

Problem Set #4
(due May 18, 2001)

Question 1:

Answer TRUE, FALSE, UNCERTAIN and explain your reasoning briefly.

(a) Walgreen's and Jewel Osco which basically sell the same selection of goods are often located right next to each other. Hotelling's (straight line) model can explain this empirical behavior.

(b) A monopolist can always do better by charging the consumer a two-part tariff than by charging a single price.

(c) The price of a bottle of water in the symphony center during a concert is \$3. The same bottle of water in the Jazz Showcase costs \$2. If both audiences have the same elasticity of demand, this implies that the average classical music lover values a bottle of water more than the average jazz fan going to the Jazz Showcase.

Question 2 (Price Discrimination)

A monopolist faces two groups of consumers. The first group has an inverse demand function of $P_1 = 50 - q$

the second one $P_2 = 100 - q$. Marginal cost of production is zero.

(a) If the monopolist can separate the consumer by observable characteristics (students versus general public) how should it set its prices?

(b) Redo part (a) if marginal costs are $c = 30$.

Question 3 (Price Discrimination)

Consider a market for copying machines with a single supplier ('the monopolist') that consists of equal number of two types of consumers. Type A consumers have the following monthly demand for copies: $P = 100 - q_A$. Type B consumers have the following monthly demand for

copies: $P = 100 - 2q_B$

Assume the monthly fixed cost of renting the machine are 500 and the marginal cost per copy is zero

(a) In general, what conditions must be satisfied in order for price discrimination to work?

(b) Suppose now the firm could discriminate perfectly between types A and B (use first-degree price discrimination). What is the maximum rental price the monopolist could charge A and B?

(c) If the monopolist could charge a single price only, which one would it choose? (*Hint*: Would she want to sell to both types of consumers or supply only one type?).

(d) Suppose now the monopolist can charge a two-part tariff consisting of a fixed rental fee for the machine and a marginal price per copy made. What is the optimal two-part tariff? (*Hint*: The rental fee is linked to the consumer surplus of the low-value consumer type). Calculate the profits for the monopolist and compare to (c).

Question 4 (Location and Price competition in the Hotelling model)

In class, we analyzed the two stage-game in a general way. Let us now look at a concrete solvable example. The structure of the game is as follows:

stage 1: firm A and B choose the location (a and b) along the Hotelling line (with 0 and 1 as endpoints). For convenience we assume that firm A will always locate at the point closer to zero (so it is always to the left of firm B).

stage 2: firm A and B compete in prices (p_a, p_b)

Assume that marginal costs of production are zero ($c = 0$). Consumers are located uniformly along the line (the same number of consumers is at each point along the line). Denote the location of a consumer by t where $t \in [0, 1]$. Each consumer has total valuation of v for the ideal product (a product located at t). 'Transport costs' for each consumer per unit of distance are given by d .

(a) Write down the problem of a consumer trying to decide whether to buy from firm a or b. Then solve for the location t^* of the marginal consumer who is just indifferent between the two firms.

(b) Derive the demand function for firm a and b.

(c) Set up the maximization problem for firm a in stage 2 and solve for the optimal prices: $p_a^*(a,b), p_b^*(a,b)$. Are the prices going to be equal?

(d) Plug your result from (c) into the demand function you derived in (b). Show that the direct demand effect is always positive and give a short intuition.

(e) Using the demand function from (b) and the optimal prices you derived in (c), show that the strategic effect from the lecture is negative and give an intuition for this result.

Question 5 (Bertrand competition with product differentiation)

Two firms, A and B, produce differentiated products and are engaged in Bertrand competition.

Suppose the demand function for the firm a is:

$$q_a = 24 - \theta p_a + p_b \quad \#$$

and similarly for firm b is

$$q_b = 24 - \theta p_b + p_a \quad \#$$

θ is a measure of how different the firms' products are. Marginal costs are assumed to be zero.

(a) Set up the maximization problem for firm a, derive the reaction function $p_a(p_b)$ and solve for the optimal prices and equilibrium profits.

(b) Compare your results from part (a) to the standard Bertrand case.

(c) Calculate the elasticity of demand under standard Bertrand competition and Bertrand competition with product differentiation. How does the elasticity with product differentiation depend on θ ? How would you therefore interpret θ ?