q_u^B$.⁴ Hence, we must discuss how scarce capacity in tier 2 is allocated in the event of a disruption. While there are many possible allocations, we select the uniform rule, which has the desirable property of simultaneously minimizing shortage gaming and maximizing total supply chain profit in a number of settings (Sprumont 1991, Cachon and Lariviere 1999). A uniform allocation splits the disrupted capacity of the unreliable tier 2 supplier equally between tier 1 suppliers, unless one of the tier 1 suppliers requested less than half the disrupted capacity, in which case the other tier 1 supplier is allotted all unused capacity. In other words, supplier i ’s allocation in the event of a disruption is $\min\{q_u^i, \hat{K}_i\}$, where $\hat{K}_i \equiv K/2 + (K/2 - q_u^i)^+$ is the effective disrupted capacity. Under this allocation rule, the unique Nash equilibrium of the shortage game between tier 1 suppliers is, essentially, for both suppliers to operate as if they have a dedicated tier 2 unreliable supplier with exogenously fixed capacity equal to \hat{K}_i (see the appendix for details), leading to the following result.

Theorem 4. *In a diamond-shaped supply chain, the optimal sourcing strategy of tier 1 supplier $i \in \{A, B\}$ with access to unreliable tier 2 supplier $j \in \{1, 2\}$ is*

$$(q_u^i, q_r^i)^* = \begin{cases} (Q_i, 0) & \text{if } c_u \leq p_i < c_u + \frac{c_r - c_u}{\lambda_j}, \\ (\hat{K}_i, Q_i - \hat{K}_i) & \text{if } p_i \geq c_u + \frac{c_r - c_u}{\lambda_j}. \end{cases}$$

Note that this is nearly identical to the suppliers' optimal strategy in a V-shaped supply chain derived in Theorem 1, except for the disrupted capacity \hat{K}_i ; in particular, the manufacturer may still only induce single sourcing from the unreliable tier 2 supplier or dual sourcing from both tier 2 suppliers, and the prices necessary to achieve each outcome are identical to the case with independent tier 2 suppliers.

5.2. The Manufacturer's Optimal Sourcing Strategy

As a result of Theorem 4, there are once again five possible unique structures that the manufacturer can induce, graphically depicted in Figure 4. We derive the manufacturer's optimal sourcing strategy by following the same procedure as in Section 4, beginning with the case of minor disruptions.

Theorem 5. If tier 2 suppliers are shared and disruptions are minor ($K \geq D$), the manufacturer's optimal sourcing strategy is $\{(p_A, Q_A), (p_B, Q_B)\}^* = \{(c_u, \theta D), (c_u, (1 - \theta)D)\}$ for any $\theta \in [0, 1]$.

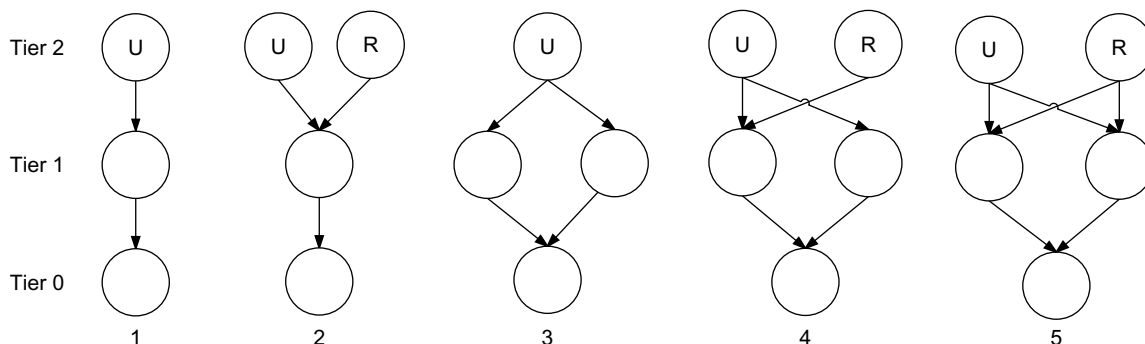
This is similar to the optimal sourcing strategy for minor disruptions with independent tier 2 suppliers (Theorem 2), with two key differences. First, with shared tier 2 suppliers, what qualifies as a "minor" disruption is a greater capacity than with independent tier 2 suppliers; in fact, the disrupted capacity must be greater than the manufacturer's demand to achieve this case, implying disruptions are effectively meaningless. Second, with shared tier 2 suppliers, any division of sourcing quantities between tier 1 suppliers will result in the same manufacturer profit, including single sourcing. Thus, while minor disruptions can be completely overcome by the manufacturer with either independent or shared tier 2 suppliers, dual sourcing with a specific division of sourced quantities is necessary in the former case, while single sourcing suffices in the latter. As a result, we say that a *manufacturer single sourcing* (SS) strategy is optimal in this case. Moving next to the case of severe disruptions, we have the following.

Theorem 6. Define $\bar{\pi}_j \equiv c_u + ((c_r - c_u)/\lambda_j^2)((D - K/2)/(D - K))$. In a diamond-shaped supply chain with shared unreliable supplier $j \in \{1, 2\}$, if disruptions are severe ($K < D$), the manufacturer's optimal sourcing strategy is

$$\{(p_A, Q_A), (p_B, Q_B)\}^* = \begin{cases} \{(c_u, \theta D), (c_u, (1 - \theta)D)\} \text{ for any } \theta \in [0, 1] \\ \text{if } c_u \leq \pi < \bar{\pi}_j, \\ \left\{ \left(c_u + \frac{c_r - c_u}{\lambda_j}, D - K/2 \right), (c_u, K/2) \right\} \\ \text{or} \\ \left\{ (c_u, K/2), \left(c_u + \frac{c_r - c_u}{\lambda_j}, D - K/2 \right) \right\} \\ \text{if } \pi \geq \bar{\pi}_j. \end{cases}$$

Note that, once again, while dual sourcing is an optimal strategy at low unit revenues, single sourcing performs just as well. In other words, in a diamond-shaped supply chain, manufacturer dual sourcing has *no* explicit value at low margins—regardless of the severity of disruptions—because tier 1 suppliers have access to identical tier 2 suppliers. There is no tier 2 supplier that supplier A can access that supplier B cannot; hence, there is no compelling reason to dual source in tier 1. In particular, if there are economies of scale with tier 1 suppliers or other reasons to value single sourcing, the manufacturer may wish to adopt a sourcing strategy employing just one tier 1 supplier. Moreover, when the unreliable tier 2 supplier is shared, inventory mitigation—sourcing more from tier 1 than total demand D —is also not valuable, since this strategy is only beneficial if it is possible that one tier 1 supplier delivers while the other does not. Hence, the manufacturer never needs to adopt direct disruption mitigation efforts in a diamond-shaped supply chain at low unit revenues.⁵ At high unit revenues, however, the manufacturer does find dual sourcing to be optimal, as it provides a way to lower the average unit procurement cost when supplier mitigation is also employed.

Figure 4. The Five Distinct Structures That the Manufacturer Can Induce in a Diamond-Shaped Supply Chain



Note. R, reliable; U, unreliable.

Our final result in this section considers the loss in profit that the manufacturer suffers from failing to use supplier mitigation.

Corollary 2. Let $L(X)$ be as defined in Corollary 1. In a diamond-shaped supply chain with shared unreliable tier 2 supplier j and under severe disruptions, the percentage profit loss from employing single sourcing is $L(SS) \leq \lambda_j(1 - K/D)$.

As with the case of a V-shaped supply chain, the profit loss depends on the likelihood as well as the severity of the disruption. Continuing with the same example as before, if there is a 10% chance of disruption, the worst-case profit loss from ignoring supplier mitigation is 10% of the optimal profit. However, unlike the V-shaped supply chain case, with a diamond-shaped supply chain, there is no intermediate strategy that performs well, i.e., achieving the $\lambda_1\lambda_2$ profit loss that dual sourcing and inventory mitigation would yield with independent suppliers. This once again illustrates the critical importance of inducing tier 1 suppliers to mitigate disruption when there is overlap in tier 2.

6. Comparing Supply Chain Configurations

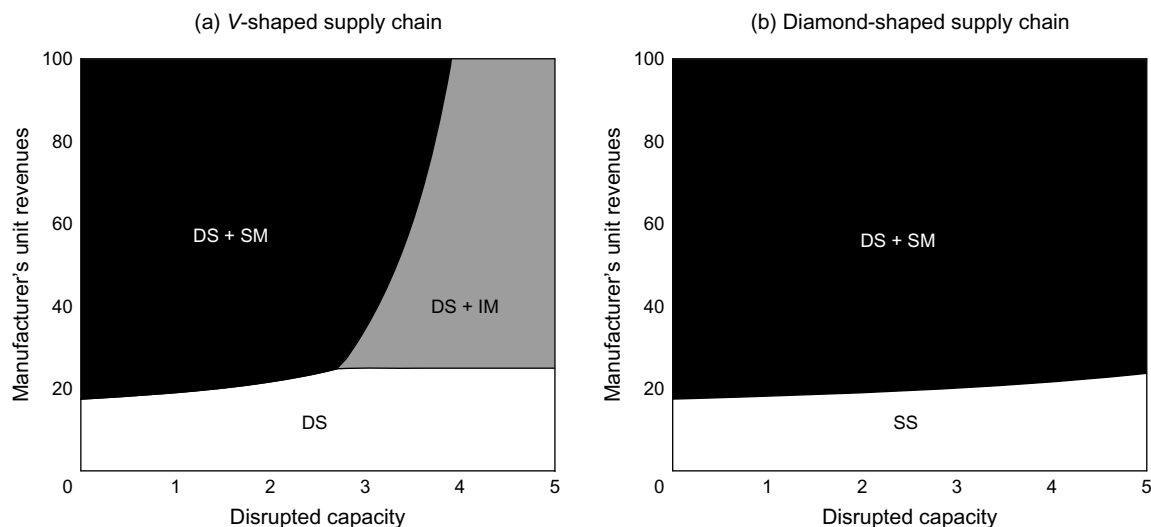
We now compare the V-shaped and diamond-shaped configurations to see how the manufacturer is impacted by overlap in the upper tiers of the network.⁶ First, we note that by comparing Theorems 3 and 6, it can be seen that increased overlap in tier 2 decreases the manufacturer's reliance on direct mitigation efforts (dual sourcing and inventory mitigation). In addition, we have the following result.

Corollary 3. If $\lambda_1 = \lambda_2$, the manufacturer finds supplier mitigation optimal at lower unit revenues in a diamond-shaped supply chain than in a V-shaped supply chain.

In other words, if $\lambda_1 = \lambda_2$, in a diamond-shaped supply chain, the manufacturer should rely more on supplier mitigation and less on manufacturer mitigation than in a V-shaped supply chain. Returning to our motivating example of an unexpectedly high degree of overlap in the automotive supply chain discovered following the 2011 Tōhoku earthquake, this suggests that an appropriate reaction from the downstream manufacturer is to reduce the emphasis placed on its own direct mitigation efforts and instead offer higher prices to its tier 1 suppliers; these suppliers, facing higher margins and potentially greater losses in the event of a disruption, will in turn invest more in their own risk mitigation efforts, thereby lowering the overall risk to the downstream manufacturer. The presence of disruption risk and overlap in tier 2 leads the manufacturer to offer higher prices to tier 1 suppliers than would otherwise be optimal; hence, it is crucial for the manufacturer to consider risk originating throughout the supply chain, even if it is not optimal to directly mitigate that risk.

The manufacturer's optimal strategy as a function of both unit revenue and disrupted capacity is depicted graphically in Figure 5 for the case of severe disruptions. In the figure, darker shading corresponds to greater disruption risk mitigation efforts. As the figure shows, an increase in either unit revenues or the severity of disruptions leads the manufacturer to increase mitigation efforts, either directly (via inventory mitigation) or indirectly (via induced supplier mitigation). The result of Corollary 3 can be seen in panel (b) of Figure 5, which depicts the optimal

Figure 5. The Manufacturer's Optimal Sourcing Strategy as a Function of Unit Revenues (π) and Disrupted Capacity (K)



Note. In the figure, $c_u = 5$, $c_r = 5.5$, $\lambda_1 = \lambda_2 = 0.2$, and $D = 10$.

strategy in a diamond-shaped supply chain: compared with panel (a), which depicts the optimal strategy in a V-shaped supply chain, the region of DS + SM optimality is larger in panel (b), and there are no regions of DS + IM or DS optimality (in the strict sense)—SS performs equally well. Importantly, Corollary 3 assumes that the two unreliable tier 2 suppliers have identical disruption risk; i.e., $\lambda_1 = \lambda_2$. In general, the result continues to hold if differences in risk are small, but if tier 2 suppliers are sufficiently heterogeneous, a different picture may emerge.

Corollary 4. Consider the case $K = 0$, and define $\bar{\Lambda} \equiv \frac{1}{2}[\sqrt{(f-1)^2 + 4f\lambda_2^2} - f + 1]$, where $f \equiv (c_r - c_u)/(c_u\lambda_2(1 - \lambda_2))$. If $\lambda_1 < \max(\lambda_2^2, \bar{\Lambda})$, the manufacturer finds supplier mitigation optimal (i) at the lowest unit revenues in a diamond-shaped supply chain with tier 2 supplier 2 shared; (ii) at moderate unit revenues in a V-shaped supply chain; and (iii) at the highest unit revenues in a diamond-shaped supply chain with tier 2 supplier 1 shared. If $\lambda_1 > \max(\lambda_2^2, \bar{\Lambda})$, the order of (ii) and (iii) is reversed. Furthermore, these results continue to hold provided disruptions are sufficiently severe (i.e., K is sufficiently small).

The key intuition for this result is that the manufacturer always pays a lower price for supplier mitigation when the more risky tier 2 supplier is in the supply chain. Interestingly, however, a diamond-shaped supply chain with supplier 2 shared and a V-shaped supply chain are not necessarily equivalent, because the manufacturer has at its disposal an alternative strategy in a V-shaped supply chain—namely, inventory mitigation. Hence, while overlap in tier 2 makes the manufacturer value supplier mitigation more, ceteris paribus, which tier 2 supplier is shared also makes a difference. If the tier 1 suppliers share the most risky supplier, the manufacturer will engage in supplier mitigation over the widest range of unit revenues; if they share the least risky unreliable

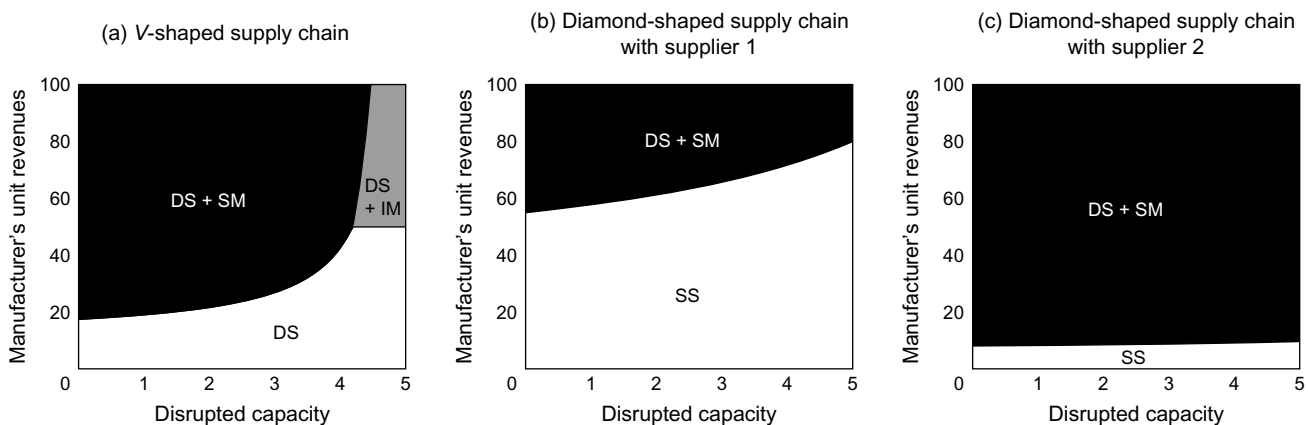
supplier, the manufacturer will engage in supplier mitigation only at very high unit revenues. The results of this corollary are depicted in Figure 6, which shows the manufacturer's optimal strategy using the same parameter values as Figure 5, except for $\lambda_1 = 0.1$ and $\lambda_2 = 0.4$.⁷ These parameter values are chosen such that the probability of two simultaneous disruptions ($\lambda_1\lambda_2$) is identical to the case in Figure 5, but in Figure 6, the individual tier 2 suppliers have different disruption probabilities; hence, the suppliers are “more heterogeneous” in the latter example.⁸

In the next result, we say that the manufacturer is “more likely to use supplier mitigation” if the parameter region under which supplier mitigation is optimal grows as a parameter changes.

Corollary 5. If $K < D/2$, in any supply chain configuration, the manufacturer is more likely to use supplier mitigation as unit revenues increase (π increases) and disruptions become more severe (K decreases), and in a V-shaped supply chain, as tier 2 suppliers become more heterogeneous (λ_1 decreases and λ_2 increases, holding the probability of simultaneous disruptions, $\lambda_1\lambda_2$, constant).

The first part of the corollary can be seen from the manufacturer's optimal strategy in Theorems 3 and 6. The second part can be seen in Figure 6. More severe disruptions simultaneously make supplier mitigation more attractive (as the loss during a disruption is greater) and inventory mitigation less attractive (as the amount of excess inventory the manufacturer must purchase increases), and as a result, supplier mitigation is adopted at lower unit revenues as disruptions become more severe in all configurations. The impact of tier 2 heterogeneity on the manufacturer's incentive to use supplier mitigation is more nuanced. In Figure 6(a), note that compared to Figure 5(a), the manufacturer's region of DS + SM optimality is weakly larger when unreliable tier 2 suppliers are heterogeneous than when they are homogeneous. In fact, it can

Figure 6. The Manufacturer's Optimal Sourcing Strategy as a Function of Unit Revenues (π) and Disrupted Capacity (K)



Notes. In the figure, $c_u = 5$, $c_r = 5.5$, $\lambda_1 = 0.1$, $\lambda_2 = 0.4$, and $D = 10$.

be shown that if $c_r < \bar{c}_r$, the manufacturer's optimal strategy is insensitive to changes in λ_1 and λ_2 , holding their product constant (see Lemma 4 in the appendix). The reason for this is that increased tier 2 heterogeneity simultaneously increases profit when using supplier mitigation (because it results in a lower price necessary to induce supplier mitigation) and increases profit when using manufacturer dual sourcing (because it reduces the risk of a disruption at the less risky tier 2 supplier). These two effects precisely counteract one another, leading to no impact of tier 2 heterogeneity on the manufacturer's strategy if reliable suppliers are cheap. However, if $c_r > \bar{c}_r$, i.e., reliable suppliers are costly, greater heterogeneity favors supplier mitigation over inventory mitigation because of the increased risk of disruption at the riskier tier 2 supplier in the latter strategy; for a similar reason, heterogeneity also favors dual sourcing over inventory mitigation, and the sum of these effects implies that as suppliers become more heterogeneous, the manufacturer relies less on inventory mitigation and more on supplier mitigation and dual sourcing. Thus, as λ_1 decreases and λ_2 increases, holding $\lambda_1 \lambda_2$ constant, the region of DS+IM optimality in Figure 6(a) will shrink, and the regions of DS+SM and DS optimality will grow.

Finally, we compare the manufacturer's profits in the possible supply chain configurations.

Corollary 6. (i) If $0 < K < D$, manufacturer profit is strictly greatest in a V-shaped supply chain.

(ii) Among the two diamond-shaped supply chains, the manufacturer's profit is larger when the less risky tier 2 supplier is shared if and only if unit revenues are small; i.e., $\pi \leq c_u + ((c_r - c_u)/\lambda_1 \lambda_2)((D - K/2)/(D - K))$.

Part (i) shows that, while there may be other factors that influence the manufacturer's preferences between the supply chain configurations in practice—for instance, if a single tier 2 supplier can achieve greater economies of scale than two independent tier 2 suppliers, the diamond shape may be more profitable; conversely, if product quality risk is a significant concern, the V shape may be preferable—the corollary confirms that, ceteris paribus, the manufacturer does indeed prefer a V-shaped supply chain purely from a disruption risk mitigation perspective.⁹ Part (ii) confirms that, when faced with a choice between diamond-shaped supply chains, the manufacturer would prefer having the less risky supplier shared at low unit revenues and the riskier supplier shared at high revenues. The underlying reason is that having a less risky supplier is advantageous for strategies that rely more extensively on sourcing from unreliable suppliers, which are prevalent at low unit revenues. By contrast, at larger unit revenues, the presence of a riskier unreliable supplier is preferable, since this makes it cheaper for the manufacturer to induce sourcing from

the reliable supplier. While intuitive, both of these results will play a critical role in our analysis in the next section.

7. The Supply Chain Game

As the results in the previous sections show, the optimal sourcing strategy may vary substantially depending on the degree of overlap in the tier 2 supplier base. Aside from the observations regarding the behavior of the manufacturer's optimal sourcing strategy, this fact has two additional implications: first, because the manufacturer's optimal sourcing strategy influences which tier 1 suppliers are selected and at what price they are compensated, the tier 1 suppliers may prefer one supply chain configuration over the other; second, these preferences may not align with the manufacturer's preference, which is always (per Corollary 6) for a V-shaped supply chain. As a result, if tier 2 suppliers are chosen strategically by tier 1 suppliers, there is a potential for the supply chain to be uncoordinated—for the tier 1 suppliers to select tier 2 suppliers that do not maximize the manufacturer's profit—an issue that we investigate in this section. Specifically, we analyze a model in which, prior to contractual offers from the manufacturer, the tier 1 suppliers engage in a simultaneous move supplier selection game in which they choose which unreliable tier 2 supplier to establish a relationship with, determining the supply chain configuration (either V shaped or diamond shaped).¹⁰ After the preferred tier 2 suppliers have been selected, the remainder of the game proceeds as in our base model; i.e., the manufacturer offers contracts to tier 1 suppliers and induces a particular supply chain structure. This sequence of events is reflective of our motivating example of the automotive supply chain, in which tier 2 supplier selection and approval is a long-term decision, while the manufacturer's day-to-day procurement contracts are short- or medium-term decisions. In other words, having analyzed the manufacturer's optimal sourcing strategy, given a fixed supply chain configuration, in this section we consider the critical question of which supply chain—V-shaped or diamond-shaped—the tier 1 suppliers would form given the manufacturer's optimal sourcing strategy.

If the manufacturer treats tier 1 suppliers asymmetrically—i.e., by offering one a high price to induce supplier mitigation and the other a low price to induce passive acceptance—and is indifferent between which supplier receives which contract (as in the diamond-shaped supply chain; Theorem 6) we assume the manufacturer randomly selects tier 1 suppliers to fulfill each role; hence, during the supplier selection game, the ex ante expected profit of a tier 1 supplier is simply the average profit of the suppliers in tier 1. We also assume all information and parameters in the game are public knowledge; i.e., tier 1 suppliers know the

Table 2. The Supply Chain Game

Tier 1 supplier A	Tier 1 supplier B	
	Supplier 1	Supplier 2
Supplier 1	{1, 1}	{1, 2}
Supplier 2	{2, 1}	{2, 2}

manufacturer's unit revenues and demand when they choose their tier 2 suppliers. Let $\{i, j\}$ denote the supply chain configuration where tier 1 supplier A sources from unreliable tier 2 supplier i , and supplier B sources from unreliable tier 2 supplier j , $i, j = 1, 2$. If the tier 1 suppliers select the same unreliable tier 2 supplier, then the resultant supply chain configuration is diamond-shaped (either $\{1, 1\}$ or $\{2, 2\}$). If tier 2 suppliers select different unreliable tier 2 suppliers, then the supply chain is V-shaped (either $\{1, 2\}$ or $\{2, 1\}$). Consequently, the tier 1 suppliers play a two-by-two normal form game, depicted in Table 2. The pure-strategy equilibria to this game will thus determine the supply chain configuration. Our primary result in this section characterizes the equilibria to this game.

Theorem 7. Let $n = L$ if $c_r \leq \bar{c}_r$ and $n = H$ if $c_r > \bar{c}_r$. If disruptions are severe ($K < D/2$), we have the following:

- (i) If $\pi < \bar{\pi}_2$, all supply chain configurations are equilibria to the supply chain game.
- (ii) If $\bar{\pi}_2 \leq \pi < \min(\bar{\pi}_n, \bar{\pi}_1)$, then both $\{1, 1\}$ and $\{2, 2\}$ are equilibria to the supply chain game. Here, $\{1, 1\}$ is a weak Nash equilibrium while $\{2, 2\}$ is a strict Nash equilibrium.
- (iii) If $\min(\bar{\pi}_n, \bar{\pi}_1) \leq \pi < \bar{\pi}_1$, $\{2, 2\}$ is the unique Nash equilibrium to the supply chain game.
- (iv) If $\min(\bar{\pi}_n, \bar{\pi}_1) \leq \pi < \bar{\pi}_n$, then both $\{1, 1\}$ and $\{2, 2\}$ are equilibria to the supply chain game. Here, $\{1, 1\}$ is both the payoff-dominant and risk-dominant equilibrium.
- (v) If $\pi \geq \max(\bar{\pi}_n, \bar{\pi}_1)$, $\{2, 2\}$ is an equilibrium to the supply chain game, and $\{1, 1\}$ is also an equilibrium if the tier 2 supplier risk is sufficiently heterogeneous. If both equilibria exist, $\{1, 1\}$ is the payoff-dominant and, if tier 2 supplier risk is sufficiently heterogeneous, risk-dominant equilibrium.

In the theorem, note that cases (iii) and (iv) are mutually exclusive and depend on the ordering of thresholds $\bar{\pi}_n$ and $\bar{\pi}_1$ (described in detail in Lemma 3 in the appendix); in turn, these depend on the relative level of heterogeneity, cost, and disrupted capacity. Observe that unless the manufacturer's unit revenues are very small, the only equilibria to the supply chain game are the diamond-shaped supply chains. Combined with Corollary 6, this implies that for all $\pi > \bar{\pi}_2$, the tier 1 suppliers select a supply chain configuration that is strictly suboptimal for the manufacturer.¹¹ The reason for this is that, in any V-shaped supply chain, there is at most one tier 1 supplier that is employed to engage in

supplier mitigation, earning a high price from the manufacturer and positive profit; this is, per Theorem 3, the tier 1 supplier with access to the riskiest tier 2 supplier. The tier 1 supplier that is not selected for supplier mitigation—which is the supplier with access to the lower risk tier 2 supplier—is paid a low price by the manufacturer, earning zero profit. This supplier thus has incentive to deviate from his action and source instead from the tier 2 supplier with greater risk, leading to a diamond-shaped supply chain. Note that both diamond-shaped supply chains can occur in equilibrium, and in particular, the shared supply chain with the riskier tier 2 supplier ($\{2, 2\}$) is always an equilibrium for any π , because of the forces just described. This tier 2 supplier, it should be emphasized, is seemingly “dominated” by the less risky tier 2 supplier: they have the same cost, but supplier 2 has higher risk. Nevertheless, it is always an equilibrium for the tier 1 suppliers to jointly select not only the same tier 2 supplier but also the riskiest tier 2 supplier.

The theorem also shows that heterogeneity among the unreliable tier 2 suppliers worsens the coordination problem between the manufacturer and tier 1 suppliers. In particular, not only do the tier 1 suppliers select a diamond-shaped supply chain, but also—under heterogeneity—they often select the worst diamond-shaped supply chain, from the manufacturer's viewpoint. To see this, note that in case (ii), where the unique strict equilibrium is $\{2, 2\}$, the manufacturer's profit would actually be larger under the alternative diamond-shaped configuration $\{1, 1\}$ —merely a weak Nash equilibrium—when unit revenues are close to $\bar{\pi}_2$ (as per Corollary 6). Weak Nash equilibria are typically unstable, which suggests that the tier 1 suppliers are more likely to select $\{2, 2\}$, which indeed is the worst overall supply chain configuration for the manufacturer. Similarly, in case (iv) (and also in (v), under sufficient heterogeneity), $\{1, 1\}$ is the payoff and risk-dominant equilibrium; equilibrium selection approaches such as evolutionary game theory (Samuelson 1997) suggest that such equilibria tend to be reached over their dominated counterparts, implying that $\{1, 1\}$ is the more likely equilibrium to occur in practice. While this may seem beneficial to the manufacturer, in fact, it is not; at these unit revenues, in either diamond-shaped configuration, the manufacturer will induce supplier mitigation but will have to pay a higher price to achieve this in $\{1, 1\}$, yielding lower profits (see Corollary 6). Hence, in this regime, the manufacturer actually prefers $\{2, 2\}$ to $\{1, 1\}$, but the tier 1 suppliers are likely to select $\{1, 1\}$.

Taken together, these results suggest that the strategic decisions of tier 1 suppliers may have unfortunate long-term repercussions for the formation and evolution of the industry's supply base. Tier 1 suppliers have an incentive to select the same tier 2

suppliers, often the worst such configuration from the manufacturer's perspective, meaning the industry over time may gravitate toward a tier 2 supply base made up of few tier 2 suppliers, resulting in a less competitive, costlier, more risky, and more correlated tier 2 supplier base, potentially to the detriment of the manufacturer's profitability. This is consistent with several observations concerning the evolution of the Japanese automotive supply chain over the last 20 years (Sasaki 2013).

In our final result in this section, we define a *preference conflict* between the manufacturer and tier 1 suppliers to mean a scenario when the manufacturer's strictly preferred supply chain configuration (a V-shaped supply chain) is *not* an equilibrium to the supply chain game—in essence, cases (ii)–(v) of Theorem 7. Given this definition, we have the following.

Corollary 7. *A preference conflict between the manufacturer and tier 1 suppliers is more likely as unit revenues increase (π increases), disruptions become more severe (K decreases), and tier 2 suppliers become more heterogeneous (λ_1 decreases and λ_2 increases, holding $\lambda_1 \lambda_2$ constant).*

The first part of the corollary (the impact of π) can be seen directly from Theorem 7. The second part follows from the behavior of the equilibrium thresholds (in particular, $\bar{\pi}_2$) as K decreases, which is derived analytically in Lemma 4 of the appendix,¹² and illustrated graphically in Figure 7.

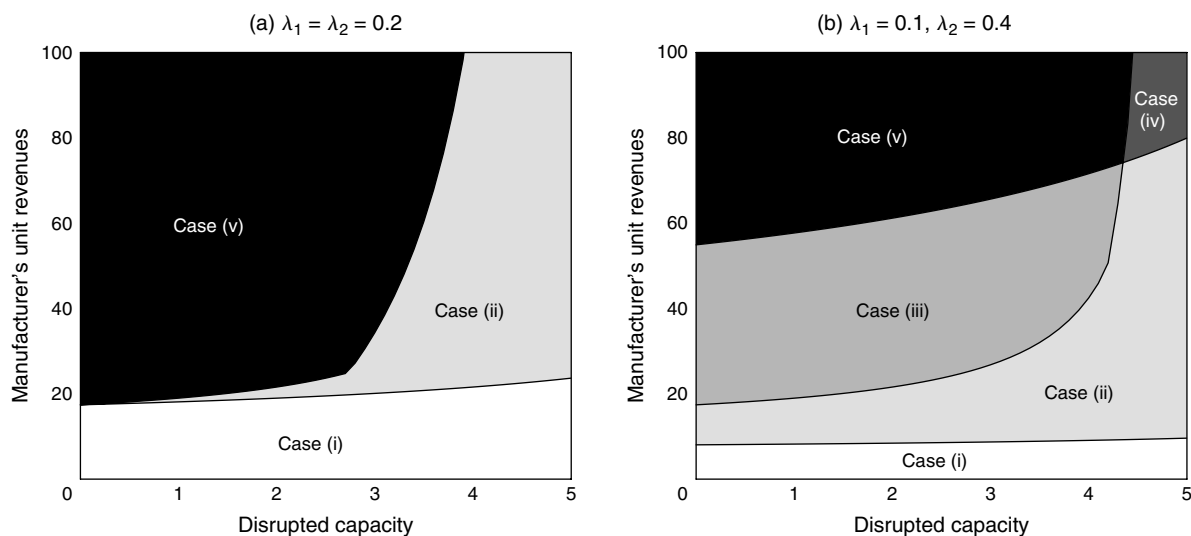
As disruptions become more severe, the size of equilibrium region for case (i) shrinks, leading to a greater likelihood of a preference conflict. The reason for this, per Corollary 5, is that the manufacturer is more likely to use supplier mitigation as K decreases, which in turn gives the tier 1 suppliers greater incentive to select

a diamond-shaped supply chain. Finally, the corollary shows that greater tier 2 heterogeneity increases the prevalence preference conflicts; this also follows from Corollary 5, since greater heterogeneity implies (in a V-shaped supply chain) that the manufacturer is more likely to use supplier mitigation, which in turn strengthens the tier 1 suppliers' incentives to deviate from a V shape and select a diamond-shaped supply chain. In Figure 7, this can be seen by noting that in panel (b), with heterogeneous tier 2 suppliers, the size of equilibrium region for case (i) is smaller than in panel (a), with homogeneous tier 2 suppliers. Hence, we conclude that preference conflicts between the manufacturer and tier 1 suppliers worsen as the manufacturer becomes more likely to induce supplier mitigation, either because unit revenues are higher, disruptions become more severe, or tier 2 suppliers are more heterogeneous.

8. Penalty Contracts

In the preceding analysis, we restricted our attention to price and quantity contracts of the form (p, Q) . While these are commonly observed in practice, they expose the manufacturer to a moral hazard problem, leading to positive agency costs when inducing supplier mitigation. Moreover, when tier 1 suppliers are capable of strategically choosing the supply chain configuration, they almost always select a diamond-shaped configuration, while the manufacturer always strictly prefers a V-shaped configuration. From these results, three questions arise: First, is it possible to overcome the moral hazard problem using a simple coordinating contract? Second, do the results derived in Sections 3–5 continue to hold qualitatively if a coordinating contract is used? And third, can this contract overcome the

Figure 7. Supply Chain Game Equilibrium Cases from Theorem 7 as a Function of Unit Revenues (π) and Disrupted Capacity (K)



Note. In the figure, $c_u = 5$, $c_r = 5.5$, and $D = 10$.

coordination problem from Section 7 when the supply chain configuration is endogenous? In this section, we demonstrate that the answer to all of these questions is yes, if the coordination mechanism used is a *penalty contract*. Specifically, we consider contracts of the form (p, Q, f) , where f is a per-unit fee for each unit ordered by the manufacturer that the contracted tier 1 supplier fails to deliver. The following theorem demonstrates that these contracts are effective both at eliminating moral hazard and at solving the preference conflict problem discussed in Section 7.

Theorem 8. *When the manufacturer uses an optimal penalty contract:*

- (i) *In either a V-shaped supply chain or a diamond-shaped supply chain, the manufacturer's optimal sourcing strategy is the same as in the price–quantity contract case, except for the fact that supplier mitigation is adopted at lower unit revenues.*
- (ii) *The manufacturer extracts all profit, and tier 1 suppliers always receive zero profit, regardless of the supply chain configuration, and hence tier 1 suppliers are indifferent between the V-shaped and diamond-shaped supply chains.*

Part (i) of Theorem 8 shows that if penalty contracts are employed, the manufacturer adopts supplier mitigation at lower unit revenues than under simple price and quantity contracts; otherwise, the manufacturer's preferences between the various mitigation strategies and induced supply chain structures are qualitatively identical to our earlier results.¹³ This increased reliance on supplier mitigation stems from the fact that penalty contracts enable the manufacturer to extract all surplus from tier 1 suppliers even when inducing them to dual source, making supplier mitigation relatively more attractive than in our earlier setting. Beyond this effect, however, all the qualitative observations made in Sections 3–6 continue to hold,¹⁴ confirming that our basic insights persist even in the absence of a moral hazard problem. There is, however, one critical difference when penalty contracts are used, as shown in part (ii) of the theorem. Since the manufacturer can extract the entire profit in all induced structures by using an appropriate penalty, tier 1 suppliers become indifferent between all supply chain configurations. In the context of the supply chain game analyzed in Section 7, this implies that a manufacturer who prefers a V-shaped supply chain can (profitably) make a small side payment to tier 1 suppliers to induce them to make this selection, thus eliminating the perverse incentives generating the moral hazard problem and effectively coordinating the supply chain.

We note that penalty contracts are commonly discussed as mechanisms to induce increased risk mitigation efforts by suppliers (Kleindorfer and Saad 2005, Yang et al. 2009, Hwang et al. 2014). However, it is important to recognize that such contracts may

face implementation challenges in practice. First, while buyers and suppliers may be willing to accept penalty contracts when disruptions arise endogenously (e.g., because of supplier malfeasance or negligence, such as from industrial accidents or poor quality control), buyers are often loathe to penalize suppliers because their supply chain experienced an exogenous natural disaster. This is especially true when buyers and suppliers have long-term relationships, as many of the firms in our example of the Japanese automotive industry do. Second, our motivating example of the 2011 Tōhoku earthquake shows precisely the opposite behavior of penalties occurred in practice: rather than punish suppliers for failing to deliver after the earthquake, Toyota took an active role in assisting them to quickly recover, providing manpower and financial assistance that subsidized the recovery efforts of the suppliers—in effect, a *payment* from the manufacturer to the suppliers in the wake of a disruption. Thus, penalties may come with a credibility problem: once a disruption has occurred, it is in the manufacturer's best interests to help suppliers quickly recover via direct financial and resource assistance (or else the manufacturer will experience even greater lost sales), meaning a punishment in the form of a financial penalty is not credible. This dynamic is not captured in our stylized single period model, but nonetheless, it suggests that penalty contracts may have limitations and might not be easily implemented in practice.

9. Conclusion

In this paper, we have analyzed the sourcing strategy of a manufacturer subject to disruptions in a multitier supply chain. In contrast to the existing disruption risk literature, we have considered the impact of risk originating in tier 2 of the supply chain, rather than tier 1. We find that the degree of overlap in tier 2 (whether unreliable tier 2 suppliers are shared by tier 1 suppliers) has a substantial impact on the manufacturer's optimal sourcing strategy, as greater overlap causes the manufacturer to rely less on direct mitigation efforts (dual sourcing and procuring excess inventory from tier 1) and more on indirect methods (inducing tier 1 suppliers to mitigate risk via contract terms). Moreover, we showed that the manufacturer and tier 1 suppliers may have conflicting preferences regarding the configuration of the supply network, potentially leading to a coordination problem in the supply chain if tier 1 suppliers strategically choose tier 2 suppliers; generally speaking, tier 1 suppliers select a diamond-shaped supply chain while the manufacturer prefers a V-shaped supply chain. In essence, the tier 1 suppliers have an interest in exploiting the manufacturer's desire to mitigate disruption risk by creating the most correlated supply chain possible; the manufacturer's own disruption risk mitigation efforts thus condemn

it to face a highly correlated supply chain configuration. The manufacturer can rectify this coordination problem by using penalty contracts to eliminate moral hazard and hence the perverse incentives that tier 1 suppliers feel to select a diamond-shaped supply chain.

We also showed that both the manufacturer's reliance on supplier mitigation and the potential for preference conflicts between the tier 1 suppliers and the manufacturer grow as the manufacturer's unit revenues increase, as disruptions become more severe, and as unreliable tier 2 suppliers become more heterogeneous. The impact of heterogeneous risk is especially intriguing, as it suggests that tier 1 suppliers are more likely to create a diamond-shaped supply chain if tier 2 suppliers are more "different."¹⁵ This phenomenon occurs because of a focusing effect generated by heterogeneity: in essence, if tier 2 suppliers were homogeneous, tier 1 suppliers would in many cases be indifferent between them, making overlap unlikely. However, as tier 2 becomes more heterogeneous, the incentives felt by tier 1 suppliers to select the same tier 2 supplier grow stronger, leading to a diamond-shaped supply chain. This indicates that manufacturers should find it most critical to confront and remedy the coordination problem precisely when unreliable tier 2 suppliers have especially heterogeneous levels of risk, e.g., if some are located in a coastal region prone to earthquakes and tsunamis while others are located in interior regions that, although still subject to geological events, are less inherently "risky" than their coastal counterparts.

There are several assumptions of our model that bear discussion. First, we focused on a single-period model rather than one in which demand (and disruptions) occur over multiple periods (Tomlin 2006). In a multi-period model, inventory mitigation—essentially, holding safety stocks—becomes a more valuable strategy, as excess inventory from one period can be held to use in subsequent periods. Nevertheless, we expect that the same sort of progression of the optimal sourcing strategy persists even over multiple periods; i.e., the manufacturer moves from direct mitigation to supplier mitigation as overlap increases. Indeed, Toyota itself is following the strategy of greater supplier mitigation following the Tōhoku earthquake, as it has worked with several key tier 1 suppliers to have them dual source and hold inventories, despite the fact that the latter strategy is a significant departure from the Toyota Production System philosophy of zero inventory (Greimel 2012). Our model thus supports the optimality of Toyota's increased reliance on supplier mitigation in the wake of the 2011 disaster.

Second, we have assumed that no emergency backup supply is available. In practice, it may be possible for tier 1 suppliers to find secondary sources of component supply if tier 2 suppliers disrupt, an extension

that we have analyzed but omit for the sake of brevity; details may be found in the supplemental appendix. As one might expect, the availability of such an option means that the manufacturer is less likely to use supplier mitigation, but otherwise, the same progression of optimal mitigation strategies holds. Interestingly, if the cost of emergency supply is sufficiently low, the manufacturer induces single sourcing from each tier 1 supplier for any unit revenues, regardless of the supply chain configuration; this implies that for components where backup supply is readily available (e.g., commodity products), the optimal manufacturer strategy is independent of the supply chain configuration. Conversely, knowing the supply chain configuration and engaging tier 1 suppliers in risk mitigation are critically important if emergency supply is costly, e.g., for specialized or custom products such as semiconductor components. This result illustrates that it is important for manufacturers to focus their limited resources on understanding and managing disruption risk in the upper tiers of their supply chain primarily for specialized, noncommodity components, while a more traditional manufacturer-only mitigation approach may be employed for more standard products with readily available backup supply.

Third, we have assumed that the manufacturer knows the configuration of the supply chain when making its optimal sourcing decision. In practice, this may not be the case—even Toyota, one of the most powerful manufacturers in the world, was unable to persuade almost half of its tier 1 suppliers to share the identities of their tier 2 suppliers. Thus, an interesting question remains unsolved by our analysis: How can a manufacturer manage disruption risk if it does not know the configuration of its supply network? Future work might explore this question, using our analysis of the optimal sourcing strategy under public information as a baseline for comparison.

Finally, we have assumed that the only mechanism by which the manufacturer can influence tier 1 sourcing decisions are purchasing contracts, specifically price-and-quantity contracts or penalty contracts. While this may be true in many cases—especially if the manufacturer has only vague knowledge about its tier 2 suppliers, such as in the case of Toyota and Boeing during the development of its 787 (Tang and Zimmerman 2009)—some manufacturers may have better awareness of their supply chains and better relationships with tier 2 and higher suppliers, allowing them to directly supervise and work with upper tiers to improve resiliency (e.g., Honda, which often contracts directly with suppliers in tiers 2 and 3; see Choi and Linton 2011). This is an intriguing aspect of the disruption risk management problem to explore in future work; in a different context, Huang et al. (2015) consider a similar trade-off between direct

manufacturer control and delegation of upper-tier supplier management.

Viewed as a whole, our results illustrate the significant impact that an extended supply chain can have on optimal sourcing and disruption mitigation efforts. While much of the existing work on disruption risk management has focused on immediate suppliers (Aydin et al. 2010), the trend toward longer and longer global supply chains has exposed firms to risk at multiple levels, necessitating an understanding of how tier 1 sourcing decisions impact risk originating at higher levels of the supply chain. Taken in that light, our model provides a first step at understanding how to manage disruption risk and optimal sourcing in multitier supply networks.

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Appendix A. Proofs

Proof of Theorem 1. For convenience, we suppress the dependence on i and j in the proof. From Equation (1), we may make several observations. First, the supplier will always source enough inventory to cover demand in the nondisrupted state; i.e., $q_u + q_r \geq Q$. Second, the supplier will never source more inventory than covers demand in the disrupted state; i.e., $q_r + \min\{K, q_u\} \leq Q$. Third, the supplier always sources at least $q_u \geq K$ units from the unreliable tier 2 supplier, as this quantity is risk free and less expensive than sourcing from the reliable tier 2 supplier. Hence, profit may be written $\Pi^S(q_u, q_r) = \lambda[p(q_r + K) - c_r q_r - c_u K] + (1 - \lambda)[pQ - c_u q_u - c_r q_r]$. Because of the linear nature of this function, it must be true that either $q_u = K$ and $q_r = Q - K$ or $q_u = Q$ and $q_r = 0$ (or some convex combination of the two) maximizes profit. Under the former strategy, profit is $\Pi^S(K, Q - K) = (p - c_r)(Q - K) + (p - c_u)K$; under the latter strategy, profit is $\Pi^S(Q, 0) = (1 - \lambda)(p - c_u)Q + \lambda(p - c_u)K$. Comparing the profit functions yields the optimal strategy in the theorem.

Proof of Theorem 2. By Lemma 1, when $K \geq D/2$, structure 3 can achieve riskless supply at a unit cost of c_u , which is the lowest cost and highest revenue outcome and is hence optimal.

Proof of Theorem 3. Observe that structure 4 dominates 2 and 5, because all three structures have perfectly reliable supply, but structure 4 achieves this at a lower average unit cost. Structure 3 dominates structure 1, since the strategy of dual sourcing outperforms single sourcing by ensuring a greater level of minimum supply. Since all profit functions are linearly increasing in π , determining the preference between

structures 3 and 4 reduces to determining the threshold π for which the structures are equivalent. First, by Lemma 1, structure 4 dominates structure 3 without inventory mitigation if

$$(\pi - c_u)D - \frac{c_r - c_u}{\lambda_2}(D - K) > (\pi - c_u)((1 - \lambda_1)D + \lambda_1 2K).$$

This reduces to $\pi > ((c_r - c_u)/\lambda_2 \lambda_1)((D - K)/(D - 2K)) + c_u \equiv \bar{\pi}_L$. In addition, structure 4 dominates structure 3 with inventory mitigation if

$$(\pi - c_u)D - \frac{c_r - c_u}{\lambda_2}(D - K) > (1 - \lambda_1 \lambda_2)(\pi - c_u)D + \lambda_1 \lambda_2 (\pi - c_u) 2K - (1 - \lambda_1)(1 - \lambda_2)c_u(D - 2K).$$

This reduces to $\pi > ((c_r - c_u)/\lambda_1 \lambda_2^2)((D - K)/(D - 2K)) - (1 - \lambda_1 - \lambda_2)c_u/\lambda_1 \lambda_2 \equiv \bar{\pi}_H$. Finally, note that within structure 3, inventory mitigation is optimal if $\pi > c_u/\lambda_1$.

Hence, there are two cases. If $c_r \leq \bar{c}_r$, then by Lemma 3(i), $c_u/\lambda_1 \geq \bar{\pi}_L \geq \bar{\pi}_H$, and structure 4 becomes optimal before inventory mitigation becomes optimal within structure 3, leading to case (i). If $c_r > \bar{c}_r$, then $c_u/\lambda_1 < \bar{\pi}_L < \bar{\pi}_H$ by Lemma 3(i), so inventory mitigation within structure 3 becomes optimal before structure 4 becomes optimal, leading to case (ii).

Proof of Corollary 1. (i) Let $\Pi^M(X)$ be the manufacturer's expected profit under strategy $X \in \{\text{DS}, \text{DS} + \text{IM}, \text{DS} + \text{SM}\}$, such that the maximum percentage profit loss from using strategy X is $L(X) = 1 - \lim_{\pi \rightarrow \infty} \Pi^M(X)/\Pi^M(\text{DS} + \text{SM})$. Then, from Lemma 1,

$$L(\text{DS}) = 1 - \lim_{\pi \rightarrow \infty} \frac{(\pi - c_u)((1 - \lambda_1)D + \lambda_1 2K)}{(\pi - c_u)D - ((c_r - c_u)/\lambda_2)(D - K)} = \lambda_1 \left(1 - \frac{2K}{D}\right).$$

(ii) When using strategy DS + IM,

$$L(\text{DS} + \text{IM}) = 1 - \lim_{\pi \rightarrow \infty} \frac{((1 - \lambda_1 \lambda_2)(\pi - c_u)D + \lambda_1 \lambda_2 (\pi - c_u) 2K - (1 - \lambda_1)(1 - \lambda_2)c_u(D - 2K)) \cdot \left((\pi - c_u)D - \frac{c_r - c_u}{\lambda_2}(D - K)\right)^{-1}}{(\pi - c_u)D - ((c_r - c_u)/\lambda_2)(D - K)} = \lambda_1 \lambda_2 \left(1 - \frac{2K}{D}\right),$$

which completes the proof.

Proof of Theorem 4. With a uniform allocation rule, supplier i 's allocation in the event of a disruption from the shared unreliable tier 2 supplier is $\min\{q_u^i, K/2 + (K/2 - q_u^i)^+\}$. This implies that supplier i 's profit is

$$\begin{aligned} \pi_i(q_u^i, q_r^i) &= \lambda_j \left[p_i \min \left\{ Q_i, q_r^i + \min \left\{ q_u^i, \frac{K}{2} + \left(\frac{K}{2} - q_u^i \right)^+ \right\} \right\} \right. \\ &\quad \left. - c_u \min \left\{ q_u^i, \frac{K}{2} + \left(\frac{K}{2} - q_u^i \right)^+ \right\} \right] \\ &\quad + (1 - \lambda_j) [p_i \min(Q_i, q_r^i + q_u^i) - c_u q_u^i] - c_r q_r^i. \end{aligned}$$

The following must be true: $q_r^i + q_u^i \geq Q_i$, $q_u^i \geq K/2 + (K/2 - q_u^i)^+ \equiv \hat{K}_i$, and $q_r^i + \min\{q_u^i, K/2 + (K/2 - q_u^i)^+\} \leq Q_i$. Using these facts, profit is

$$\pi_i(q_u^i, q_r^i) = \lambda_j [p_i q_r^i + p \hat{K}_i - c_u \hat{K}_i] + (1 - \lambda_j) [p_i Q_i - c_u q_u^i] - c_r q_r^i.$$

Observing that the profit function is linear in supplier i 's sourcing quantities, we have two cases: either the supplier will source $q_u^i = Q_i$ and $q_r^i = 0$, yielding profit $\pi_i(q_u^i, q_r^i) = \lambda_j[p_i - c_u]\hat{K}_i + (1 - \lambda_j)[p_i - c_u]Q_i$, or the supplier will source $q_u^i = \hat{K}_i$ and $q_r^i = Q_i - \hat{K}_i$, yielding profit $\pi_i(q_u^i, q_r^i) = p_i Q_i - c_u \hat{K}_i - c_r(Q_i - \hat{K}_i)$. The latter is preferred if $p_i \geq c_u + (c_r - c_u)/\lambda_j$, which is identical to the condition derived with independent tier 2 suppliers (Theorem 1) and is independent of \hat{K}_i and hence q_u^i . Consequently, the supplier has two sourcing strategies (single source or dual source), with the conditions given in the theorem.

Proof of Theorem 5. The result follows from part (iii) of Lemma 2. When disruptions are minor ($K \geq D$), the manufacturer's profit under the contract $\{(c_u, \theta D), (c_u, (1 - \theta)D)\}$ is $\Pi^M = (\pi - c_u)D$, which corresponds to the minimal possible sourcing cost.

Proof of Theorem 6. From Lemma 2, structures 1 and 3 are equivalent, and structures 2 and 5 are clearly dominated by structure 4, so the manufacturer's choice is effectively between structures 1 and 3 and structure 4. The latter is preferred if $\pi \geq c_u + ((c_r - c_u)/\lambda_j^2)((D - K/2)/(D - K)) = \bar{\pi}_j$.

Proof of Corollary 2. Let $\Pi^M(X)$ be the manufacturer's expected profit under strategy $X \in \{DS, DS + SM\}$, such that the maximum percentage profit loss from using strategy DS is $L(DS) = 1 - \lim_{\pi \rightarrow \infty} \Pi^M(DS)/\Pi^M(DS + SM)$. Then, from Lemma 2,

$$L(DS) = 1 - \lim_{\pi \rightarrow \infty} \frac{\lambda_j(\pi - c_u)K + (1 - \lambda_j)(\pi - c_u)D}{(\pi - c_u - (c_r - c_u)/\lambda_j)(D - K/2) + (\pi - c_u)K/2} \\ = \lambda_j \left(1 - \frac{K}{D}\right).$$

Proof of Corollary 3. By Lemma 3(i), note that $\bar{\pi}_L > \bar{\pi}_H \Leftrightarrow c_r < \bar{c}_r$. This implies that the manufacturer prefers inducing supplier mitigation under a V-shaped supply chain if and only if $\pi > \max(\bar{\pi}_L, \bar{\pi}_H)$. To show that this occurs at higher profit margins than in a diamond-shaped supply chain, it thus suffices to show that $\max(\bar{\pi}_L, \bar{\pi}_H) \geq \bar{\pi}_1, \bar{\pi}_2$.

If $\lambda_1 = \lambda_2$, then Lemma 3(ii) implies that $\max(\bar{\pi}_L, \bar{\pi}_H) \geq \bar{\pi}_1, \bar{\pi}_2$ holds at $K = 0$, and Lemma 3(v) implies that $\bar{\pi}_H, \bar{\pi}_L$ will grow faster than $\bar{\pi}_1, \bar{\pi}_2$, so that $\max(\bar{\pi}_L, \bar{\pi}_H) \geq \bar{\pi}_1, \bar{\pi}_2$ will continue to hold for any $K \geq 0$, proving the result.

Proof of Corollary 4. Consider the case $\lambda_1 < \lambda_2$, and $K = 0$. Recall that the manufacturer prefers to induce supplier mitigation under a V-shaped supply chain if and only if $\pi > \max(\bar{\pi}_L, \bar{\pi}_H)$ (see the proof of Corollary 3). By Lemma 3(iii), two cases can arise, depending on whether $\max(\bar{\pi}_L, \bar{\pi}_H)$ is larger or smaller than $\bar{\pi}_1$. When $\bar{\pi}_2 < \max(\bar{\pi}_L, \bar{\pi}_H) < \bar{\pi}_1$ holds, the manufacturer prefers supplier mitigation (a) at the lowest unit revenues when the tier 2 supplier 2 is shared, (b) at intermediate unit revenues in a V-shaped supply chain, and (c) at the highest unit revenues when tier 2 supplier 1 is shared. When $\bar{\pi}_2 < \bar{\pi}_1 < \max(\bar{\pi}_L, \bar{\pi}_H)$ holds, the order of (b) and (c) is flipped. Since these inequality hold strictly at $K = 0$, they will continue to hold for sufficiently small $K > 0$, proving the results for sufficiently severe disruptions.

We now show that $\lambda_1 < \max(\lambda_2^2, \bar{\Lambda})$ implies $\max(\bar{\pi}_L, \bar{\pi}_H) < \bar{\pi}_1$. By Lemma 3(iii), we have that $\max(\bar{\pi}_L, \bar{\pi}_H) < \bar{\pi}_1$ holds if and only if $c_r < \bar{c}_r$. This condition holds trivially when

$\lambda_1 \leq \lambda_2^2$, since $\bar{c}_r = \infty$. Thus, let us assume $\lambda_1 > \lambda_2^2$, in which case $\bar{c}_r \equiv c_u + c_u((\lambda_1 \lambda_2(1 - \lambda_1)(1 - \lambda_2))/(\lambda_1 - \lambda_2^2))$. The equation $c_r = \bar{c}_r$ has two real solutions for λ_1 , of which the positive one is exactly

$$\bar{\Lambda} = \frac{1}{2}[\sqrt{(f - 1)^2 + 4f\lambda_2^2} - f + 1],$$

where $f \equiv (c_r - c_u)/(c_u \lambda_2(1 - \lambda_2))$. Therefore, since $\partial \bar{c}_r / \partial \lambda_1 = -(c_u \lambda_2(1 - \lambda_2)(\lambda_1^2 + \lambda_2^2 - 2\lambda_1 \lambda_2^2))/(\lambda_1 - \lambda_2^2)^2 < 0$, it can be readily seen that $c_r < \bar{c}_r$ for any $\lambda_1 < \bar{\Lambda}$. The same argument also shows that $\lambda_1 > \max(\lambda_2^2, \bar{\Lambda})$ implies $\max(\bar{\pi}_L, \bar{\pi}_H) > \bar{\pi}_1$, which completes the proof.

We note that a more general result is also possible, for any value of K . In particular, note that having the order (i)–(iii), is equivalent to having $\bar{\pi}_2 \leq \max(\bar{\pi}_L, \bar{\pi}_H) \leq \bar{\pi}_1$. By Lemma 3(vi), we always have $\bar{\pi}_2 \leq \bar{\pi}_1$ and $\bar{\pi}_2 \leq \bar{\pi}_1$. Thus, it is necessary and sufficient to have $\bar{\pi}_L \leq \bar{\pi}_1$ and $\bar{\pi}_H \leq \bar{\pi}_1$. By parts (vii) and (ix) of Lemma 3, this is equivalent to

$$\bar{\pi}_L \leq \bar{\pi}_1 \Leftrightarrow \lambda_1 \leq \frac{(2D - K)(D - 2K)}{2(D - K)^2} \lambda_2, \\ \bar{\pi}_H \leq \bar{\pi}_1 \Leftrightarrow \lambda_1 \leq \max\left(\frac{(2D - K)(D - 2K)}{2(D - K)^2} \lambda_2^2, \bar{\Lambda}\right),$$

where $\bar{\Lambda}$ is the positive root of the equation $c_r = c_u + c_u((2\lambda_1 \lambda_2(1 - \lambda_1)(1 - \lambda_2)(D - K)(D - 2K))/(2\lambda_1(D - K)^2 - \lambda_2^2(2D - K)(D - 2K)))$ in λ_1 . The proof for the latter equivalence follows a similar logic to the case $K = 0$, by arguing that the positive root is always at most λ_2 and that the right-hand side of the equation is always decreasing in λ_1 . Full details are omitted for space considerations.

Proof of Corollary 5. The behavior as a function π follows immediately from Theorems 3 and 6. To see that the likelihood of using SM increases as K decreases, note from Lemma 4 that $\partial \bar{\pi}_H / \partial K \geq \partial \bar{\pi}_{L,1} / \partial K \geq \partial \bar{\pi}_2 / \partial K \geq 0$; i.e., all the supplier mitigation thresholds are increasing in K . Moreover, $\partial \bar{c}_r / \partial K \leq 0$, which implies that as K decreases, holding all else constant, the “costly” reliable supplier threshold \bar{c}_r grows. This means it is possible to transition from the costly to the “cheap” case as K decreases, which further increases the chance of employing SM. Finally, to see that the likelihood of using SM increases as suppliers become more heterogeneous, note that, from Lemma 4, holding $\lambda_1 \lambda_2$ constant, $\partial \bar{c}_r / \partial \lambda_2 \geq 0$, $\partial \bar{\pi}_L / \partial \lambda_2 = 0$, and $\partial \bar{\pi}_H / \partial \lambda_2 \leq 0$ if $c_r \geq \bar{c}_r$. Hence, the SM thresholds in a V-shaped supply chain are (weakly) decreasing in tier 2 heterogeneity, and moreover, the costly reliable supplier threshold is increasing in tier 2 heterogeneity, confirming the result.

Proof of Corollary 6. (i) The result clearly holds for all $D/2 < K < D$. Hence, we need only show it for $0 < K < D/2$. First, we note that in either a V- or diamond-shaped supply chain, per Lemmas 1 and 2, the manufacturer chooses between structures 3 and 4. Hence, a sufficient condition for optimal manufacturer profit in a V-shaped supply chain to strictly exceed optimal profit in a diamond-shaped supply chain is for the profit *within each structure* to be strictly greater in a V-shaped supply chain. Manufacturer profit in structure 3 is strictly greater in a V-shaped supply chain if

$$(\pi - c_u)((1 - \lambda_1)D + \lambda_1 2K) > (\pi - c_u)((1 - \lambda_j)D + \lambda_j K)$$

for $j = 1, 2$. (Note that we assume the manufacturer does not use inventory mitigation in a V -shaped supply chain; allowing this option only increases the left-hand side of the above inequality.) This inequality is strict for all $K > 0$. In structure 4, manufacturer profit is greater in a V -shaped supply chain if

$$(\pi - c_u)D - \frac{c_r - c_u}{\lambda_2}(D - K) > (\pi - c_u)D - \frac{c_r - c_u}{\lambda_j}\left(D - \frac{K}{2}\right),$$

which always holds for $j = 1, 2$ and $K > 0$.

(ii) Note first that $\bar{\pi}_2 \leq \bar{\pi}_1$, by Lemma 3(vi). From Theorem 6 and Lemma 2, for $\pi \leq \bar{\pi}_2$, the manufacturer's profit when the shared tier 2 supplier is j is given by $(\pi - c_u) \cdot ((1 - \lambda_j)D + \lambda_j K)$. Since $\lambda_1 \leq \lambda_2$ and $D \geq K$, we readily see that the profit is larger when supplier 1 is shared. Similarly, for $\bar{\pi}_2 < \pi \leq \bar{\pi}_1$, the condition that profit is larger when supplier 1 is shared is equivalent to

$$(\pi - c_u)((1 - \lambda_1)D + \lambda_1 K) \geq (\pi - c_u)D - \frac{c_r - c_u}{\lambda_2}\left(D - \frac{K}{2}\right) \\ \Leftrightarrow \pi \leq c_u + \frac{(c_r - c_u)(D - K/2)}{\lambda_1 \lambda_2 (D - K)}.$$

It can be readily checked that this threshold is always greater than $\bar{\pi}_2$ and always smaller than $\bar{\pi}_1$, provided that $\lambda_1 \leq \lambda_2$. Finally, when $\pi > \bar{\pi}_1$, the profit when the shared tier 2 supplier is j is given by $(\pi - c_u)D - ((c_r - c_u)/\lambda_j)(D - K/2)$, so that the configuration where supplier 2 is shared yields larger profit, as $\lambda_2 \geq \lambda_1$.

Proof of Theorem 7. From Theorems 3 and 6, there are five cases:

(i) If $\pi < \bar{\pi}_2$, under any supply chain configuration, the manufacturer will induce passive acceptance by its tier 1 suppliers. This leaves all tier 1 suppliers with zero profit; hence tier 1 suppliers are indifferent between tier 2 suppliers, and any supply chain configuration is an equilibrium.

(ii) If $\bar{\pi}_2 \leq \pi < \min(\bar{\pi}_n, \bar{\pi}_1)$, the manufacturer will induce supplier mitigation only if the tier 1 suppliers share supplier 2. Let $\Pi_2^D = ((c_r - c_u)/\lambda_2)(D - K/2) - c_u K/2 - c_r(D - K)$ be a tier 1 supplier's profit in a diamond-shaped supply chain when supplier mitigation is induced and the unreliable supplier is tier 2 supplier 2, and the manufacturer orders all product from that tier 1 supplier; the expected payoff is thus $\Pi_2^D/2$. Then the payoffs to the supply chain game are $(0, 0)$ in all configurations except for $\{2, 2\}$, where they are $(\Pi_2^D/2, \Pi_2^D/2)$. Consequently, there are two equilibria: either both tier 1 suppliers select supplier 1 or both select supplier 2. The former is a weak Nash equilibrium, since tier 1 suppliers are indifferent between their actions, while the latter is a strict Nash equilibrium.

(iii) If $\min(\bar{\pi}_n, \bar{\pi}_1) \leq \pi < \bar{\pi}_1$, the manufacturer will induce supplier mitigation if the tier 1 suppliers share unreliable tier 2 supplier 2 or if they select different tier 1 suppliers; otherwise, the manufacturer induces passive acceptance. Let $\Pi_2^V = ((c_r - c_u)/\lambda_2)(D - K) - c_u K - c_r(D - 2K)$ be a tier 1 supplier's profit in a V -shaped supply chain when supplier mitigation is induced. Thus, the payoffs to the supply chain game are $(0, 0)$ in $\{1, 1\}$, $(\Pi_2^V, 0)$ in $\{2, 1\}$, $(0, \Pi_2^V)$ in $\{1, 2\}$, and $(\Pi_2^D/2, \Pi_2^D/2)$ in $\{2, 2\}$. Consequently, the unique Nash equilibrium to this game is for both tier 1 suppliers to select the dominated tier 2 supplier (supplier 2).

(iv) If $\min(\bar{\pi}_n, \bar{\pi}_1) \leq \pi < \bar{\pi}_n$, the manufacturer will induce supplier mitigation in any diamond-shaped supply chain but not in a V -shaped supply chain. Let $\Pi_1^D = ((c_r - c_u)/\lambda_1) \cdot (D - K/2) - c_u K/2 - c_r(D - K)$ be the tier 1 supplier's profit when engaged in supplier mitigation in a diamond-shaped supply chain and sourcing from an unreliable tier 2 supplier 1. Note that $\Pi_1^D \geq \Pi_2^D$. Thus the payoffs to the supply chain game are $(\Pi_1^D/2, \Pi_1^D/2)$ in $\{1, 1\}$, $(0, 0)$ in $\{2, 1\}$ and $\{1, 2\}$, and $(\Pi_2^D/2, \Pi_2^D/2)$ in $\{2, 2\}$. Consequently, either diamond-shaped supply chain is an equilibrium. Clearly, $\{1, 1\}$ is payoff dominant; to see that $\{1, 1\}$ is risk dominant, note that risk dominance holds (Samuelson 1997) if $(0 - \Pi_1^D/2)(0 - \Pi_1^D/2) \geq (0 - \Pi_2^D/2)(0 - \Pi_2^D/2)$, which holds for any $\lambda_1 \leq \lambda_2$.

(v) If $\pi \geq \max(\bar{\pi}_n, \bar{\pi}_1)$, the manufacturer will induce supplier mitigation under all supply chain configurations. The payoffs to the supply chain game are $(\Pi_1^D/2, \Pi_1^D/2)$ in $\{1, 1\}$, $(\Pi_2^V, 0)$ in $\{2, 1\}$, $(0, \Pi_2^V)$ in $\{1, 2\}$, and $(\Pi_2^D/2, \Pi_2^D/2)$ in $\{2, 2\}$. Here, there is a distinction between the cases, depending on whether $\Pi_1^D/2 > \Pi_2^V$ holds.

(v-a) If $\Pi_1^D/2 > \Pi_2^V$, there are two Nash equilibria to the game: $\{1, 1\}$ and $\{2, 2\}$. As in case (iv), $\{1, 1\}$ is payoff dominant; it is risk dominant if and only if $(\Pi_2^V - \Pi_1^D/2)^2 \geq (\Pi_2^D)^2/4$. Both of these conditions are met if, all else being equal, λ_2 is sufficiently high relative to λ_1 .

(v-b) If $\Pi_1^D/2 < \Pi_2^V$, there is a unique Nash equilibrium to the game: $\{2, 2\}$.

Proof of Corollary 7. From Theorem 7, a preference conflict exists in every case except case (i). Thus, to show the result, we show that case (i) is less likely in each instance. This immediately follows for increases in π . For decreases in K , note that $\bar{\pi}_2$ increases in K per part (iii) of Lemma 4, proving the result. For increased heterogeneity, per part (iv) of Lemma 4, $\bar{\pi}_2$ is decreasing in λ_2 , holding the product $\lambda_1 \lambda_2$ constant, which proves the result.

Proof of Theorem 8. (i) We show the result only for the case of a V -shaped supply chain; the proof with a diamond-shaped supply chain is similar and is omitted. With a contract (p, Q, f) , the profit of tier 1 supplier i sourcing from tier 2 supplier j is

$$\Pi^i(q_u^i, q_r^i) \\ = \lambda_j [(p_i + f) \min\{q_r^i + \min\{K, q_u^i\}, Q_i\} - c_r q_r^i - c_u \min\{K, q_u^i\}] \\ + (1 - \lambda_j) [(p_i + f) \min\{q_u^i + q_r^i, Q_i\} - c_u q_u^i - c_r q_r^i] - f Q_i.$$

The last term is a constant; the rest of the expression is identical to the (p, Q) contract case with p replaced by $p + f$. Hence, the tier 1 supplier's optimal sourcing strategy is

$$(q_u^i, q_r^i)^* = \begin{cases} (Q_i, 0) & \text{if } c_u \leq p_i + f < c_u + \frac{c_r - c_u}{\lambda_j}, \\ (K, Q_i - K) & \text{if } p_i + f \geq c_u + \frac{c_r - c_u}{\lambda_j}. \end{cases}$$

In the latter case, tier 1 supplier profit reduces to $\Pi^i(q_u^i, q_r^i) = p_i Q_i - c_r(Q_i - K) - c_u K$. Consequently, the manufacturer has two strategies: it can induce passive acceptance by a tier 1 supplier, by offering a contract price $p_i = c_u$ and no penalty ($f = 0$), or it can induce dual sourcing, by offering a contract price plus penalty $(p_i + f)$ equal to $c_u + (c_r - c_u)/\lambda_j$.

To ensure participation of the tier 1 supplier, the manufacturer must also ensure nonnegative expected profit, meaning $p_i \geq c_u + (1 - K/Q_i)(c_r - c_u)$. The optimal price is clearly the lowest price that achieves this outcome, i.e., $p_i = c_u + (1 - K/Q_i)(c_r - c_u) < c_u + (c_r - c_u)/\lambda_j$, with a fee equal to $f = ((c_r - c_u)/\lambda_j)(1 - \lambda_j(1 - K/Q))$, such that the sum of the price and fee is $c_u + (c_r - c_u)/\lambda_j$ and the tier 1 supplier earns zero profit. This implies that inducing passive acceptance is identical to the (p, Q) case, but inducing supplier mitigation is less expensive for the manufacturer than in the (p, Q) case. Hence, manufacturer profit when employing either DS or DS + IM is unchanged, but profit when employing DS + SM is increased; hence the manufacturer pursues supplier mitigation at lower unit revenues than with (p, Q) contracts but otherwise follows an identical strategy.

(ii) Because the manufacturer leaves the tier 1 suppliers with zero expected profit in any structure, the result follows.

Appendix B. Supporting Results

Lemma 1. In a V-shaped supply chain, we have the following:

(i) In structure 1, the optimal contract is $\{(c_u, D), (c_u, 0)\}$, and the optimal expected profit is $\Pi^M = (\pi - c_u)((1 - \lambda_1)D + \lambda_1 K)$.

(ii) In structure 2, the optimal contract is $\{(c_u, 0), (c_u + (c_r - c_u)/\lambda_2, D)\}$, and the optimal expected profit is $\Pi^M = (\pi - c_u - (c_r - c_u)/\lambda_2)D$.

(iii-a) In structure 3, if $K \geq D/2$, the optimal contracts are $\{(c_u, \theta D), (c_u, (1 - \theta)D)\}$ for any $\theta \in [0, 1]$ such that neither quantity exceeds K , and the optimal expected profit is $\Pi^M = (\pi - c_u)D$.

(iii-b) In structure 3, if $K < D/2$ and if $\pi < c_u/\lambda_1$, the optimal contracts are $\{(c_u, D - K), (c_u, K)\}$, and the optimal expected profit is $\Pi^M = (\pi - c_u)((1 - \lambda_1)D + \lambda_1 2K)$.

(iii-c) In structure 3, if $K < D/2$ and if $\pi > c_u/\lambda_1$, the optimal contract is $\{(c_u, D - K), (c_u, D - K)\}$, and the optimal expected profit is $\Pi^M = (1 - \lambda_1 \lambda_2)(\pi - c_u)D + \lambda_1 \lambda_2(\pi - c_u)2K - (1 - \lambda_1)(1 - \lambda_2)c_u(D - 2K)$.

(iv) In structure 4, the optimal contract is $\{(c_u, K), (c_u + (c_r - c_u)/\lambda_2, D - K)\}$, and the optimal expected profit is $\Pi^M = (\pi - c_u)D - ((c_r - c_u)/\lambda_2)(D - K)$.

(v) In structure 5, the optimal contracts are $\{(c_u + (c_r - c_u)/\lambda_1, 0), (c_u + (c_r - c_u)/\lambda_2, D)\}$, and the optimal expected profit is $\Pi^M = (\pi - c_u - (c_r - c_u)/\lambda_2)D$.

Proof of Lemma 1. Parts (i), (ii), (iv), and (v) follow immediately from Theorem 1 and Equation (2). For part (iii-a), structure 3 is induced by offering a price c_u to both tier 1 suppliers, implying that in Equation (2), $f_1 = (1 - \lambda_1)(1 - \lambda_2)$, $f_2 = \lambda_1(1 - \lambda_2)$, $f_3 = (1 - \lambda_1)\lambda_2$, and $f_4 = \lambda_1\lambda_2$. Two observations can be made to simplify Equation (2): at optimality, $Q_A + Q_B \geq D$, and $Q_i + K \leq D$ for $i = A, B$. Furthermore, observe that if $K > D/2$, the manufacturer can simply source $D/2$ from each tier 1 supplier and fully eliminate disruption risk. More generally, any sourcing strategy in which $Q_A = \theta D$, $Q_B = (1 - \theta)D$, for some $\theta \in [0, 1]$, and in which neither quantity exceeds K , results in profit equal to $(\pi - c_u)D$. If, on the other hand, $K < D/2$, it is clearly optimal for the manufacturer to source at least K from each supplier (because this is the “risk-free” quantity that each supplier delivers for certain), resulting in expected profit

$$\begin{aligned} \Pi^M(\{(c_u, Q_A), (c_u, Q_B)\}) \\ = \lambda_1 \lambda_2 (\pi - c_u) 2K + \lambda_1 (1 - \lambda_2) (\pi - c_u) (K + Q_B) \end{aligned}$$

$$\begin{aligned} + \lambda_2 (1 - \lambda_1) (\pi - c_u) (K + Q_A) \\ + (1 - \lambda_1) (1 - \lambda_2) (\pi D - c_u (Q_A + Q_B)). \end{aligned}$$

It can be checked that $\partial \Pi^M / \partial Q_i = (1 - \lambda_i)(\lambda_{-i}\pi - c_u)$ for $i = A, B$. Hence, if $\lambda_i \pi > c_u$, $i = 1, 2$, the manufacturer will source as much as possible from each supplier, satisfying the constraints $Q_A + Q_B \geq D$, $K \leq Q_A$, $K \leq Q_B$, $D - K \geq Q_A$, and $D - K \geq Q_B$. This implies $Q_A^* = Q_B^* = D - K$, resulting in the optimal expected profit given in part (iii-c) the lemma. Conversely, if $c_u/\lambda_2 < \pi < c_u/\lambda_1$, the manufacturer will source as much as possible from supplier A and as little as possible from supplier B, resulting in $Q_A^* = D - K$, $Q_B^* = K$. Finally, if $c_u/\lambda_2 > \pi$, the manufacturer wants to source as little as possible while satisfying all the constraints. This implies $Q_A^* = D - K$, $Q_B^* = K$.

Lemma 2. In a diamond-shaped supply chain with shared unreliable tier 2 supplier $j \in \{1, 2\}$, we have the following:

(i) In structure 1, the optimal contract is $\{(c_u, D), (c_u, 0)\}$ or $\{(c_u, 0), (c_u, D)\}$, and the optimal expected profit is $\Pi^M = (\pi - c_u) \cdot ((1 - \lambda_j)D + \lambda_j K)$.

(ii) In structure 2, the optimal contract is $\{(c_u, 0), (c_u + (c_r - c_u)/\lambda_j, D)\}$ or $\{(c_u + (c_r - c_u)/\lambda_j, D), (c_u, 0)\}$, and the optimal expected profit is $\Pi^M = (\pi - c_u - (c_r - c_u)/\lambda_j)D$.

(iii) In structure 3, if $D \leq K$, the optimal contracts are $\{(c_u, \theta D), (c_u, (1 - \theta)D)\}$ for any $\theta \in [0, 1]$, and the optimal expected profit is $\Pi^M = (\pi - c_u)D$. If $D > K$, the optimal contracts are $\{(c_u, \theta D), (c_u, (1 - \theta)D)\}$ for any $\theta \in [0, 1]$, and the optimal expected profit is $\Pi^M = (\pi - c_u)((1 - \lambda_j)D + \lambda_j K)$.

(iv) In structure 4, the optimal contracts are $\{(c_u + (c_r - c_u)/\lambda_j, D - K/2), (c_u, K/2)\}$ or $\{(c_u, K/2), (c_u + (c_r - c_u)/\lambda_j, D - K/2)\}$, and the optimal expected profit is $\Pi^M = (\pi - c_u)D - ((c_r - c_u)/\lambda_j) \cdot (D - K/2)$.

(v) In structure 5, the optimal contract is $\{(c_u, 0), (c_u + (c_r - c_u)/\lambda_j, D)\}$ or $\{(c_u + (c_r - c_u)/\lambda_j, D), (c_u, 0)\}$, and the optimal expected profit is $\Pi^M = (\pi - c_u - (c_r - c_u)/\lambda_j)D$.

Proof of Lemma 2. Parts (i), (ii), and (v) follow immediately. For part (iii), in structure 3, there are two cases. If $D \leq K$, the manufacturer can achieve risk-free sourcing by splitting his order between the two tier 1 suppliers in any way. Conversely, if $D > K$, under the uniform allocation rule the manufacturer can divide its sourcing quantities between tier 1 suppliers in any way and achieve expected profit

$$\begin{aligned} \Pi^M(\{(c_u, Q_A), (c_u, Q_B)\}) \\ = \lambda_j (\pi - c_u) K + (1 - \lambda_j) (\pi \min\{D, Q_A + Q_B\} - c_u (Q_A + Q_B)). \end{aligned}$$

In that case, any $Q_A + Q_B = D$ is optimal, and manufacturer profit is as given in the lemma. For part (iv), under the uniform allocation rule, to induce supplier A to source from the reliable tier 2 supplier, the manufacturer must allocate at least $K/2$ units of inventory to supplier A (see Theorem 4). If the manufacturer allocates such that $Q_i \geq K/2$ for $i = A, B$, then supplier A will source $K/2$ units from the unreliable tier 2 supplier and $Q_A - K/2$ from the reliable tier 2 supplier, while supplier B will source Q_B from the unreliable tier 2 supplier (see Theorem 4). The manufacturer's profit will be

$$\begin{aligned} \Pi^M = \lambda_j (\pi - c_u) K_s + (1 - \lambda_j) (\pi \min\{D, Q_A + Q_B\} - c_u Q_B) \\ - \left(c_u + \frac{c_r - c_u}{\lambda_j} \right) Q_A. \end{aligned}$$

It is straightforward to see that $\partial \Pi^M / \partial Q_B < 0$, and $Q_A + Q_B \leq D$ at optimality, suggesting the solution $Q_A = D - K/2$, $Q_B = K/2$. Conversely, if the manufacturer allocates $Q_B < K/2$, profit will be

$$\Pi^M = \pi D - c_u Q_B - \left(c_u + \frac{c_r - c_u}{\lambda_j} \right) (D - Q_B).$$

Since $c_u < c_u + (c_r - c_u)/\lambda_j$, the optimal allocation is clearly $Q_B = K/2$, leading to the optimal contracts and profits as stated in the lemma.

Lemma 3. The following identities hold for the thresholds \bar{c}_r , $\bar{\pi}_L$, $\bar{\pi}_H$, $\bar{\pi}_1$, and $\bar{\pi}_2$:

- (i) The following are equivalent: $c_u/\lambda_1 > \bar{\pi}_L \Leftrightarrow c_u/\lambda_1 > \bar{\pi}_H \Leftrightarrow \bar{\pi}_L > \bar{\pi}_H \Leftrightarrow c_r < \bar{c}_r$.
- (ii) If $K = 0$ and $\lambda_1 = \lambda_2$, then $\bar{\pi}_1 = \bar{\pi}_2 = \bar{\pi}_L$.
- (iii) If $K = 0$ and $\lambda_1 < \lambda_2$, then
 - (iii-a) $\bar{\pi}_H < \bar{\pi}_2 < \bar{\pi}_L < \bar{\pi}_1$ if $c_r < c_u + c_u \lambda_2 (1 - \lambda_2)$,
 - (iii-b) $\bar{\pi}_2 < \bar{\pi}_H < \bar{\pi}_L < \bar{\pi}_1$ if $c_u + c_u \lambda_2 (1 - \lambda_2) < c_r < \bar{c}_r$,
 - (iii-c) $\bar{\pi}_2 < \bar{\pi}_L < \bar{\pi}_H < \bar{\pi}_1$ if $\bar{c}_r < c_r < \bar{c}_r$, and
 - (iii-d) $\bar{\pi}_2 < \bar{\pi}_L < \bar{\pi}_1 < \bar{\pi}_H$ if $\bar{c}_r < c_r$,

where

$$\bar{c}_r = \begin{cases} c_u + c_u \frac{\lambda_1 \lambda_2 (1 - \lambda_1)(1 - \lambda_2)}{\lambda_1 - \lambda_2^2} & \text{if } \lambda_1 > \lambda_2^2, \\ \infty & \text{otherwise.} \end{cases}$$

- (iv) In the limit as $K \nearrow D/2$, $\bar{\pi}_2 \leq \bar{\pi}_1 \leq \bar{\pi}_L, \bar{\pi}_H$.
- (v) If $\lambda_2 \leq 2\lambda_1$, $0 \leq d\bar{\pi}_2/dK \leq d\bar{\pi}_1/dK \leq d\bar{\pi}_L/dK \leq d\bar{\pi}_H/dK$.
- (vi) Both $\bar{\pi}_2 \leq \bar{\pi}_1$ and $\bar{\pi}_2 \leq \bar{\pi}_L$.
- (vii) The inequality $\bar{\pi}_L \leq \bar{\pi}_1$ holds if and only if $\lambda_1 \leq ((2D - K)(D - 2K)/(2(D - K)^2))\lambda_2$.
- (viii) The inequality $\bar{\pi}_H \leq \bar{\pi}_2$ holds if and only if $c_r \leq c_u + c_u((2\lambda_2(1 - \lambda_1)(1 - \lambda_2)(D - 2K)(D - K))/(2(D - K)^2 - \lambda_1(D - 2K)(2D - K)))$.
- (ix) The inequality $\bar{\pi}_H > \bar{\pi}_1$ holds if and only if $\lambda_1 > ((2D - K)(D - 2K))/(2(D - K)^2)\lambda_2^2$ and $c_r > c_u + c_u((2\lambda_1\lambda_2 \cdot (1 - \lambda_1)(1 - \lambda_2)(D - K)(D - 2K))/(2\lambda_1(D - K)^2 - \lambda_2^2(2D - K)(D - 2K)))$.

Proof of Lemma 3. Part (i) follows by showing that each condition is exactly equivalent to $c_r < \bar{c}_r$. To prove (ii) and (iii), note first that when $K = 0$, the thresholds become $\bar{\pi}_L = c_u + (c_r - c_u)/(\lambda_1 \lambda_2)$, $\bar{\pi}_H = (c_r - c_u)/(\lambda_1 \lambda_2^2) - (1 - \lambda_1 - \lambda_2)c_u/(\lambda_1 \lambda_2)$, $\bar{\pi}_1 = c_u + ((c_r - c_u)/\lambda_1^2)$, and $\bar{\pi}_2 = c_u + ((c_r - c_u)/\lambda_2^2)$. Thus, $\bar{\pi}_1 = \bar{\pi}_2 = \bar{\pi}_L$ if $\lambda_1 = \lambda_2$, proving (ii). To prove (iii), note first that $\lambda_1 < \lambda_2$ implies $\bar{\pi}_2 < \bar{\pi}_L < \bar{\pi}_1$. Furthermore, $\bar{\pi}_H < \bar{\pi}_2$ is equivalent to

$$\frac{c_r - c_u}{\lambda_1 \lambda_2^2} - \frac{(1 - \lambda_1 - \lambda_2)c_u}{\lambda_1 \lambda_2} < c_u + \frac{c_r - c_u}{\lambda_2^2} \Leftrightarrow c_r < c_u + \lambda_2(1 - \lambda_2)c_u.$$

The latter threshold is always smaller than $\bar{c}_r \equiv c_u + \lambda_2 \cdot (1 - \lambda_1)c_u$, since $\lambda_1 < \lambda_2$. Therefore, since $\bar{\pi}_H < \bar{\pi}_L \Leftrightarrow c_r < \bar{c}_r$, by part (i), we immediately obtain the results in (ii-a) and (ii-b).

Finally, note that $\bar{\pi}_H < \bar{\pi}_1$ is equivalent to

$$\frac{c_r - c_u}{\lambda_1 \lambda_2^2} - \frac{(1 - \lambda_1 - \lambda_2)c_u}{\lambda_1 \lambda_2} < c_u + \frac{c_r - c_u}{\lambda_1^2} \Leftrightarrow (c_r - c_u)(\lambda_1 - \lambda_2^2) \leq c_u \lambda_1 \lambda_2 (1 - \lambda_1)(1 - \lambda_2),$$

which reduces to $c_r < \bar{c}_r$. Since $\bar{c}_r \geq \bar{c}_r$ always holds, this yields the results in (ii-c) and (ii-d).

To prove (iv), observe that as $K \searrow D/2$, $\bar{\pi}_L$ and $\bar{\pi}_H$ go to infinity, whereas $\bar{\pi}_1$ and $\bar{\pi}_2$ remain finite, immediately leading to the result.

To prove (v), by differentiating each threshold with respect to K , we have $d\bar{\pi}_1/dK = ((c_r - c_u)/\lambda_1^2)(D/2(D - K)^2)$, $d\bar{\pi}_2/dK = ((c_r - c_u)/\lambda_2^2)(D/2(D - K)^2)$, $d\bar{\pi}_L/dK = ((c_r - c_u)/\lambda_1 \lambda_2)(D/(D - 2K)^2)$, and $d\bar{\pi}_H/dK = ((c_r - c_u)/\lambda_1 \lambda_2^2) \cdot (D/(D - 2K)^2)$. It follows that $d\bar{\pi}_2/dK \leq d\bar{\pi}_1/dK$ and $d\bar{\pi}_L/dK \leq d\bar{\pi}_H/dK$. Observe that $D/(D - 2K)^2 > D/(2(D - K)^2)$ and $(d/dK)(D/(D - 2K)^2 - D/(2(D - K)^2)) > 0$. Hence, a sufficient condition for $d\bar{\pi}_1/dK \leq d\bar{\pi}_L/dK$ is if this inequality holds when $K = 0$. In that case, the derivatives are $d\bar{\pi}_1/dK = ((c_r - c_u)/\lambda_1^2)(1/(2D))$ and $d\bar{\pi}_L/dK = ((c_r - c_u)/(\lambda_1 \lambda_2))(1/D)$. The inequality holds if $\lambda_2 \leq 2\lambda_1$.

To see (vi), note first that $\bar{\pi}_2 \leq \bar{\pi}_1$ always holds when $\lambda_1 \leq \lambda_2$. Also, $\bar{\pi}_2 \leq \bar{\pi}_L$ is equivalent to

$$c_u + \frac{c_r - c_u}{\lambda_2^2} \frac{D - K/2}{D - K} \leq c_u + \frac{c_r - c_u}{\lambda_1 \lambda_2} \frac{D - K}{D - 2K} \Leftrightarrow \lambda_1(2D - K)(D - 2K) \leq 2\lambda_2(D - K)^2,$$

which always holds when $\lambda_1 \leq \lambda_2$.

Inequality (vii) follows since $\bar{\pi}_L \leq \bar{\pi}_1$ is equivalent to

$$c_u + \frac{c_r - c_u}{\lambda_1 \lambda_2} \frac{D - K}{D - 2K} \leq c_u + \frac{c_r - c_u}{\lambda_1^2} \frac{D - K/2}{D - K} \Leftrightarrow 2(D - K)^2 \lambda_1 \leq (2D - K)(D - 2K) \lambda_2.$$

To prove (viii), note that $\bar{\pi}_H \leq \bar{\pi}_2$ is equivalent to

$$\frac{c_r - c_u}{\lambda_1 \lambda_2^2} \frac{D - K}{D - 2K} - c_u \frac{1 - \lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \leq c_u + \frac{c_r - c_u}{\lambda_2^2} \frac{D - K/2}{D - K} \Leftrightarrow (c_r - c_u) \left[\frac{D - K}{D - 2K} - \frac{\lambda_1(2D - K)}{2(D - K)} \right] \leq c_u \lambda_2(1 - \lambda_1)(1 - \lambda_2).$$

Since the terms in parentheses multiplying $c_r - c_u$ on the left are always positive, this yields the desired inequality.

Finally, to prove (ix), note that $\bar{\pi}_H > \bar{\pi}_1$ is equivalent to

$$\frac{c_r - c_u}{\lambda_1 \lambda_2^2} \frac{D - K}{D - 2K} - c_u \frac{1 - \lambda_1 - \lambda_2}{\lambda_1 \lambda_2} > c_u + \frac{c_r - c_u}{\lambda_1^2} \frac{D - K/2}{D - K} \Leftrightarrow (c_r - c_u) \left[\frac{D - K}{(D - 2K)\lambda_2} - \frac{\lambda_2(2D - K)}{2(D - K)\lambda_1} \right] > c_u(1 - \lambda_1)(1 - \lambda_2).$$

For this to hold, the terms in parentheses multiplying $c_r - c_u$ must be positive, yielding the desired result.

Lemma 4. The following identities hold for the thresholds \bar{c}_r , $\bar{\pi}_L$, $\bar{\pi}_H$, $\bar{\pi}_1$, $\bar{\pi}_2$:

- (i) $\partial \bar{c}_r / \partial \lambda_1 \leq 0$, $\partial \bar{\pi}_L / \partial \lambda_1 \leq 0$, $\partial \bar{\pi}_1 / \partial \lambda_1 \leq 0$, $\partial \bar{\pi}_2 / \partial \lambda_1 = 0$. Also, $\partial \bar{\pi}_H / \partial \lambda_1 \leq 0$ if $c_r \geq \bar{c}_r$.
- (ii) $\partial \bar{c}_r / \partial \lambda_2 \geq 0$, $\partial \bar{\pi}_L / \partial \lambda_2 \leq 0$, $\partial \bar{\pi}_1 / \partial \lambda_2 = 0$, $\partial \bar{\pi}_2 / \partial \lambda_2 \leq 0$. Also, $\partial \bar{\pi}_H / \partial \lambda_2 \leq 0$ if $c_r \geq \bar{c}_r$.
- (iii) $\partial \bar{\pi}_H / \partial K \geq \partial \bar{\pi}_L / \partial K \geq \partial \bar{\pi}_2 / \partial K \geq 0 \geq \partial \bar{c}_r / \partial K$. Also, $\partial \bar{\pi}_L / \partial K \geq \partial \bar{\pi}_1 / \partial K$ if and only if $\lambda_1 \geq ((D - 2K)^2 / 2(D - K)^2)\lambda_2$.
- (iv) Assume the product $\lambda_1 \lambda_2$ is held constant. Then, $\partial \bar{c}_r / \partial \lambda_2 \geq 0$, $\partial \bar{\pi}_L / \partial \lambda_2 = 0$, $\partial \bar{\pi}_1 / \partial \lambda_2 \geq 0$, $\partial \bar{\pi}_2 / \partial \lambda_2 \leq 0$, and $\partial \bar{\pi}_H / \partial \lambda_2 \leq 0$ if $c_r \geq \bar{c}_r$.

Proof of Lemma 4. (i) It can be readily checked that

$$\begin{aligned}\frac{\partial \bar{c}_r}{\partial \lambda_1} &= -\frac{c_u(D-2K)\lambda_2}{D-K} \leq 0; & \frac{\partial \bar{\pi}_L}{\partial \lambda_1} &= -\frac{(c_r - c_u)(D-K)}{(D-2K)\lambda_1^2\lambda_2} \leq 0; \\ \frac{\partial \bar{\pi}_1}{\partial \lambda_1} &= \frac{-2(c_r - c_u)(D-K/2)}{\lambda_1^3(D-K)} \leq 0,\end{aligned}$$

and π_2 does not depend on λ_1 , so $\partial \bar{\pi}_2/\partial \lambda_1 = 0$. Finally, $\partial \bar{\pi}_H/\partial \lambda_1 \leq 0$ is equivalent to $c_r \geq c_u + c_u((\lambda_2(1-\lambda_2)(D-2K))/(D-K))$. Since the latter threshold is smaller than $\bar{c}_r \equiv c_u + c_u((\lambda_2(1-\lambda_1)(D-2K))/(D-K))$, the last result follows.

(ii) Similarly, one can check that

$$\begin{aligned}\frac{\partial \bar{c}_r}{\partial \lambda_2} &= \frac{c_u(D-2K)(1-\lambda_1)}{D-K} \geq 0; & \frac{\partial \bar{\pi}_L}{\partial \lambda_2} &= -\frac{(c_r - c_u)(D-K)}{(D-2K)\lambda_1\lambda_2^2} \leq 0; \\ \frac{\partial \bar{\pi}_2}{\partial \lambda_2} &= \frac{-2(c_r - c_u)(D-K/2)}{\lambda_2^3(D-K)} \leq 0,\end{aligned}$$

and π_1 does not depend on λ_2 , so $\partial \bar{\pi}_1/\partial \lambda_2 = 0$. Finally, $\partial \bar{\pi}_H/\partial \lambda_2 \leq 0$ is equivalent to $c_r \geq c_u + c_u((\lambda_2(1-\lambda_1)(D-2K))/(2(D-K)))$. Since the latter threshold is smaller than \bar{c}_r , the last result follows.

(iii) By taking derivatives with respect to K , we have

$$\begin{aligned}\frac{\partial \bar{c}_r}{\partial K} &= -\frac{c_u D(1-\lambda_1)\lambda_2}{(D-K)^2} \leq 0; & \frac{\partial \bar{\pi}_L}{\partial K} &= \frac{(c_r - c_u)D}{(D-2K)^2\lambda_1\lambda_2} \geq 0; \\ \frac{\partial \bar{\pi}_H}{\partial K} &= \frac{(c_r - c_u)D}{(D-2K)^2\lambda_1\lambda_2^2} \geq 0; & \frac{\partial \bar{\pi}_j}{\partial K} &= \frac{(c_r - c_u)D}{2(D-K)^2\lambda_j^2} \geq 0, \quad j=1,2.\end{aligned}$$

It can be readily checked that $\partial \bar{\pi}_H/\partial K \geq \partial \bar{\pi}_{L,1}/\partial K \geq \partial \bar{\pi}_2/\partial K$, and the inequality between $\partial \bar{\pi}_L/\partial K$ and $\partial \bar{\pi}_1/\partial K$ follows.

(iv) Assume $\lambda_1\lambda_2 \equiv A$ is constant. Then,

$$\begin{aligned}\frac{\partial \bar{c}_r}{\partial \lambda_2} &= \frac{c_u(D-2K)}{D-K} \geq 0; & \frac{\partial \bar{\pi}_L}{\partial \lambda_2} &= 0; \\ \frac{\partial \bar{\pi}_1}{\partial \lambda_2} &= \frac{(c_r - c_u)(2D-K)\lambda_2}{A^2(D-K)} \geq 0; \\ \frac{\partial \bar{\pi}_2}{\partial \lambda_2} &= -\frac{(c_r - c_u)(2D-K)\lambda_2}{(D-K)\lambda_2^3} \leq 0; \\ \frac{\partial \bar{\pi}_H}{\partial \lambda_2} &= -\frac{1}{A} \left[\frac{(c_r - c_u)(D-K)}{(D-2K)\lambda_2^2} + \frac{Ac_u}{\lambda_2^2} - c_u \right].\end{aligned}$$

Thus, $\partial \bar{\pi}_H/\partial \lambda_2 \leq 0$ is equivalent to $c_r \geq c_u + c_u(D-2K) \cdot (\lambda_2^2 - A)/(D-K)$. Since $\lambda_2^2 - A \equiv \lambda_2(\lambda_2 - \lambda_1) \leq \lambda_2(1-\lambda_1)$, the latter threshold is smaller than \bar{c}_r , so that $\partial \bar{\pi}_H/\partial \lambda_2 \leq 0$ if $c_r \geq \bar{c}_r$.

Endnotes

¹ Since the reliable tier 2 suppliers are perfectly reliable, it is irrelevant whether they are shared between the tier 1 suppliers, and the key differentiating feature is whether the unreliable tier 2 suppliers are shared; hence, in the figure, we suppress the reliable tier 2 supplier of each tier 1 supplier.

² Assuming deterministic demand allows us to focus solely on the element of primary interest: the role of the supply network structure in determining sourcing and contracting decisions. In addition, incorporating stochastic demands significantly increases model complexity (see, e.g., Tomlin 2006).

³ In fact, when $p_i = c_u + (c_r - c_u)/\lambda_i$, the tier 1 supplier is indifferent between single sourcing from the unreliable supplier and dual sourcing from both suppliers; in this boundary case, any convex combination of the two strategies is equivalent. Throughout the

paper, for expositional convenience, we will assume that when in any such boundary case, the sourcing strategy with the maximum amount of inventory from the reliable supplier—the “safest” sourcing strategy—is chosen. The manufacturer clearly prefers this outcome to a passive acceptance strategy, and indeed, this outcome could be achieved if the manufacturer merely raises the price an infinitesimal amount above $c_u + (c_r - c_u)/\lambda_j$.

⁴ Note that we assume the unreliable tier 2 supplier, when disrupted, has capacity K , which is the same as the disrupted capacity of each unreliable supplier in the V-shaped supply chain. In practice, this may or may not be the case. For example, if the diamond-shaped supply chain arises from consolidation of unreliable tier 2 suppliers, we might expect that the total capacity (and hence the disrupted capacity) of the shared unreliable tier 2 supplier in a diamond-shaped supply chain is larger than that of each independent unreliable tier 2 supplier in a V-shaped supply chain. Our key results extend to the case of arbitrary disrupted capacity in the diamond-shaped supply chain, provided that capacity is no more than $2K$.

⁵ If disruption can occur in tier 1, direct manufacturer mitigation—including dual sourcing and inventory mitigation—will likely have some value. Nevertheless, the qualitative impact of increased overlap in tier 2 reducing the value of manufacturer mitigation should remain even if disruption can occur in tier 1.

⁶ A key determinant of this is the relationship between the various thresholds that determine the manufacturer’s optimal sourcing strategy in the two configurations. Lemmas 3 and 4 in the appendix derive several properties of these thresholds; in what follows, we discuss the implications of these results in a series of corollaries and examples.

⁷ Note that $\lambda_1 < \lambda_2^2$, so for sufficiently severe disruptions (K small enough), the ordering described in Corollary 4 holds.

⁸ Throughout the paper, we define “more heterogeneous tier 2 suppliers” to mean the case when λ_1 decreases and λ_2 increases while their product, $\lambda_1\lambda_2$, remains constant. This has an appealing interpretation in our model because it means the probability of a simultaneous disruption at both tier 2 suppliers is unchanged, but the individual suppliers have increasingly different probabilities of disruption as λ_1 and λ_2 move away from one another. We numerically investigated other definitions of “more heterogeneous,” such as increasing λ_2 while holding λ_1 constant and decreasing λ_1 and increasing λ_2 while holding their sum constant, and we found similar qualitative insights.

⁹ If disrupted capacity is nonzero, this preference is strict; in the special case where disrupted capacity is equal to zero, the manufacturer may achieve equal profit in the V-shaped and diamond-shaped supply chains at some unit revenues but would still always *weakly* prefer the V-shaped supply chain.

¹⁰ We restrict tier 1 suppliers to selecting exactly one unreliable supplier to keep the model parsimonious and consistent with our preceding analysis. Note that, to make positive profit, a tier 1 supplier should always select at least one unreliable supplier. While the cost of qualification and vetting makes it reasonable to assume that at most one unreliable supplier is selected in most cases, our numerical experiments suggest that allowing two or more does not substantially impact our qualitative results (from the manufacturer’s perspective, it would still generate a fixed—but considerably more complex—disruption profile in tier 2, hence only significantly complicating the analysis of the optimal sourcing decisions in the various tiers).

¹¹ Indeed, even when $\pi < \bar{\pi}_2$, a preference conflict can arise, as diamond-shaped supply chains are equilibria to the supply chain game (although the V-shaped configurations are also equilibria in this case). In addition, while we do not explicitly model asymmetric information, if tier 1 suppliers have some uncertainty about the manufacturer’s unit revenues expressed, e.g., as a prior distribution

on π , if this distribution has any mass placed above the threshold $\bar{\pi}_2$, the suppliers would strictly prefer a diamond-shaped supply chain. Thus, the selection of a diamond-shaped supply chain seems plausible in practice.

¹² Lemma 4 also provides a full characterization of how each threshold, and hence each equilibrium region in Theorem 7, changes as a function of disrupted capacity and the probability of disruptions.

¹³ It is also straightforward to show that with the optimal penalty contracts, the manufacturer makes the same sourcing choices as a centralized system (i.e., a system in which the manufacturer sources directly from tier 2). For a discussion of this issue, please see the supplemental appendix of the paper.

¹⁴ For instance, the result that increased overlap in tier 2 leads the manufacturer to favor supplier mitigation more and direct manufacturer mitigation less.

¹⁵ Indeed, while we showed this result analytically for heterogeneity in tier 2 disruption risk, in the supplemental appendix we numerically demonstrate that it—and our other key results—continues to qualitatively hold when unreliable tier 2 suppliers are heterogeneous in both cost and risk.

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