

On the Lamb Vector Divergence as a Momentum Field Diagnostic Employed in Turbulent Channel Flow

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Overview

- Kinematics
- Dynamics
- Turbulent Channel Flow DNS

Hypothesis:

“The Lamb Vector Divergence and Transport Identifies Dynamically Important Motions.”

$$\frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\omega} \times \mathbf{u} = -\nabla \left(\frac{p}{\rho} + \frac{u^2}{2} \right) - \nu \nabla \times \boldsymbol{\omega}$$

Kinematics

- Lamb Vector

$$\boldsymbol{l} = \boldsymbol{\omega} \times \boldsymbol{u} = (\nabla \times \boldsymbol{u}) \times \boldsymbol{u}$$

- Lamb Vector Divergence

$$\nabla \cdot \boldsymbol{l} = \underbrace{\boldsymbol{u} \cdot \nabla \times \boldsymbol{\omega}}_{\text{Flexion Product}} - \underbrace{\boldsymbol{\omega} \cdot \boldsymbol{\omega}}_{\text{Enstrophy}}$$

- Parity/Galilean Invariant

Kinematics

- $\nabla \cdot \mathbf{l} = \mathbf{u} \cdot \nabla \times \boldsymbol{\omega} - \boldsymbol{\omega} \cdot \boldsymbol{\omega}$
- Source/Sink Character
 - $-\boldsymbol{\omega} \cdot \boldsymbol{\omega} \leq 0$ Always (i.e. Sink)
 - $\mathbf{u} \cdot \nabla \times \boldsymbol{\omega} \geq 0$ Typically (i.e. Source)
- Positive $\nabla \cdot \mathbf{l}$ Occurs Only From $\mathbf{u} \cdot \nabla \times \boldsymbol{\omega}$
 - $\mathbf{u} \cdot \nabla \times \boldsymbol{\omega}$: High to Low Streamline Curvature.
 - Vortex “Unwinding” : Angular \rightarrow Linear Momentum

Dynamics

- $\nabla \cdot \mathbf{l} = -\nabla^2 \Phi$ where $\nabla \cdot \mathbf{u} = 0$
 - Bernoulli Function is $\Phi = p/\rho + u^2/2$
- Harmonic Interpretation
 - If $\nabla \cdot \mathbf{l} > 0$, Φ has a Local Maxima (Evacuates).
 - If $\nabla \cdot \mathbf{l} < 0$, Φ has a Local Minima (Concentrates).
- Energy Curvature (Approaches Equilibrium)
- Distinct Capacity to Alter Momentum Field

Mean Lamb Vector Divergence Decomposition

Mean Lamb Vector Divergence:

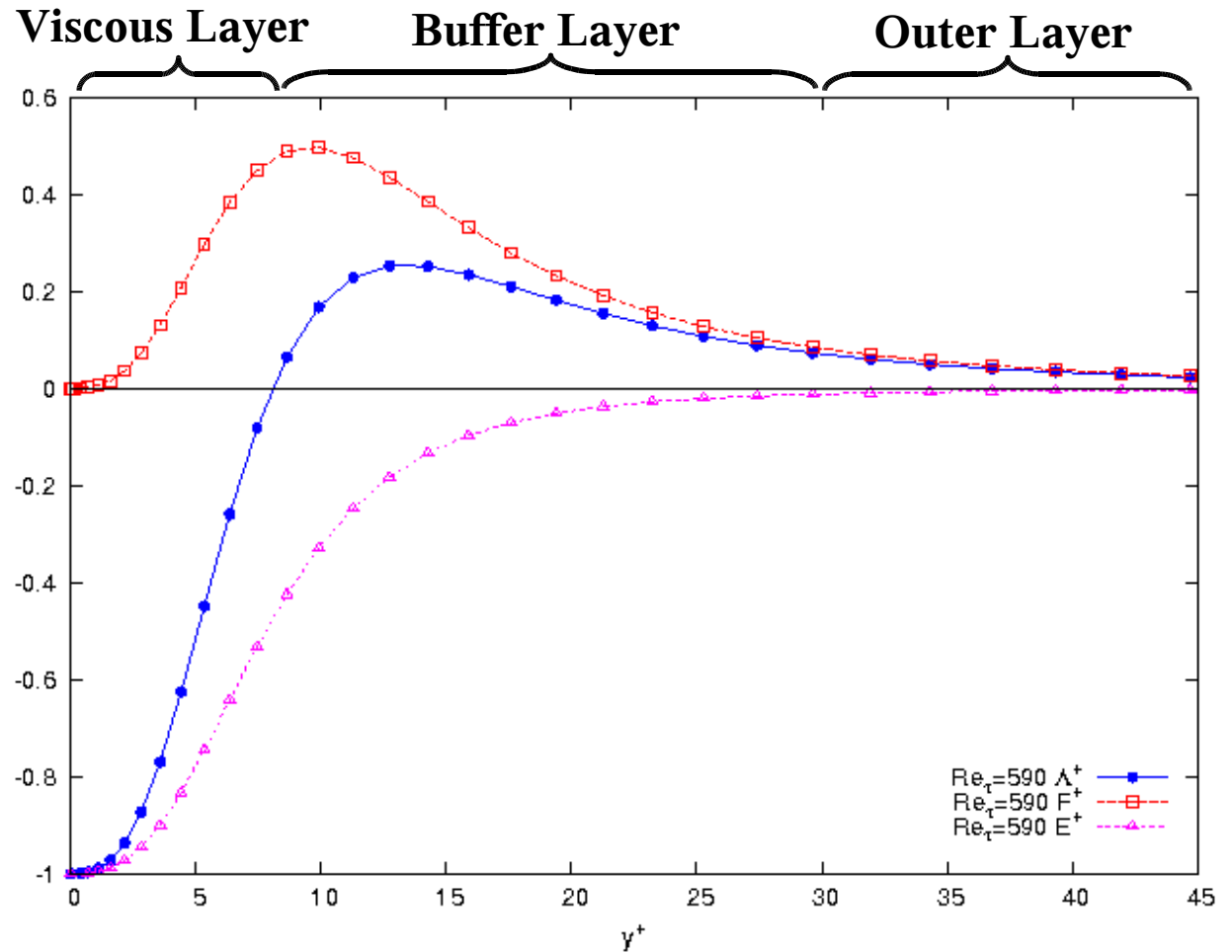
$$\Lambda^+ = F^+ + E^+$$

Mean Flexion Product:

$$F^+ = -U^+ \frac{d^2 U^+}{dy^{+2}}$$

Mean Enstrophy:

$$E^+ = - \left(\frac{dU^+}{dy^+} \right)^2$$



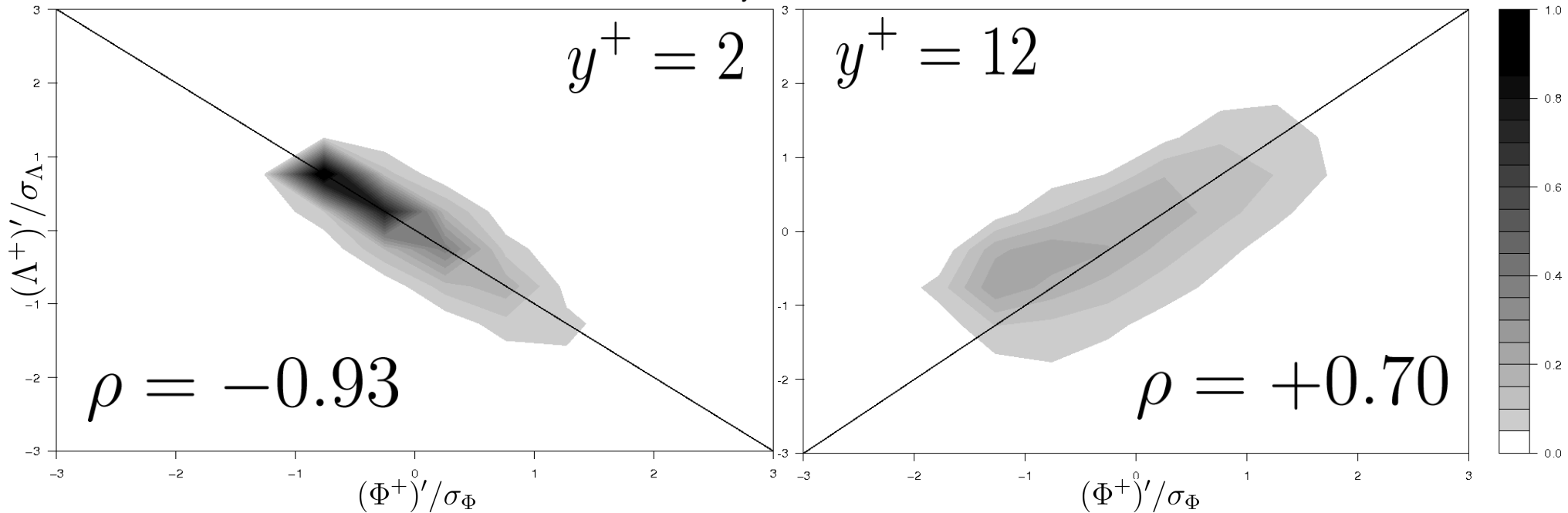
Mean Lamb Vector Divergence Components, $Re_\tau = 590$.

Data courtesy of Moser, Kim, Mansour (1999).

Correlation Between Fluctuating $\nabla \cdot \boldsymbol{l}$ and Φ

Joint PDF of Fluctuating Lamb Vector Divergence and Bernoulli Function, $\text{Re}_\tau = 180$.

Data courtesy of H.M. Blackburn.

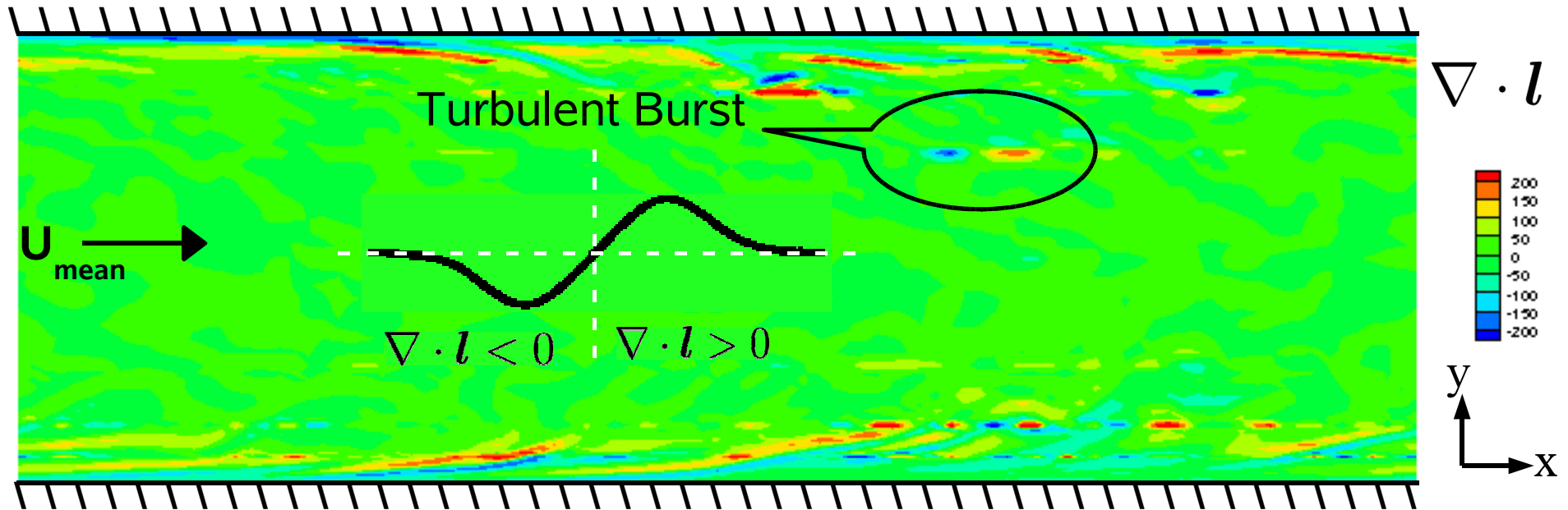


- Viscous Layer: Φ and $\nabla \cdot \boldsymbol{l}$ are Anti-correlated (Sink).
- Buffer Layer: Φ and $\nabla \cdot \boldsymbol{l}$ are Correlated (Source).

Instantaneous Lamb Vector Divergence

Instantaneous Lamb Vector Divergence Contours for a Plane Turbulent Channel Flow.

Data courtesy of H.M. Blackburn.



- Positive/Negative Character (“Unwinding”)
 - Angular ($\nabla \cdot \boldsymbol{l} < 0$) Converts to Linear ($\nabla \cdot \boldsymbol{l} > 0$) Momentum

Conclusions

- ▶ Lamb Vector Divergence:
 - ▶ Identifies Conversion of Angular to Linear Momentum
 - ▶ Interference of Dissipation and Enstrophy
 - ▶ Measures Energy Curvature (i.e. Bernoulli Function)
 - ▶ Superharmonic/Subharmonic Characterization
- ▶ $\nabla \cdot \boldsymbol{l}$ Identifies Regions Having a Distinct Capacity to Affect a Time Rate of Change of Momentum.

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Exact Lamb Vector Divergence Decomposition

Mean Lamb Vector Divergence:

$$\Lambda^+ = \overline{\nabla \cdot \mathbf{l}^+}$$

$$= F^+ + E^+ + C^+$$

Mean Flexion Product:

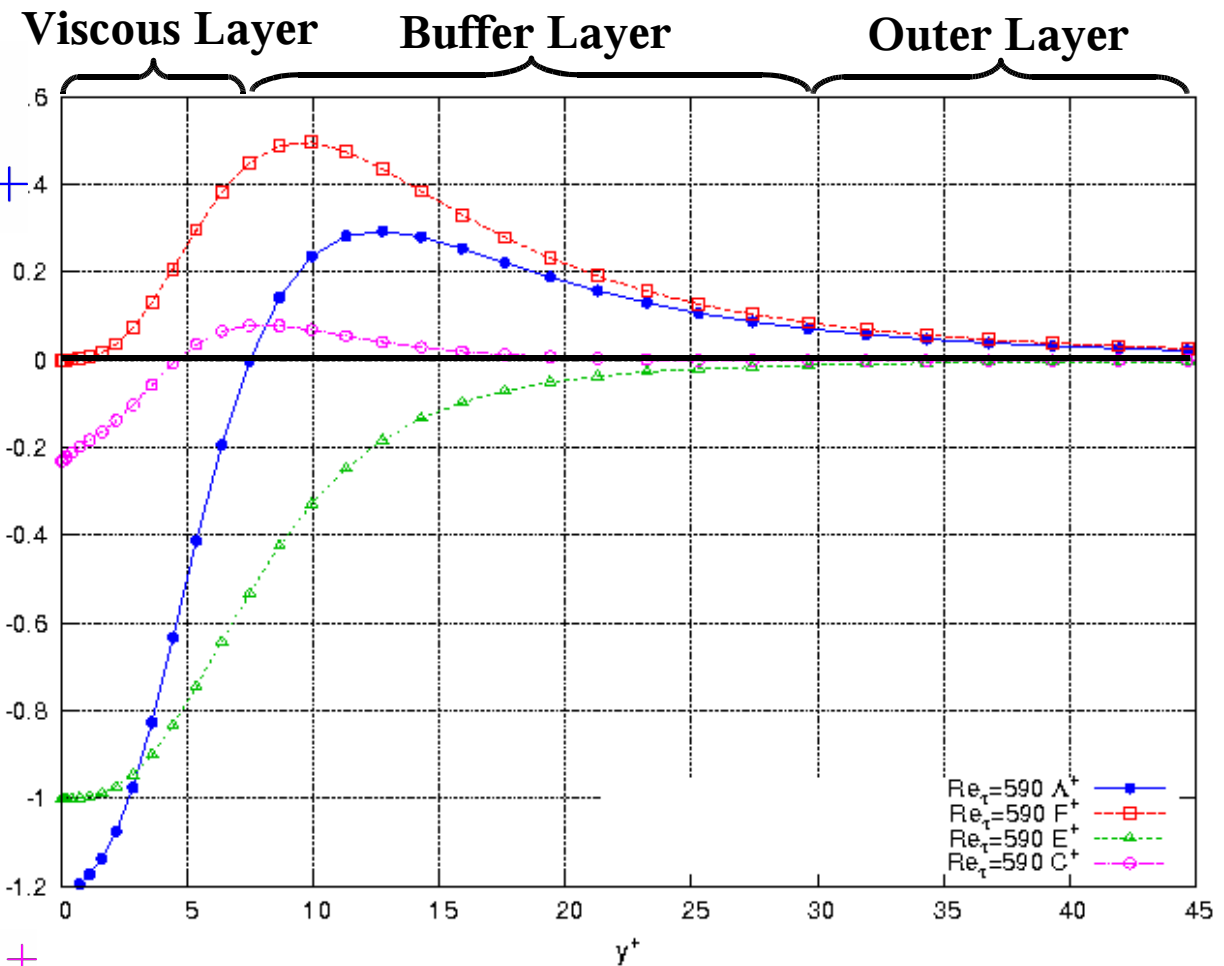
$$F^+ = -U^+ \frac{d^2 U^+}{dy^{+2}}$$

Mean Enstrophy:

$$E^+ = - \left(\frac{dU^+}{dy^+} \right)^2$$

Mean Correlation:

$$C^+ = \frac{d}{dy^+} \left(\overline{u'_x \omega'_z} - \overline{u'_z \omega'_x} \right)^+$$



Mean Lamb Vector Divergence Components, $Re_\tau = 590$.

Data courtesy of Moser, Kim, Mansour (1999).