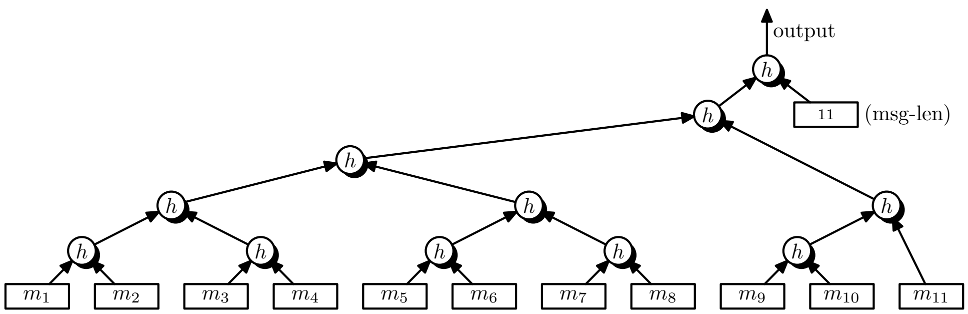
CS255: Cryptography and Computer Security **Winter 2017**

Assignment #2

Due: Tue, Feb. 21, 2017, by Gradescope (each answer on a separate page).

Problem 1. Parallel Merkle-Damgård. Recall that the Merkle-Damgård construction gives a *sequential* method for extending the domain of a CRHF. The tree construction in the figure below is a parallelizable approach: all the hash functions h within a single level can be computed in parallel. Prove that the resulting hash function defined over $(\mathcal{X}^{\leq L},\ \mathcal{X})$ is collision resistant, assuming h is collision resistant. Here h is a compression function $h : \mathcal{X}^2 \to \mathcal{X}$, and we assume the message length can be encoded as an element of $\mathcal{X}$.



More precisely, the hash function is defined as follows:

input: $m_1 \dots m_s \in \mathcal{X}^s$ for some $1 \leq s \leq L$
output: $y \in \mathcal{X}$

let $t \in \mathbb{Z}$ be the smallest power of two such that $t \geq s$ (i.e., $t := 2^{\lceil \log_2 s \rceil}$)
for $i = s+1$ to t: $m_i \leftarrow \perp$
for $i = t+1$ to $2t-1$:
  $\ell \leftarrow 2(i-t)-1,\quad r \leftarrow \ell + 1$ // *indices of left and right children*
  if $m_\ell = \perp$ and $m_r = \perp$: $m_i \leftarrow \perp$ // *if node has no children, set node to null*
  else if $m_r = \perp$: $m_i \leftarrow m_\ell$ // *if one child, propagate child as is*
  else $m_i \leftarrow h(m_\ell, m_r)$ // *if two children, hash with h*
output $y \leftarrow h\big(m_{2t-1},\ s\big)$ // *hash final output and message length*

Problem 2. In the lecture we saw that Davies-Meyer is used to convert an ideal block cipher into a collision resistant compression function. Let $E(k, m)$ be a block cipher where the message space is the same as the key space (e.g. 128-bit AES). Show that the following methods do not work:

$$f_1(x, y) = E(y, x) \oplus y \quad \text{and} \quad f_2(x, y) = E(x,\ x \oplus y)$$

That is, show an efficient algorithm for constructing collisions for f_1 and f_2. Recall that the block cipher E and the corresponding decryption algorithm D are both known to you.

Problem 3. Multicast MACs. Suppose user A wants to broadcast a message to n recipients $B_1, \ldots, B_n$. Privacy is not important but integrity is. In other words, each of $B_1, \ldots, B_n$ should be assured that the message he is receiving were sent by A. User A decides to use a MAC.

a. Suppose user A and $B_1, \ldots, B_n$ all share a secret key k. User A computes the MAC tag for every message she sends using k. Every user B_i verifies the tag using k. Using at most two sentences explain why this scheme is insecure, namely, show that user B_1 is not assured that messages he is receiving are from A.

b. Suppose user A has a set $S = \{k_1, \ldots, k_\ell\}$ of ℓ secret keys. Each user B_i has some subset $S_i \subseteq S$ of the keys. When A transmits a message she appends ℓ MAC tags to it by MACing the message with each of her ℓ keys. When user B_i receives a message he accepts it as valid only if all tags corresponding to keys in S_i are valid. Let us assume that the users $B_1, \ldots, B_n$ do not collude with each other. What property should the sets $S_1, \ldots, S_n$ satisfy so that the attack from part (a) does not apply?

c. Show that when $n = 10$ (i.e. ten recipients) it suffices to take $\ell = 5$ in part (b). Describe the sets $S_1, \ldots, S_{10} \subseteq \{k_1, \ldots, k_5\}$ you would use.

d. Show that the scheme from part (c) is completely insecure if two users are allowed to collude.

Problem 4. In lecture we saw that an attacker who intercepts a randomized counter mode encryption of the message "To:bob@gmail.com", can change the ciphertext to be an encryption of the message "To:mel@gmail.com". In this exercise we show that the same holds for randomized CBC mode encryption.

Suppose you intercept the following hex-encoded ciphertext:

54e2654a8b52038c659360ecd8638532 b365828d548b3f742504e7203be41548

You know that the ciphertext is a randomized CBC encryption using AES of the plaintext "To:bob@gmail.com", where the plaintext is encoded as ASCII bytes. The first 16-byte block is the IV and the second 16-byte block carries the message. Modify the ciphertext above so that it decrypts to the message "To:mel@gmail.com". Your answer should be the two block modified ciphertext.

Problem 5. Authenticated encryption. Let (E, D) be an encryption system that provides authenticated encryption. Here E does not take a nonce as input and therefore must be a randomized encryption algorithm. Which of the following systems provide authenticated encryption? For those that do, give a short proof. For those that do not, present an attack that either breaks CPA security or ciphertext integrity.

a. $E_1(k,m) = \big[c \leftarrow E(k,m),\ \text{output}\ \ (c,c)\ \big]$ and $D_1(k,\ (c_1,c_2)\ ) = D(k,c_1)$

b. $E_2(k,m) = \big[c \leftarrow E(k,m),\ \text{output}\ \ (c,c)\ \big]$ and $D_2(k,\ (c_1,c_2)\ ) = \begin{cases} D(k,c_1) & \text{if } c_1 = c_2 \\ \mathsf{fail} & \text{otherwise} \end{cases}$

c. $E_3(k,m) = \big(\ E(k,m),\ E(k,m)\ \big)$ and $D_3(k,\ (c_1,c_2)\ ) = \begin{cases} D(k,c_1) & \text{if } D(k,c_1) = D(k,c_2) \\ \mathsf{fail} & \text{otherwise} \end{cases}$

To clarify: $E(k,m)$ is randomized so that running it twice on the same input will result in different outputs with high probability.

d. $E_4(k,m) = \big(\ E(k,m),\ H(m)\ \big)$ and $D_4(k,\ (c_1,c_2)\ ) = \begin{cases} D(k,c_1) & \text{if } H(D(k,c_1)) = c_2 \\ \mathsf{fail} & \text{otherwise} \end{cases}$

where H is a collision resistant hash function.

Problem 6. Let (E, D) be a secure block cipher defined over $(\mathcal{K}, \mathcal{X})$ and let $(E_{\text{cbc}}, D_{\text{cbc}})$ be the cipher derived from (E, D) using randomized CBC mode. Let $H : \mathcal{X}^{\leq L} \to \mathcal{X}$ be a collision resistant hash function. Consider the following attempt at building an AE-secure cipher defined over $(\mathcal{K},\ \mathcal{X}^{\leq L}, \mathcal{X}^{\leq L+2})$:

$$E'(k,m) := E_{\text{cbc}}\big(k,\ (H(m),m)\big)\ ; \qquad D'(k,c) := \left\{ \begin{array}{l} (t,m) \leftarrow D_{\text{cbc}}(k,c) \\ \text{if } t = H(m) \text{ output } m, \text{ else } \mathsf{reject} \end{array} \right\}$$

Note that when encryting a single block message $m \in \mathcal{X}$, the output is three blocks: the IV, the CBC encryption of $H(m)$, and the CBC encryption of m. Show that (E', D') is not AE-secure by showing that it does not have ciphertext integrity. This construction was used to protect secret keys in the Android KeyStore. Your attack resulted in a compromise of the key store.

Problem 7. Let G be a finite cyclic group. Suppose the order of G is $2q$ for some odd integer q. Show that the Decision Diffie-Hellman problem does not hold in the group G.
Hint: given a tuple (g, h, u, v) try raising g, h, u, v to the power of q.