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AA283
Aircraft and Rocket Propulsion

Chapter 6 - The Turboprop Cycle

PW4077 fan front view – Denver accident

GE90 fan blade
4ft long
50 lbs

GE90 fan front view

Allison T56 developed in the 1950s

The gearbox is a complex and fairly massive component

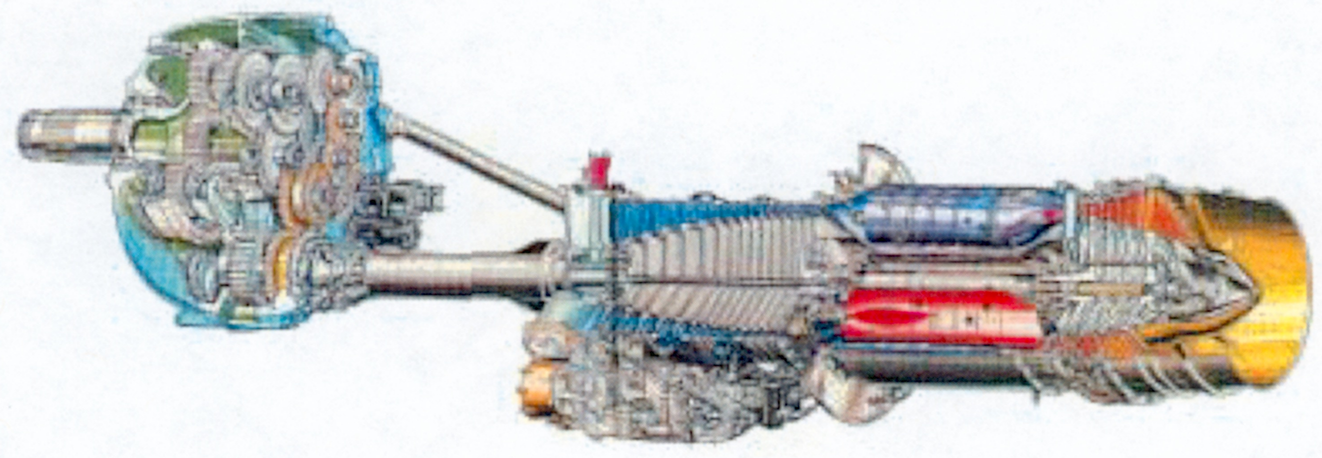
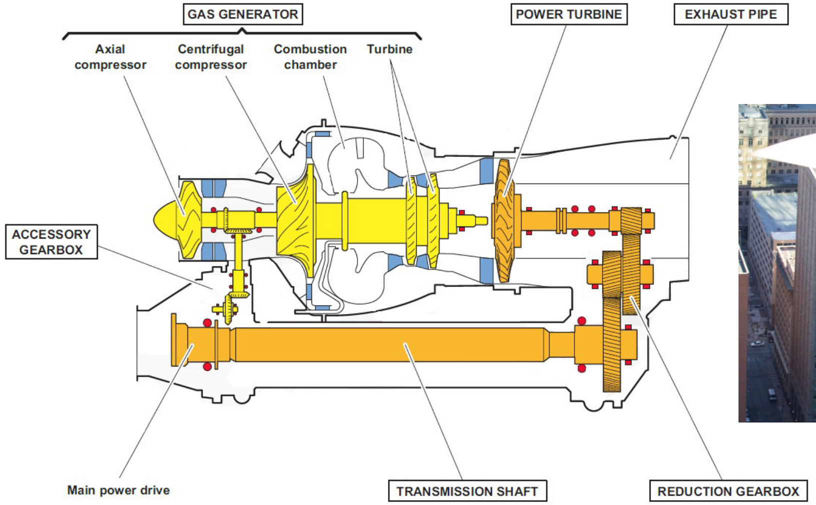


Figure 6.2 Turboprop engine with its gearbox

e2C

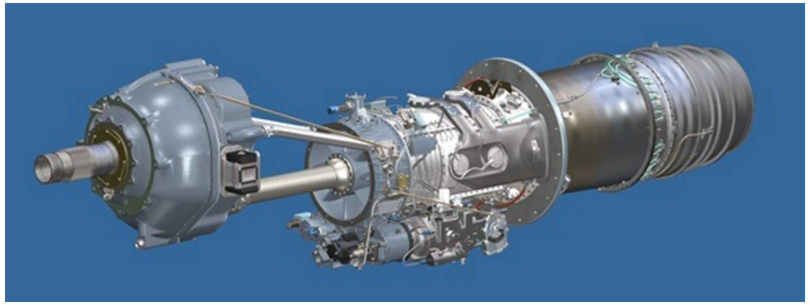
e2C+

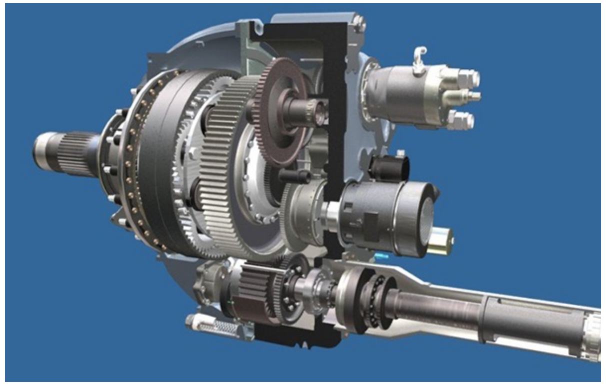
Helicopter turboshaft engine



Lockheed L-188 turboprop introduced 1957

Allison T56





Antonov 70 counter rotating turboprop

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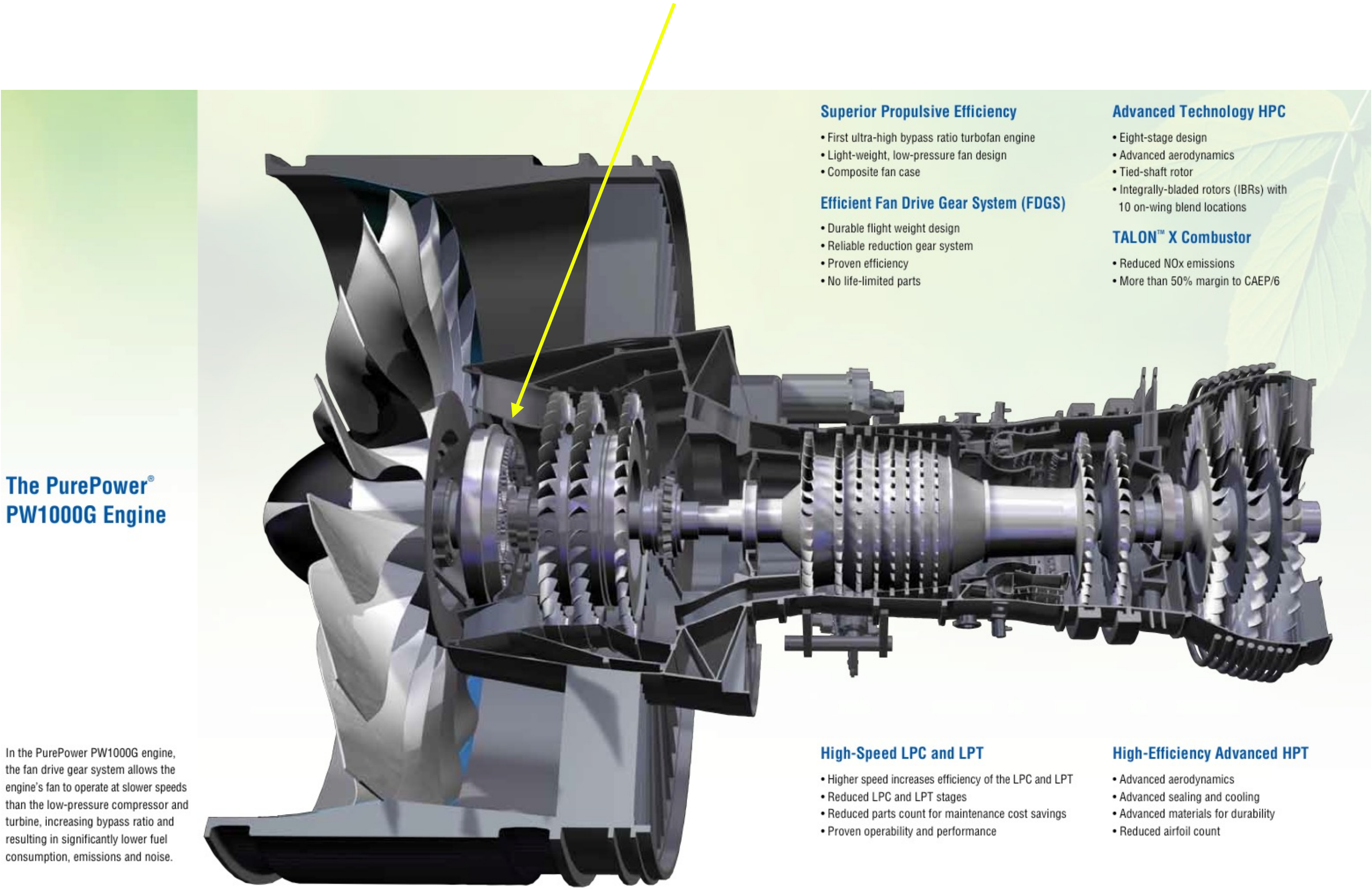
PW Geared Turbofan - note the small size of the gearbox



6.1 Propellor efficiency

Recall

$$\eta_{pr} = \frac{2}{1 + U_e/U_0} \tag{6.1}$$

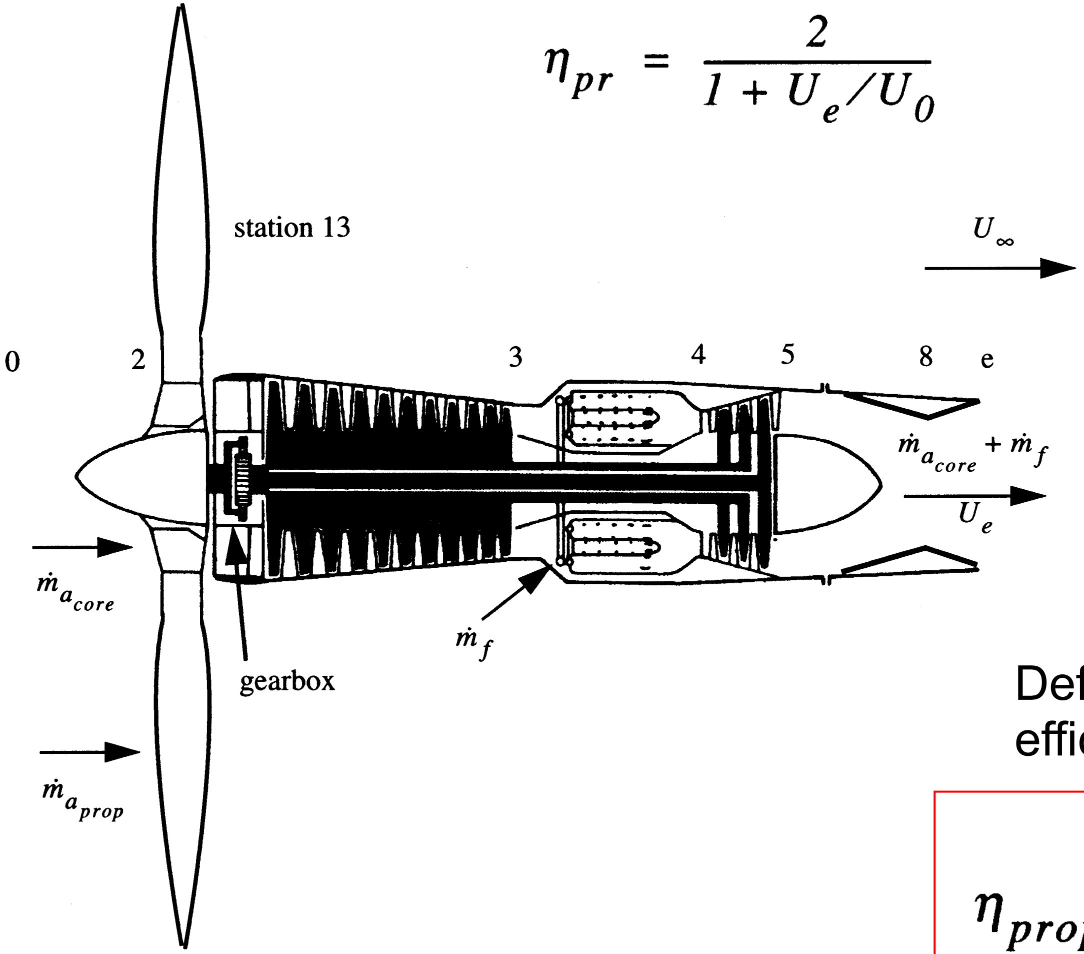
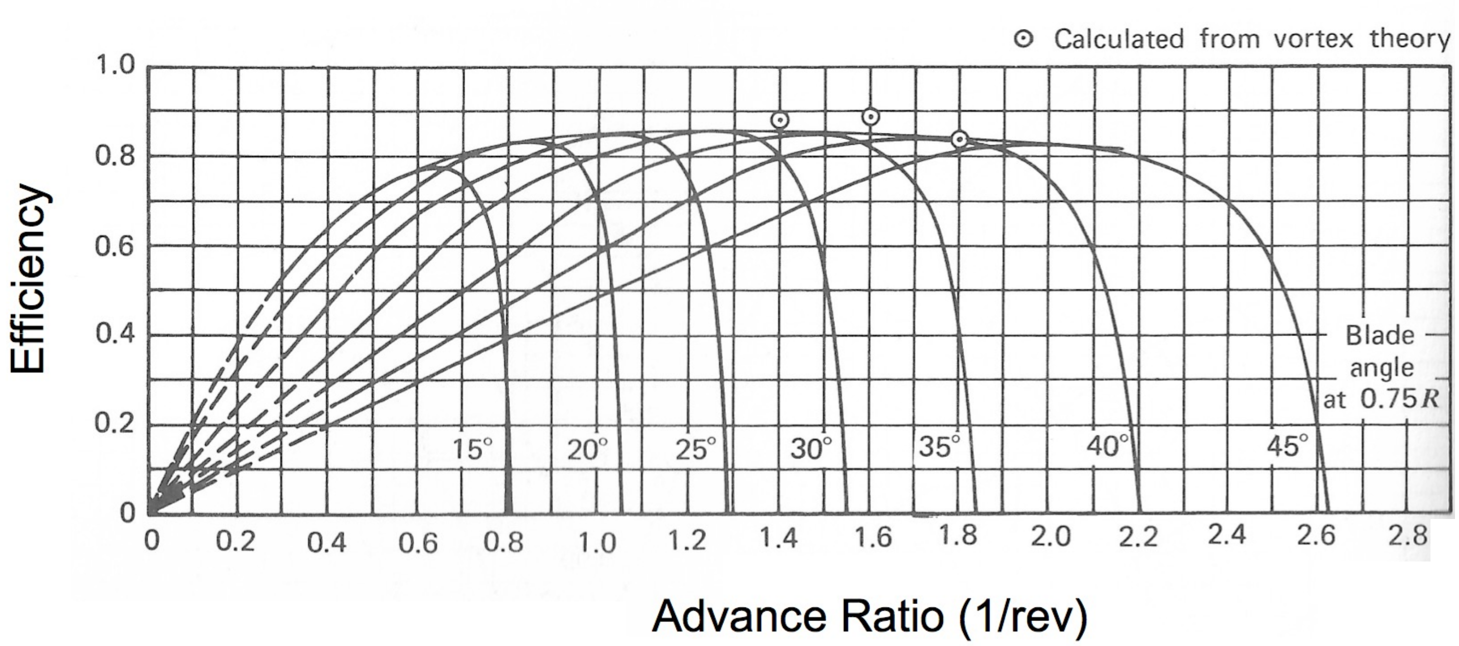


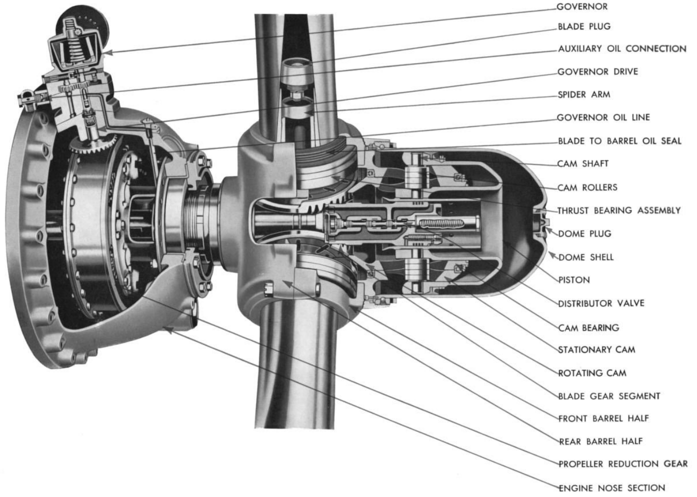
Figure 6.1 Schematic of a turboprop engine

Definition of propellor efficiency

$$\eta_{prop} = \frac{T_{prop} U_0}{W_p} \tag{6.2}$$

Effect of advance ratio on propellor efficiency





McCormick, B.W., Aerodynamics, Aeronautics, and Flight Mechanics, Wiley, 1979.

$$\text{Advance ratio} = \frac{\text{Distance in one rotation period}}{\text{Propellor diameter}}$$

Propellor thrust

$$T_{prop} = \dot{m}_{prop}(U_\infty - U_0) \tag{6.3}$$

Factor the propellor efficiency

$$\eta_{prop} = \left(\frac{\dot{m}_{prop}(U_\infty - U_0)U_0}{\frac{1}{2}\dot{m}_{prop}(U_\infty^2 - U_0^2)}\right)\left(\frac{\frac{1}{2}\dot{m}_{prop}(U_\infty^2 - U_0^2)}{W_p}\right) \tag{6.4}$$

The left factor is the propulsive efficiency

$$\eta_{prop} = \left(\frac{2U_0}{U_\infty + U_0}\right)\left(\frac{\frac{1}{2}\dot{m}_{prop}(U_\infty^2 - U_0^2)}{W_p}\right) \tag{6.5}$$

Thermodynamic interpretation of propellor efficiency. Treat the propellor as an actuator disc

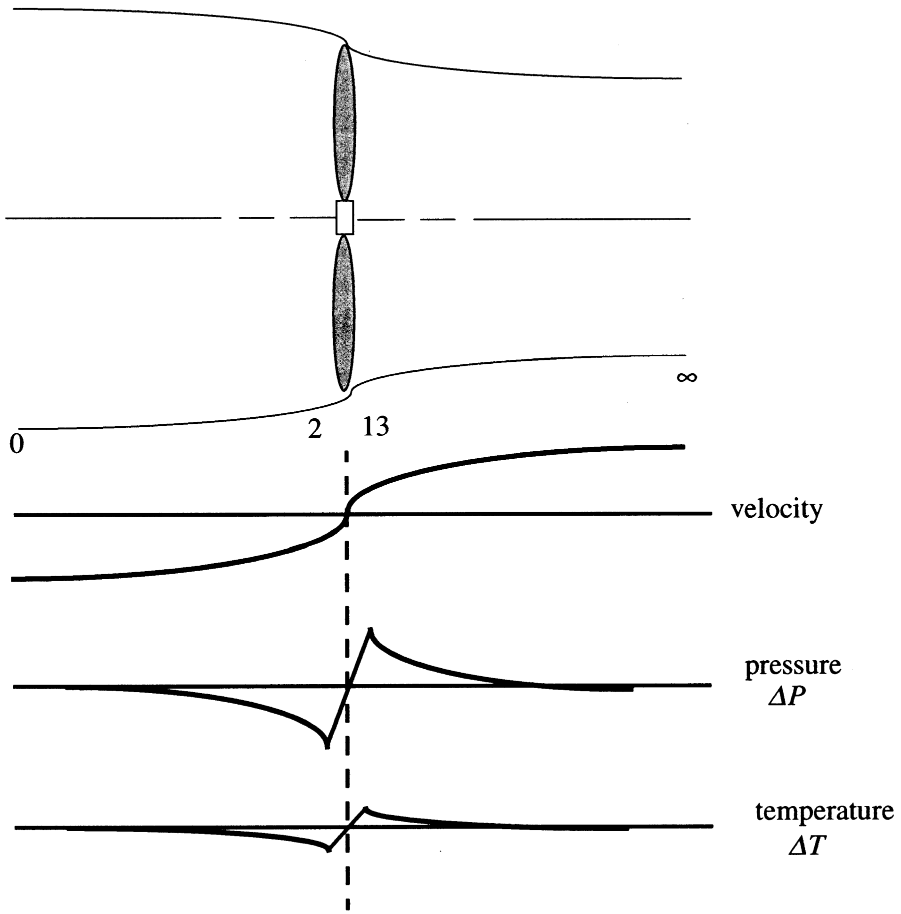
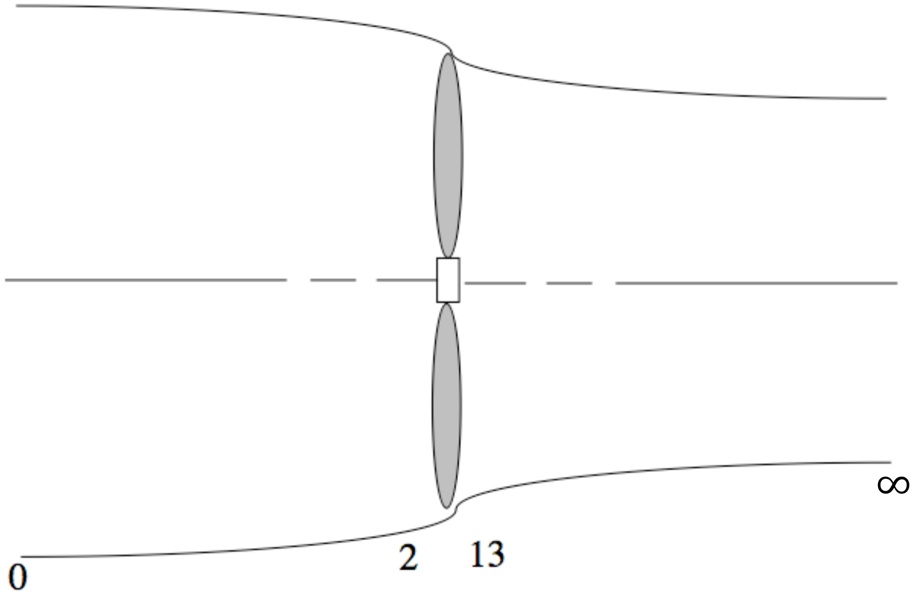


Figure 6.3 Effect of propellor actuator disc on flow velocity, pressure and temperature.

6.6 Problems

Problem 1 - Consider the propeller shown below..



Show that for small Mach number, the velocity at the propeller is approximately

$$U_2 = U_{13} = \frac{U_0 + U_\infty}{2}.$$

In other words one-half the velocity change induced by the propeller occurs upstream of the propeller. This is known as Froude's theorem and is one of the cornerstones of propeller theory.

Work done by the propellor

$$W_p = \dot{m}_{prop}(h_{t13} - h_{t2}) \tag{6.6}$$

Froude's theorem

$$U_2 = \frac{U_0 + U_\infty}{2} \tag{6.7}$$

Since the velocity is constant through the propellor

$$W_p = \dot{m}_{prop} C_p (T_{13} - T_2) \tag{6.8}$$

The pressure and temperature ratio are related by a polytropic efficiency

$$\frac{P_{t13}}{P_{t2}} = \left(\frac{T_{t13}}{T_{t2}}\right)^{\frac{\gamma \eta_{pc}}{\gamma - 1}} \tag{6.9}$$

Express (6.9) as

$$\frac{P_{13}}{P_2}\left(\frac{1+\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T_{13}}{T_2}\right)^{\frac{\gamma\eta_{pc}}{\gamma-1}}\left(\frac{1+\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\eta_{pc}} \qquad (6.10)$$

The Mach number change across the propellor is small and so (6.10) can be approximated as

$$\frac{P_{13}}{P_2} = \left(\frac{T_{13}}{T_2}\right)^{\frac{\gamma\eta_{pc}}{\gamma-1}} \qquad (6.11)$$

$$1+\left(\frac{P_{13}-P_2}{P_2}\right) = \left(1+\left(\frac{T_{13}-T_2}{T_2}\right)\right)^{\frac{\gamma\eta_{pc}}{\gamma-1}} \qquad (6.12)$$

For a lightly loaded propellor

$$\frac{P_{13} - P_2}{P_2} \cong \frac{\gamma\eta_{pc}}{\gamma - 1}\left(\frac{T_{13} - T_2}{T_2}\right) \tag{6.13}$$

$$P_{13} - P_2 \cong \eta_{pc}\rho_2 C_p (T_{13} - T_2) \tag{6.14}$$

Propellor thrust

$$T = (P_{13} - P_2)A \tag{6.15}$$

where A is the area of the actuator disc. Now form the propellor efficiency

$$\eta_{prop} = \frac{\eta_{pc}\rho_2 U_0 A C_p (T_{13} - T_2)}{\dot{m}_{prop} C_p (T_{13} - T_2)} = \frac{\eta_{pc}\rho_2 U_0 A}{\rho_2 U_2 A} \tag{6.16}$$

Using Froude's theorem the propellor efficiency becomes

$$\eta_{prop} = \left(\frac{2U_0}{U_0 + U_\infty}\right)\eta_{pc} \tag{6.17}$$

Using this result we can now interpret the energy factor in (6.5).

$$\frac{\frac{1}{2}\dot{m}_{prop}(U_\infty^2 - U_0^2)}{W_p} \cong \eta_{pc} \tag{6.18}$$

The polytropic efficiency is related to the entropy change across the propellor as follows.

$$\frac{dP}{P} = \eta_{pc}\left(\frac{\gamma}{\gamma - 1}\right)\frac{dT}{T}. \tag{6.19}$$

$$\frac{ds}{C_p} = \frac{dT}{T} - \left(\frac{\gamma - 1}{\gamma}\right)\frac{dP}{P} = (1 - \eta_{pc})\frac{dT}{T} \tag{6.20}$$

The propellor efficiency can also be written as

$$\eta_{prop} = \left(\frac{2U_0}{U_0 + U_\infty}\right)\left(1 - \frac{T}{C_p}\frac{ds}{dT}\right) \tag{6.21}$$

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6.2 Work output coefficient

Thrust equation for the turboprop

$$T_{total} = T_{core} + T_{prop}$$

$$or \qquad (6.22)$$

$$T = \dot{m}_a(U_e - U_0) + \dot{m}_f U_e + A_e(P_e - P_0) + \dot{m}_{prop}(U_\infty - U_0)$$

Substitute the propellor efficiency

$$T_{total} = \dot{m}_a(U_e - U_0) + \dot{m}_f U_e + A_e(P_e - P_0) + \eta_{prop}\frac{W_p}{U_0} \qquad (6.23)$$

Core thrust

$$\frac{T_{core}}{\dot{m}_a a_0} = M_0\left((1+f)\frac{M_e}{M_0}\sqrt{\frac{T_e}{T_0}} - 1\right) \qquad (6.24)$$

Define the work output coefficient

$$C_{total} = \frac{T_{total}U_0}{\dot{m}_a C_p T_0}. \qquad (6.25)$$

Now

$$C_{total} = C_{core} + C_{prop} \qquad (6.26)$$

Where

$$C_{core} = (\gamma - 1)M_0^2\left((1+f)\frac{M_e}{M_0}\sqrt{\frac{T_e}{T_0}} - 1\right) \qquad (6.27)$$

and

$$C_{prop} = \eta_{prop}\frac{W_p}{\dot{m}_a C_p T_0} \qquad (6.28)$$

The fuel efficiency of the turboprop is expressed in terms of the specific horsepower

$$SHP = \frac{\text{pounds of fuel burned per hour}}{\text{output horsepower}} = 3600\frac{\dot{m}_f g}{T_{total}U_0}$$

$$or \qquad (6.29)$$

$$SHP = \frac{2545}{C_p T_0}\left(\frac{f}{C_{total}}\right)$$

Where the temperature is in degrees Rankine and the heat capacity is in BTU/lbm-hr

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6.3 Turboprop power balance

Turbine-compressor-propellor matching

$$W_p = \eta_g((\dot{m}_a + \dot{m}_f)\eta_m(h_{t4} - t_{t5}) - \dot{m}_a(h_{t3} - h_{t2})) \tag{6.30}$$

Rearrange

$$C_{prop} = \eta_{prop}\eta_g((1 + f)\eta_m\tau_\lambda(1 - \tau_t) - \tau_r(\tau_c - 1)) \tag{6.31}$$

6.4 The ideal turboprop

Assume

$$\pi_d = 1 \qquad \eta_{pc} = 1 \qquad \pi_b = 1 \qquad \eta_{pe} = 1 \qquad \pi_n = 1 \tag{6.32}$$

and assume the core exhaust is fully expanded. Note that the propellor efficiency is not assumed to be one.

Exit Mach number

$$\frac{M_e^2}{M_0^2} = \left(\frac{\tau_r \tau_c \tau_t - 1}{\tau_r - 1}\right) \tag{6.33}$$

Temperature ratio

$$\frac{T_e}{T_0} = \frac{\tau_\lambda}{\tau_r \tau_c} \tag{6.34}$$

Work output coefficient of the core

$$C_{core} = 2(\tau_r - 1)\left((1+f)\left(\frac{\tau_\lambda}{\tau_r \tau_c}\right)^{1/2}\left(\frac{\tau_r \tau_c \tau_t - 1}{\tau_r - 1}\right)^{1/2} - 1\right) \tag{6.35}$$

Optimization of the ideal turboprop cycle

Question: What fraction of the total thrust should be generated by the core in order to produce the maximum work output coefficient?

$$\frac{\partial C_{total}}{\partial \tau_t} = \frac{\partial C_{core}}{\partial \tau_t} + \frac{\partial C_{prop}}{\partial \tau_t} = 0 \qquad (6.36)$$

Now

$$2(\tau_r - 1)(1 + f)\left(\frac{\tau_\lambda}{\tau_r \tau_c}\right)^{1/2}\left(\frac{1}{2}\right)\left(\frac{\tau_r \tau_c \tau_t - 1}{\tau_r - 1}\right)^{-1/2}\left(\frac{\tau_r \tau_c}{\tau_r - 1}\right) - \eta_{prop}\eta_g(1 + f)\eta_m \tau_\lambda = 0 \qquad (6.37)$$

or

$$(\tau_\lambda \tau_r \tau_c)^{1/2}\left(\frac{\tau_r \tau_c \tau_t - 1}{\tau_r - 1}\right)^{-1/2} = \eta_{prop}\eta_g \eta_m \tau_\lambda \qquad (6.38)$$

Notice that the propellor, gearbox and shaft efficiencies enter the analysis as one product. Let

$$\eta = \eta_{prop}\eta_g\eta_m \qquad (6.39)$$

$$\tau_t\Big|_{max\ thrust\ turboprop} = \frac{1}{\tau_r\tau_c} + \frac{(\tau_r - 1)}{\eta^2\tau_\lambda} \qquad (6.40)$$

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What core velocity ratio does this correspond to ?

$$\left(\frac{U_e}{U_0}\right)^2\Bigg|_{max\ thrust\ turboprop} = \frac{\tau_\lambda}{\tau_r\tau_c}\left(\frac{1}{\tau_r - 1}\right)\left(\tau_r\tau_c\tau_t\big|_{max\ thrust\ turboprop} - 1\right) \tag{6.41}$$

Substitute (6.40)

$$\frac{U_e}{U_0}\Bigg|_{max\ thrust\ turboprop} = \frac{1}{\eta} \tag{6.42}$$

An ideal turboprop with a polytropic efficiency of one would have

$$U_e\big|_{max\ thrust\ turboprop} = \frac{U_0 + U_\infty}{2} \tag{6.43}$$

In other words the core would become indistinguishable from the actuator disc.

Ideal turboprop - compression for maximum thrust

$$\frac{\partial C_{prop}}{\partial \tau_c} = \eta_{prop}\eta_g\left(-\eta_m\tau_\lambda\left(\frac{\partial \tau_t}{\partial \tau_c}\right) - \tau_r\right) = \eta_{prop}\eta_g\left(-\eta_m\tau_\lambda\left(\frac{-1}{\tau_r\tau_c^2}\right) - \tau_r\right) \qquad (6.44)$$

Maximum is achieved with

$$\tau_c = \frac{\sqrt{\eta_m\tau_\lambda}}{\tau_r} \qquad (6.45)$$

Same result as turbojet

6.5 Turbine sizing for the non-ideal turboprop

Recall (6.36)

$$\frac{\partial C_{total}}{\partial \tau_t} = \frac{\partial C_{core}}{\partial \tau_t} + \frac{\partial C_{prop}}{\partial \tau_t} = 0 \qquad (6.46)$$

Assume the core exhaust is fully expanded. The velocity ratio across the non-ideal core is

$$\left(\frac{U_e}{U_0}\right)^2 = \frac{1}{\tau_r - 1}\left(\frac{\tau_\lambda}{\tau_r \tau_c}\right)\left(\tau_r \tau_c \tau_t - \frac{\tau_c^{1-\eta_{pc}} \tau_t^{\left(1 - \frac{1}{\eta_{pt}}\right)}}{(\pi_d \pi_b \pi_n)^{\frac{\gamma - 1}{\gamma}}}\right) \qquad (6.47)$$

The core work output coefficient is

$$C_{core} = 2(\tau_r - 1)\left(\frac{(1+f)}{(\tau_r - 1)^{1/2}}\left(\frac{\tau_\lambda}{\tau_r \tau_c}\right)^{1/2}\left(\tau_r \tau_c \tau_t - \frac{\tau_c^{1-\eta_{pc}} \tau_t^{\left(1 - \frac{1}{\eta_{pt}}\right)}}{(\pi_d \pi_b \pi_n)^{\frac{\gamma - 1}{\gamma}}}\right)^{1/2} - 1\right) \qquad (6.48)$$

The condition (6.46) becomes

$$(\tau_r - 1)^{1/2}\left(\frac{\tau_\lambda}{\tau_r\tau_c}\right)^{1/2}\left(\tau_r\tau_c\tau_t - \frac{\tau_c^{1-\eta_{pc}}\tau_t^{\left(1-\frac{1}{\eta_{pt}}\right)}}{(\pi_d\pi_b\pi_n)^{\frac{\gamma-1}{\gamma}}}\right)^{\frac{-1}{2}} \times$$

$$\left(\tau_r\tau_c - \left(1 - \frac{1}{\eta_{pt}}\right)\frac{\tau_c^{1-\eta_{pc}}\tau_t^{-\frac{1}{\eta_{pt}}}}{(\pi_d\pi_b\pi_n)^{\frac{\gamma-1}{\gamma}}}\right) - \quad (6.49)$$

$$\eta_{prop}\eta_g\eta_m\tau_\lambda = 0$$

Various parameters in (6.47) are specified and the turbine temperature ratio is determined implicitly.

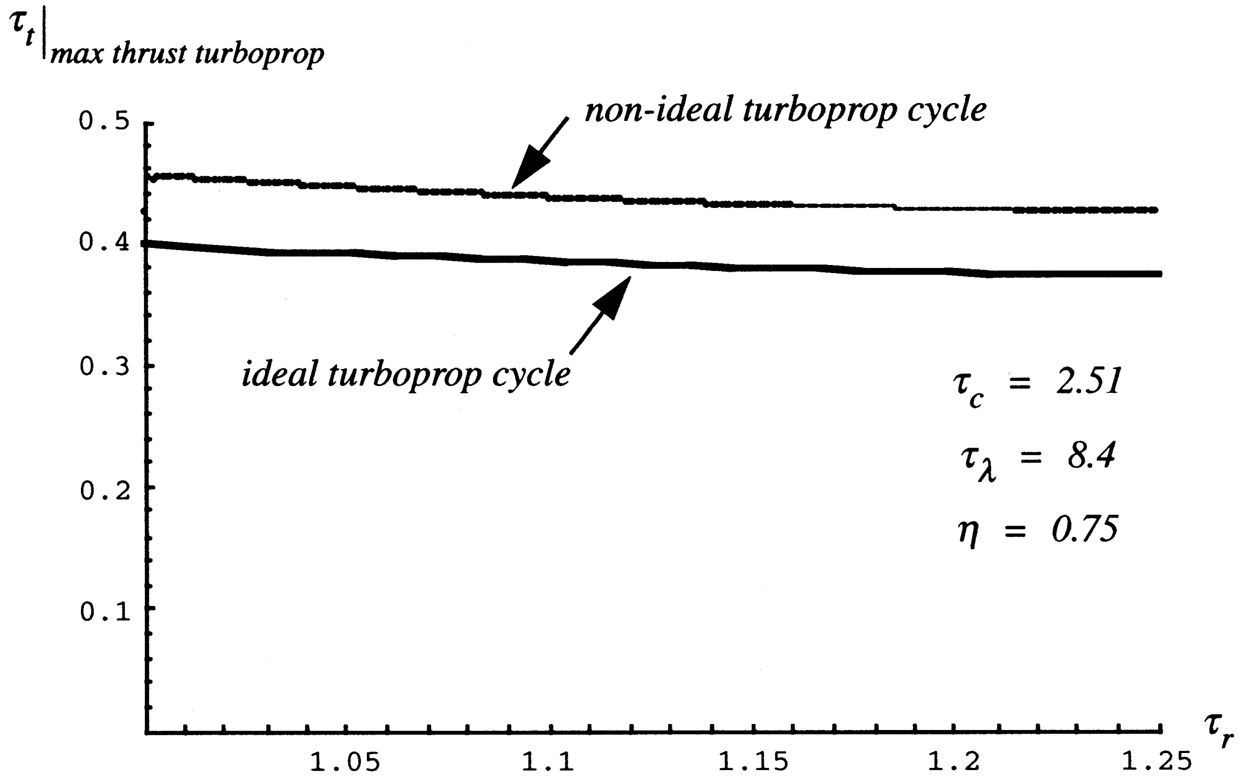

Figure 6.4 *Comparison of turbine selection for the the ideal and non-ideal turboprop cycle. Parameters of the nonideal cycle are* $\pi_d = 0.97$, $\eta_{pc} = 0.93$, $\pi_b = 0.96$, $\eta_{pe} = 0.95$, $\pi_n = 0.98$.

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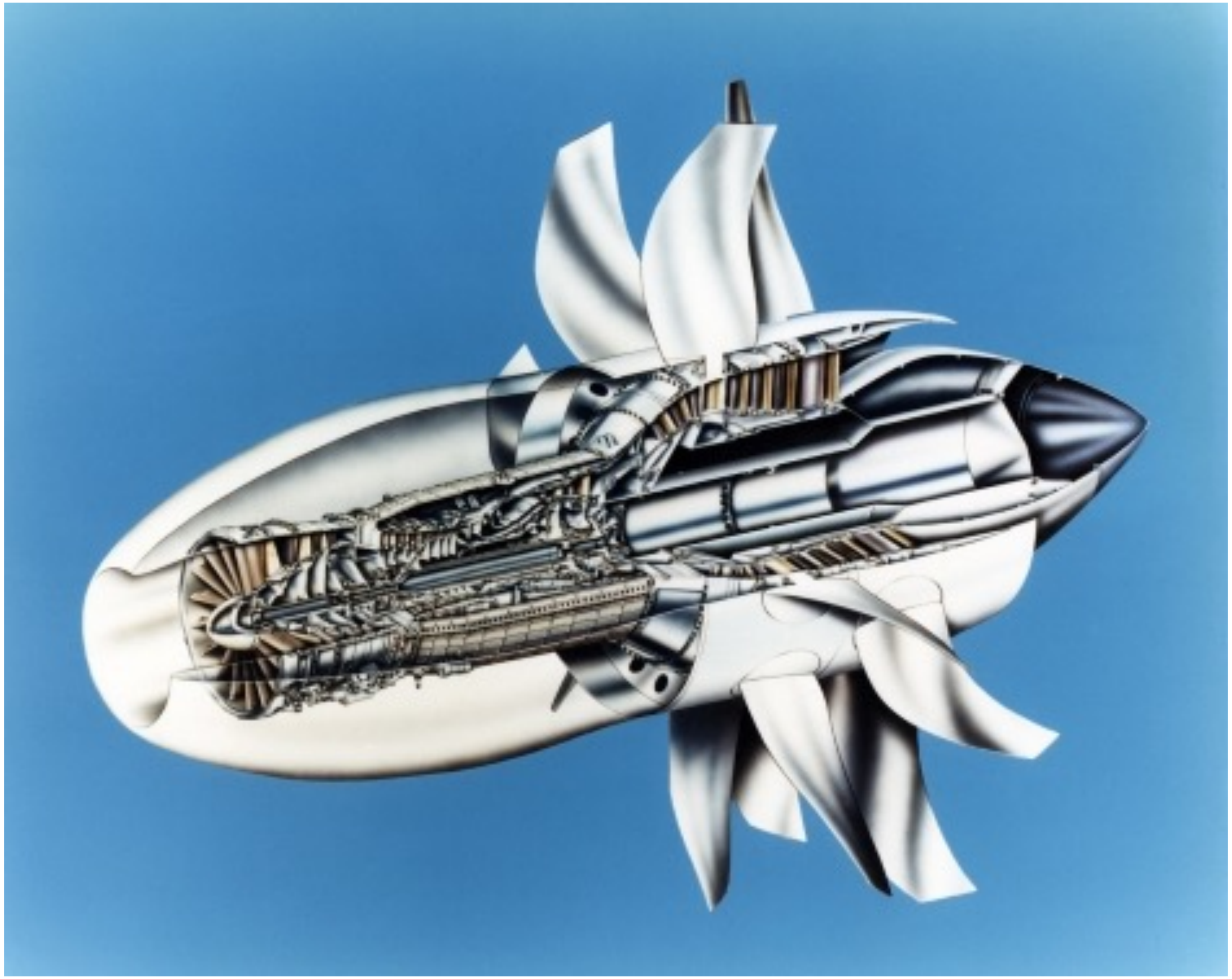
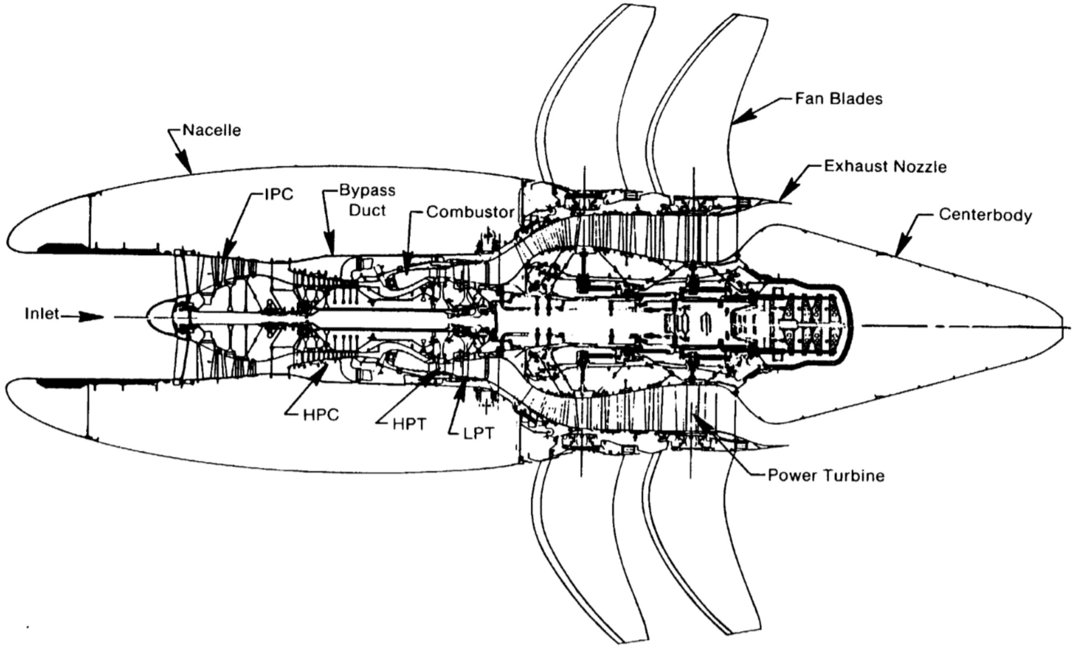
GE 36 - Unducted Fan counter rotating turboprop concept demonstrator

An idea tested in the 1980s

GE 36 25,000 lbs thrust

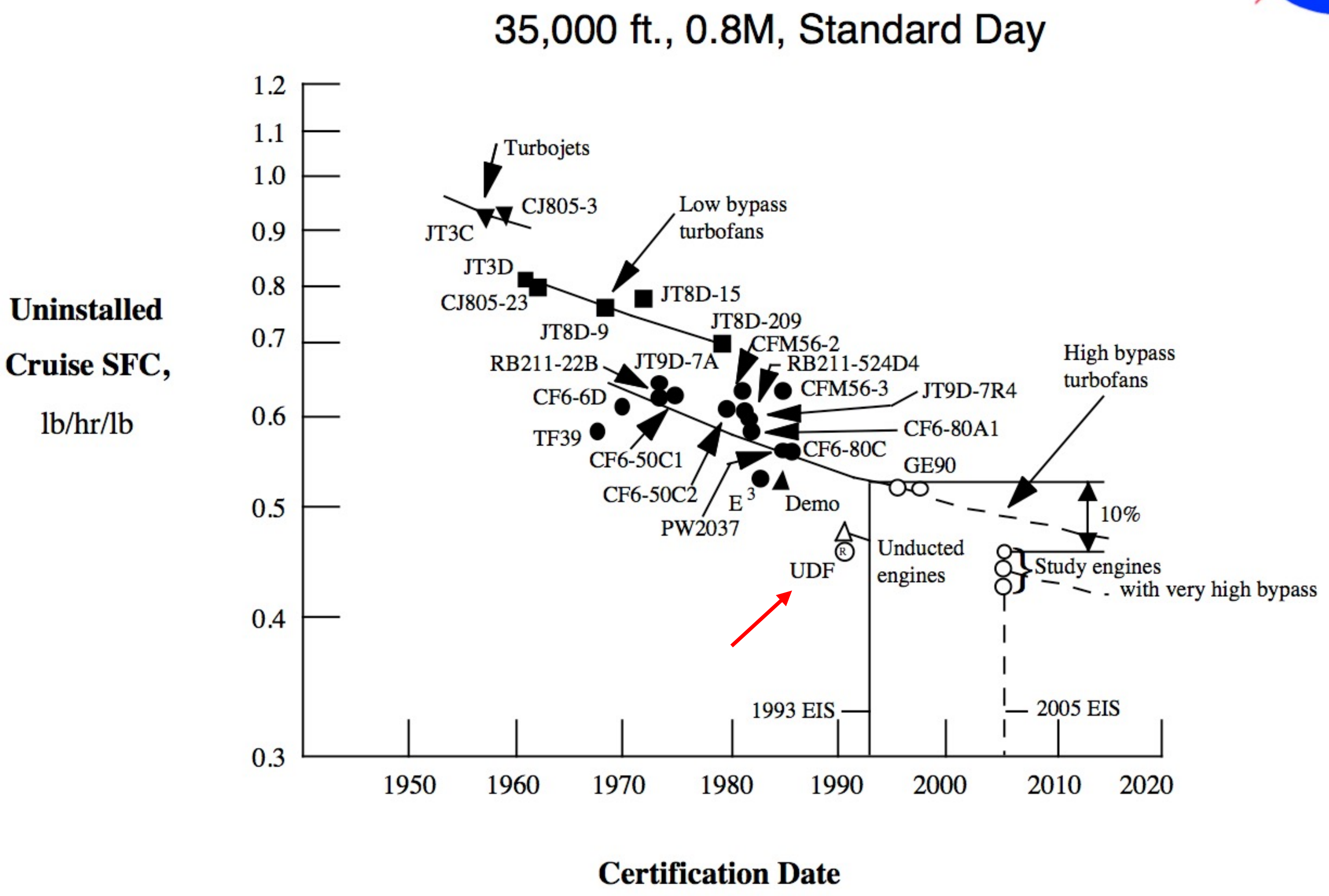


Core engine GE F404 10,000 lbs thrust



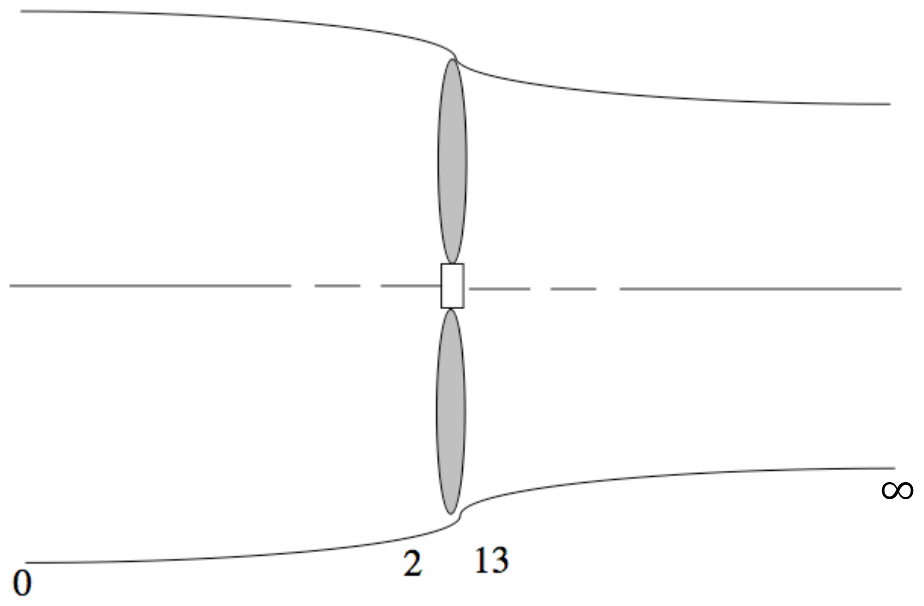
Effective Bypass ratio = 30

State-of-the-Art Subsonic Engine SFC



6.6 Problems

Problem 1 - Consider the propeller shown below..

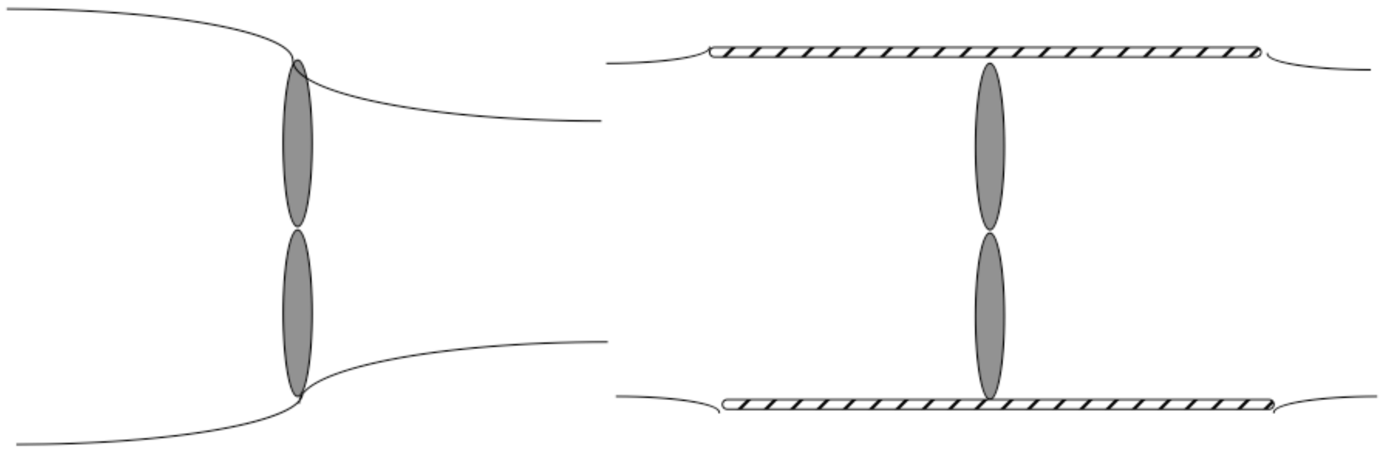


Use the Bernoulli relation for small Mach number and the thrust equation for a propellor to show that the velocity at the propeller is approximately

$$U_2 = U_{13} = \frac{U_0 + U_\infty}{2}. \tag{6.50}$$

In other words one-half the velocity change induced by the propeller occurs upstream of the propeller. This is known as Froude's theorem and is one of the cornerstones of propeller theory.

Problem 2 - Compare ducted versus unducted fans. Let the fan area be the same in both



cases. Show that for the same power input that the ducted case produces more thrust.

Problem 3 - Use Matlab or Mathematica to develop a program that reproduces Figure 6.4.

Problem 4 - An ideal turboprop engine operates at a free stream Mach number, $M_0 = 0.7$. The propeller efficiency is, $\eta_{prop} = 0.8$ and the gearbox and shaft efficiencies are both 1.0. The turbine is chosen to maximize the total work output coefficient. The compressor is chosen according to, $\tau_c = \sqrt{\tau_\lambda}/\tau_r$ and $\tau_\lambda = 6$. Determine the dimensionless thrust, $T/P_0 A_0$ where A_0 is the capture area corresponding to the air flow through the core engine. Assume $f << 1$. Is the exit nozzle choked?.

Problem 5 - A non-ideal turboprop engine operates at a free stream Mach number, $M_0 = 0.6$. The propeller efficiency is, $\eta_{prop} = 0.8$ and the gearbox and shaft efficiencies are both 1.0. The operating parameters of the engine are $\tau_\lambda = 7$, $\tau_c = 2.51$, $\pi_d = 0.97$, $\eta_{pc} = 0.93$, $\pi_b = 0.96$, $\eta_{pe} = 0.95$, $\pi_n = 0.98$. Determine the dimensionless thrust, $T/P_0 A_0$ where A_0 is the capture area corresponding to the air flow through the core engine. Do not assume $f \ll 1$.

Problem 6 - A turboprop engine operates at a free stream Mach number, $M_0 = 0.6$. The propeller efficiency is $\eta_{prop} = 0.85$, the gearbox efficiency is $\eta_g = 0.95$, and the shaft efficiency is $\eta_m = 1$. All other components operate ideally and the exhaust is fully expanded $P_e = P_0$. The operating parameters of the engine are $\tau_\lambda = 7$ and $\tau_c = 2.51$. Assume the turbine is sized to maximize C_{total} and assume $f \ll 1$. Determine the total work output coefficient C_{total} and dimensionless thrust T/P_0A_0. The ambient temperature and pressure are $T_0 = 216\,K$ and $P_0 = 2 \times 10^4 N/m^2$.

Problem 7 - A propulsion engineer is asked by her supervisor to determine the thrust of a turboprop engine at cruise conditions. The engine is designed to cruise at $M_0 = 0.5$. At that Mach number the engine is known to be operating close to its maximum total work output coefficient C_{total}. She responds by asking the supervisor to provide some data on the operation of the engine at this condition. List the minimum information she would need in order to provide a rough estimate of the thrust of the engine. What assumptions would she need to make in order to produce this estimate?

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Problem 8 - Figure 6.7 shows two personal air vehicles. On the left is the Kitty Hawk 'Cora' unveiled a few years ago and designed to lift off vertically using 12 electric fans. On the right is the Robinson R22 with one large rotor. Each is designed to carry two passengers. The maximum lift-off mass of the R22 is $623\,kg$ and the rotor area is $46.2\,m^2$. I will assume the maximum liftoff mass of the Cora to be $727\,kg$. This is the maximum take off mass of a Cessna 150, which has a wing span of $10.11\,m$ compared to the Cora wing span of $11.0\,m$. But the powerplants are completely different, and the battery system and electric motors required by the Cora probably makes this guess of the lift off mass somewhat low. From the picture, I estimate the total area of the fans to be $15\,m^2$. Consider both machines in hover, $U_0 = 0$, and treat each lift system as an actuator disc. That is, the flow through the rotor/fans can be treated as uniform over the area. The ambient pressure and temperature are $288.15\,K$ and $101325\,N/m^2$. The air density is $\rho_0 = 1.2250\,kg/m^3$ and the gravitational acceleration is $9.80\,m/sec^2$. For each case determine, in our propellor notation, each of the following items.

1) $P_{13} - P_2$

2) U_∞

3) The flow of kinetic energy far below the vehicle induced by each lift system. Express your answer in $Joules/sec$.

Figure 6.7: *Two personal air vehicles capable of vertical take-off and landing.*