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Self-Similar Diffusion Flame Including Effects of Streamwise Diffusion

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Abstract—Using Lie group theory, a self-similar solution, representing a diffusion flame in which both cross stream and streamwise diffusion is included is obtained. The solution arises in problems related to conjugate heat transfer and pollutant dispersion applications that are governed by the same partial differential equation. Results including flame shape, flame height dependence on Peclet number and overall stoichiometry, are compared with other known solutions. A unique feature of the solution is the existence of a maximum in the size of the back diffusion region as a function of Peclet number. This could be important in very low speed burner applications.

Key words: Group theory Diffusion Flame Laminar Self-Similar Streamwise Diffusion

INTRODUCTION

Exact analytical solutions to combustion-fluid mechanics problems are rare. In addition to providing useful insight, analytical solutions are valuable in evaluating direct numerical simulation codes and in providing initial conditions suitable for simulations. A classic example is the laminar diffusion flame solution developed by Burke and Schumann (1928) to predict diffusion flame heights in low speed burners. The important physics built in their model is the balance of axial convection with radial or cross stream diffusion, leading to a linear dependence of flame height on Peclet number. With the neglect of streamwise diffusion, their model is necessarily limited to large Peclet number flows. However, large Peclet number flows tend to be turbulent with flame heights independent of Peclet number. Another problem in which streamwise diffusion is important is in the analysis of flame structure to understand why unity Lewis number flames are not susceptible to opening of the flame tip (Im *et al.*, 1990). Chung and Law (1984) modified the Burke-Schumann problem to include streamwise diffusion. In addition, their analysis includes preferential diffusion effects. A problem that they allude to in some detail is the possibility of the occurrence of back diffusion (of both heat and mass) upstream of the burner exit, and the difficulty in treating it mathematically. They circumvent this experimentally by placing a porous plate at the exit to suppress back diffusion. In their analysis, they are thus justified in prescribing constant concentration (and temperature) at the jet exit. Mahalingam *et al.* (1990) developed a self-similar solution in which a simple closed form expression for the flame shape was obtained. However, their analysis excludes streamwise diffusion. Experimental measurements by Eckbreth and Hall (1979) at very short distances downstream of the burner exit in a propane-air diffusion flame are suggestive of strong diffusional effects. In this paper, streamwise and cross stream diffusion effects are included and a self-similar diffusion flame solution is obtained using a fundamental approach based on Lie group theory. The solution is compared with other diffusion flame solutions, and related to well established solutions that arise in conjugate heat transfer problems and problems related to pollutant dispersion. The solution provides new insight into the nature and extent of back diffusion.
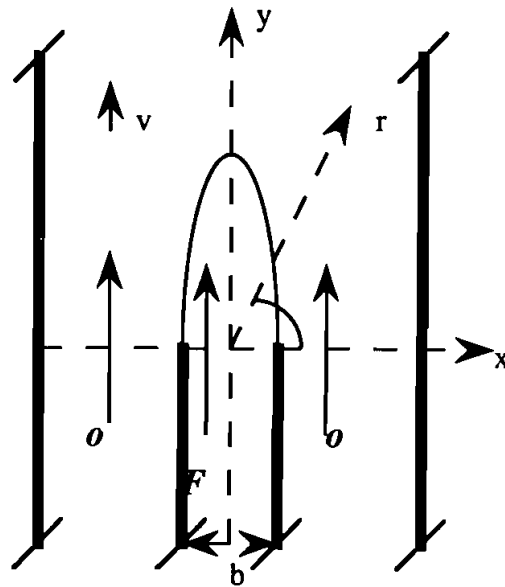
FIGURE 1   Schematic of two-dimensional Burke-Schumann flow configuration. The limit considered is when the outer slot width approaches infinity and the inner slot is replaced by a line source of fuel.

PROBLEM FORMULATION

The problem considered is a low Mach number laminar diffusion flame established in a coflowing, two-dimensional slot burner. Fuel and oxidizer flow through the inner and outer slots and meet at the mouth of the inner slot as in Fig. 1. The limiting case, when the outer slot width approaches infinity and the inner slot is replaced by a line source of fuel at the origin, is the focus of the present study. The notation used in this section closely follows Williams (1985). Radiation, diffusion due to pressure gradients, Soret and Dufour effects are all assumed negligible. The diffusivities of all species and temperature are assumed equal. Bulk viscosity and buoyancy are neglected, and convection velocity in the cross stream direction is assumed to be negligibly small. The thin flame approximation is made so that no penetration of fuel or oxidizer across the infinitesimally thin flame surface occurs. The reader is referred to Clarke (1965) for a thorough mathematical description of this model. Chemistry is modeled by a single, irreversible step between fuel (F) and oxidizer (O) reacting to yield a product (P):

$$\nu_F F + \nu_O O \longrightarrow \nu_p P,$$
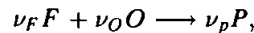
where the ν_i are the stoichiometric coefficients. Let $Y_{F,0}$ nd $Y_{O,0}$ represent the fuel and oxidizer mass fractions in their unmixed state. The following quantities are defined:

$$\alpha_F \equiv -\frac{Y_F}{W_F \nu_F}, \quad \alpha_O \equiv -\frac{Y_O}{W_O \nu_O}, \quad \beta \equiv \alpha_O - \alpha_F, \tag{1}$$

where Y_F and Y_O are the fuel and oxidizer mass fractions, respectively, and W_F, W_O are their respective molecular weights. The quantity β is the coupling function in the

Schvab-Zeldovich formulation. Under the stated assumptions, for two dimensional flow, β is governed by the conserved scalar equation

$$\frac{\partial}{\partial y}\rho v\beta = \frac{\partial}{\partial x}\rho D\frac{\partial \beta}{\partial x} + \frac{\partial}{\partial y}\rho D\frac{\partial \beta}{\partial y}, \tag{2}$$

where ρ is the density, v is the streamwise velocity in the y direction and D is the diffusion coefficient. Note that β ranges from $\beta_{O,0} \equiv -Y_{O,0}/W_O\nu_O$ to $\beta_{F,0} \equiv Y_{F,0}/W_F\nu_F$. Since β decays asymptotically to $-Y_{O,0}/W_O\nu_O$, rather than to zero in the far field, it is convenient to define the mixture fraction

$$\phi \equiv \frac{\beta - \beta_{O,0}}{\beta_{F,0} - \beta_{O,0}}, \tag{3}$$

so that ϕ goes to zero in the far field (pure oxidizer) and $\phi = 1$ in the pure fuel stream. In the thin flame approximation, the surface $\beta = 0$ or $\phi = \alpha_c$, where α_c defined by

$$\alpha_c \equiv \frac{Y_{O,0}}{W_O\nu_O}/(\frac{Y_{F,0}}{W_F\nu_F} + \frac{Y_{O,0}}{W_O\nu_O}), \tag{4}$$

identifies the flame surface. Clearly ϕ satisfies Eq. (2). By making a further assumption that ρv and ρD are constants, and defining

$$c \equiv \frac{\rho v}{\rho D} \tag{5}$$

one gets,

$$c\frac{\partial \phi}{\partial y} = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}. \tag{6}$$

The objective in this paper is to seek a self-similar solution to Eq. (6). The limit considered is the situation wherein the outer slot width approaches infinity and fuel issuing from the inner slot is modeled by placing a line source of fuel at the origin. It is worth noting that King (1914) used the Boussinesq transformation, applicable to two-dimensional irrotational flow, to transform the full version of Eq. (2) including streamwise convection, into Eq. (6) with stream and potential functions as the independent variables. Several solution methods (see Morse and Feshbach, 1953) are possible for problems of this type. In this paper, the problem is approached using Lie group theoretical methods. The reader may refer to Bluman and Kumei (1989) for an excellent textbook treatment of the theory and a comprehensive list of references. The solution obtained is related to other known solutions to convection-diffusion problems.

APPLICATION OF LIE GROUP THEORY

Assume a solution to Eq. (6) of the form

$$\phi(x,y) = \exp(\omega y)f(x,y). \tag{7}$$

By choosing $\omega = c/2$, it follows that $f(x,y)$ satisfies the more symmetrical form

$$f_{xx} + f_{yy} - \frac{c^2}{4}f = 0, \tag{8}$$

where f_{xx} implies differentiation of f twice with respect to x. The basic idea now is to seek various Lie group transformations that leave Eq. (8) invariant. The infinitesimals

associated with the transformation lead to a set of determining equations that are always linear. Solutions of this determining set yield similarity variables, reducing Eq. (8) to two ordinary differential equations. This is achieved by the methods outlined in Bluman and Cole (1974) and more recently by Bluman and Kumei (1989). The notation used in this section follows Cantwell (1978). Let $g(x,y)$ be a solution to Eq. (8) which may be written as:

$$S(g, g_{xx}, g_{yy}) = 0, \qquad (9)$$

representing a surface in (x, y, g_{xx}, g_{yy}) space. For notational convenience the correspondence $(x,y) \leftrightarrow (x_1, x_2)$ is made. Consider the one-parameter infinitesimal transformations of the independent and dependent variables of the form:

$$x_i' = x_i + \epsilon \xi_i(x_1, x_2, g) + O(\epsilon^2)$$

$$f' = g(x_1, x_2) + \epsilon \eta(x_1, x_2, g) + O(\epsilon^2), \qquad (10)$$

where the infinitesimals ξ_1, ξ_2, and η are yet unknown functions that need to be determined as part of the solution process, and ϵ is a small parameter. Equation (8) stays invariant under the transformation given by Eq. (10) provided, (Cantwell, 1978)

$$US = 0, \qquad (11)$$

where U and S are given by,

$$U = N_{xx}\frac{\partial}{\partial g_{xx}} + N_{yy}\frac{\partial}{\partial g_{yy}} - Ng = 0 \quad ; \quad S = g_{xx} + g_{yy} - \frac{c^2}{4}g, \qquad (12)$$

where N_{xx}, N_{yy} are the second extensions of the group with respect to x and y respectively. They refer to the way second partial derivatives transform under the assumed transformation given by Eq. (10). Substituting Eq. (12) in Eq. (11) one obtains for invariance,

$$N_{xx} + N_{yy} - \frac{c^2}{4}N = 0. \qquad (13)$$

Using expressions for N_{xx} and N_{yy} from Bluman and Cole (1974) (too lengthy for reproduction here) one can rearrange the equation so that each term contains products of partial derivatives of g with respect to x_1, x_2 multiplied by coefficient terms involving partial derivatives of η, ξ_1, and ξ_2 with respect to their arguments. Bluman and Kumei (1989) show that the resulting equation must hold for arbitrary values of the partial derivatives of g suggesting that the coefficient terms must each vanish separately. This results in a system of linear, homogeneous partial differential equations for η, ξ_1, and ξ_2 called the determining equations for the infinitesimals. For a convenient summary of the stepwise strategy for obtaining these determining equations the interested reader may refer to Cantwell (1978). For the present problem, the following set of determining equations result:

$$\frac{\partial^2 \eta}{\partial x_1^2} + \frac{\partial^2 \eta}{\partial x_2^2} - \frac{c^2}{4}\eta = 0 \quad ; \quad 2\frac{\partial^2 \eta}{\partial x_1 \partial f} - \frac{\partial^2 \xi_1}{\partial x_1^2} - \frac{\partial^2 \xi_1}{\partial x_2^2} = 0$$

$$\frac{\partial^2 \xi_2}{\partial x_1^2} + \frac{\partial^2 \xi_2}{\partial x_2^2} - 2\frac{\partial^2 \eta}{\partial x_2 \partial f} = 0 \quad ; \quad \frac{\partial^2 \xi_2}{\partial x_1 \partial f} + \frac{\partial^2 \xi_1}{\partial x_2 \partial f} = 0$$

$$\frac{\partial^2 \eta}{\partial f^2} - 2\frac{\partial^2 \xi_1}{\partial x_1 \partial f} = 0 \quad ; \quad \frac{\partial^2 \eta}{\partial f^2} - 2\frac{\partial^2 \xi_2}{\partial x_2 \partial f} = 0$$

$$\frac{c^2}{4}\frac{\partial \eta}{\partial f} - 2\frac{\partial \xi_1}{\partial x_1} = 0 \quad ; \quad \frac{\partial \xi_2}{\partial x_2} - \frac{\partial \xi_1}{\partial x_1} = 0 \quad ; \quad \frac{\partial \xi_2}{\partial x_1} + \frac{\partial \xi_1}{\partial x_2} = 0$$

$$\frac{\partial^2 \xi_1}{\partial f^2} = 0 \quad ; \quad \frac{\partial^2 \xi_2}{\partial f^2} = 0 \quad ; \quad \frac{\partial \xi_1}{\partial f} = 0 \quad ; \quad \frac{\partial \xi_2}{\partial f} = 0 \tag{14}$$

The general non-trivial solution to equation set (14) gives the required infinitesimals:

$$\xi_1 = k_1 y + k_2 \quad ; \quad \xi_2 = -k_1 x + k_3 \quad ; \quad \eta = k_4 f, \tag{15}$$

where k_1, k_2, k_3, and k_4 are arbitrary constants. These solutions may also be found using a symbolic package as done recently by Ma and Hui (1990) in their analysis of the unsteady boundary layer equations. In order to obtain the similarity variables, one solves for the constants associated with the characteristic equations given by (see Cantwell, 1978 or Bluman and Kumei, 1989):

$$\frac{dx}{\xi_1} = \frac{dy}{\xi_2} = \frac{df}{\eta}, \tag{16}$$

where ξ_1, ξ_2, and η are given by Eq. (15). If $k_1 = k_4 = 0$, one obtains the plane wave solution to Eq. (8). By choosing $k_2 = k_3 = 0$, one obtains the radial solution which is relevant to the present problem. Solving the first equality in Eq. (16), one obtains the similarity variable

$$r = (x^2 + y^2), \tag{17}$$

that plays the role of the independent variable. Solving the second and third expression in Eq. (16) one obtains the result

$$f(x,y) = G(r) \exp\left[-\frac{k_4}{k_1}\sin^{-1}\frac{y}{r}\right]. \tag{18}$$

The form of the solution given by Eq. (18) suggests that a separable solution to Eq. (8) can be obtained by seeking a solution of the form:

$$f(x,y) = G(r)h(y/r). \tag{19}$$

Making the above substitution yields the following two ordinary differential equations:

$$r^2 G'' + rG' - G(\frac{c^2 r^2}{4} - \delta) = 0$$

$$(1 - \frac{y^2}{r^2})h'' - \frac{y}{r}h - \delta h = 0, \tag{20}$$

where primes indicate differentiation with respect to the appropriate argument and δ is a separation constant. It is easy to verify that $\delta = -n^2$, $n = 0,1,2,3,\cdots$, is the appropriate choice that ensures that the solution is single valued and periodic in $\theta \equiv \sin^{-1} y/r$. Thus a solution to Eq. (20) is given by:

$$G_n(\frac{cr}{2}) = C_{1n}K_n(\frac{cr}{2}) + C_{2n}I_n(\frac{cr}{2})$$

$$h_n(\frac{y}{r}) = D_{3n} \exp(in\theta) + \frac{D_{4n}}{in} \exp(-in\theta) \text{ or equivalently}$$

$$h_n(\frac{y}{r}) = E_{3n} \cos n\theta + E_{4n} \sin n\theta, \qquad \qquad (21)$$

where K_n and I_n are the modified Bessel functions of the first and second kind, the symbol i is defined by $i \equiv \sqrt{-1}$, and the C's, D's, and E's are constant coefficients. Requiring that the solution be bounded as $r \rightarrow \infty$, gives $C_{2n} = 0$. One can enforce zero-gradient condition on f about the y-axis (or symmetry) by insisting that replacing θ by $(\pi - \theta)$ should leave the solution unchanged. Using this result, and combining elementary solutions given in Eq. (21) and noting the connection between f and ϕ as given by Eq. (7), one obtains the complete solution:

$$\phi(x,y) = \exp(\frac{cy}{2}) \left\{ \sum_{n=0}^{n=\infty} K_n(\frac{cr}{2})[C_{3n} \cos n\theta + C_{4n} \sin n\theta] \right\}$$

where $C_{31} = C_{33} = C_{35} = \cdots = 0$, and $C_{40} = C_{42} = C_{44} = \cdots = 0$.   (22)

It should be pointed out that Imai (1954) used a similar solution to solve the vorticity equation describing Oseen's approximation to uniform flow around a cylinder. Appropriate conditions prescribed at the origin will complete the problem solution. This is discussed in the following section.

DIFFUSION FLAME SOLUTION

King (1914) used the first term in Eq. (22) to solve for the temperature distribution in a uniform stream with a heat source at the origin. His solution was used by Hunt and Mulhearn (1973) to study pollutant dispersion from a line source. The application of group theory for the problem solution and relating the solution to the diffusion flame problem is the unique aspect of the present work.

For convenience, the solution given by Eq. (22) is written as

$$\phi = \phi_0 + \phi_1 + \phi_2 + \cdots, \qquad \qquad (23)$$

where

$$\phi_0 = \exp(\frac{cy}{2})C_{30}K_0(\frac{cr}{2}), \quad \phi_1 = \exp(\frac{cy}{2})C_{41}K_1(\frac{cr}{2})\sin\theta, \cdots \qquad (24)$$

An integral of the flux of ϕ around any closed contour C that includes the origin will yield the rate of influx of ϕ at the origin. If C excludes the origin, it is clear that the integral would be zero. A convenient contour to evaluate this integral Q is a circle of radius r, centered at the origin. Thus,

$$Q = \int_0^{2\pi} \left[D\frac{\partial\phi}{\partial r} - V\phi\sin\epsilon \right] r\,d\epsilon. \qquad (25)$$

Recognizing symmetry of the integrand about the y axis, one can write,

$$Q = 2r \int_{\pi/2}^{3\pi/2} \left[D\frac{\partial\phi}{\partial r} - V\phi\sin\epsilon \right] d\epsilon. \qquad (26)$$

Let $Q_0, Q_1, \cdots$ represent the integrals corresponding to $\phi_0, \phi_1, \cdots$. Using the result (see Abramowitz and Stegun, 1972 for example),

$$K'_\nu(z) = -K_{\nu-1}(z) - \frac{\nu}{z} K_\nu(z), \tag{27}$$

and making the change of variable $\epsilon = \pi/2 - \alpha$, one gets for Q_0,

$$Q_0 = rVC_{30}\left[K_0(\frac{cr}{2})\int_0^\pi e^{(\frac{cr}{2}\cos\alpha)}\cos\alpha d\alpha + K_1(\frac{cr}{2})\int_0^\pi e^{(\frac{cr}{2}\cos\alpha)}d\alpha\right] \tag{28}$$

Using the relations (see Abramowitz and Stegun, 1972)

$$I_\nu(z) = \frac{1}{\pi}\int_0^\pi e^{(z\cos\alpha)}\cos\nu\alpha d\alpha, \tag{29}$$

and

$$I_{\nu+1}(z)K_\nu(z) + I_\nu(z)K_{\nu+1}(z) = \frac{1}{z}, \tag{30}$$

one obtains,

$$Q_0 = 2\pi D C_{30}. \tag{31}$$

It is easy to verify that Q_1, Q_2, etc., also yield similar relations. For the present problem, only Q_0 is useful, since only ϕ_0 is integrable in r. In the neighborhood of the origin, ϕ_0 behaves like $-\ln(cr/2)$ which is the correct form of the solution in an unbounded domain, based on Green's function arguments (see Morse and Feshbach, 1953 for example). Thus the required solution for ϕ is given by

$$\phi = \frac{Q}{2\pi D}\exp(\frac{cy}{2})K_0(\frac{cr}{2}). \tag{32}$$

This solution corresponds to King's (1914) result for diffusion of heat from a line source in uniform flow that has been subsequently used in the study of pollutant dispersion (for example Hunt and Mulhearn, 1973).

DIFFUSION FLAME CHARACTERISTICS

In order to match the source solution given by Eq. (32) to the solution for the problem represented by Fig. 1, one needs to match the flux of ϕ per unit length. If b is the inner-slot width, then one requires

$$Q = vb. \tag{33}$$

A Peclet number may be defined as

$$Pe \equiv \frac{vb}{D} = cb, \tag{34}$$

and lengths may be scaled using b. Thus the locus of the flame in terms of lengths scaled with b is given by:

$$\alpha_c = \frac{Pe}{2\pi}\exp(\frac{Per\sin\theta}{2})K_0(\frac{Per}{2}). \tag{35}$$
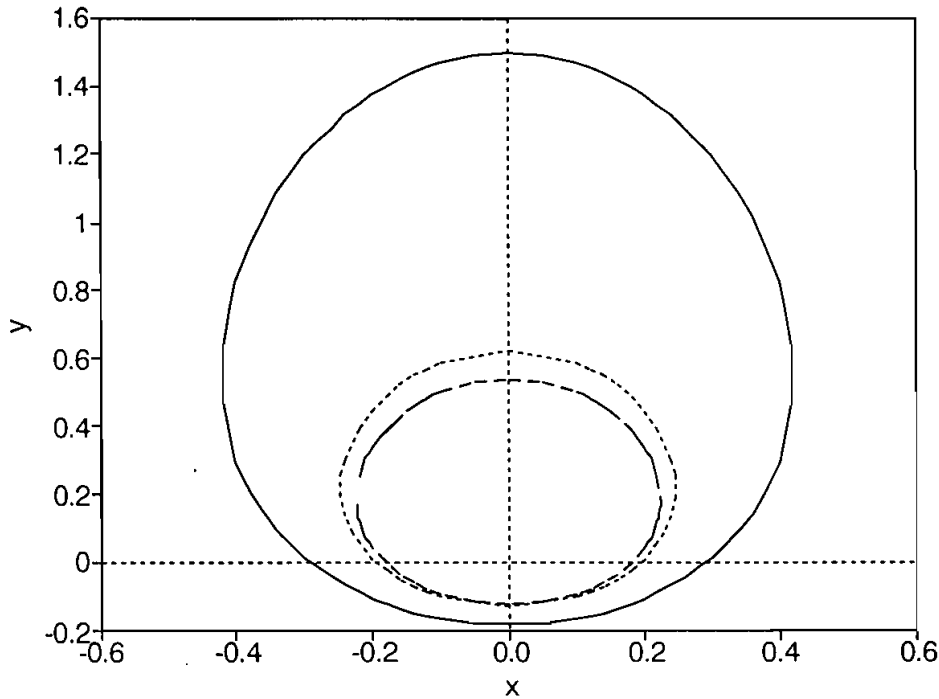
FIGURE 2   Flame contours for Peclet number, $Pe = 5.0$. _____ : $\alpha_c = 0.50$; - - - - - : $\alpha_c = 0.75$;
— · — : $\alpha_c = 0.80$.

Figure 2 shows several flame contours for different values of α_c, indicative of different fuel-oxidizer combinations and effect of diluents in the two streams. In each case, $Y_{F,0}/W_F \nu_F$ was fixed at 0.1. Higher values of α_c were thus achieved by increasing oxidizer mass fraction in the unmixed state. Several interesting features are observed in Fig. 2. Due to back-diffusion, the flame exists for $y < 0$. For fixed Peclet number the volume enclosed by the flame sheet decreases with increasing oxidizer concentration. The flame height is obtained by setting $\theta = \pm\pi/2$ and $r = h_\pm$ where h_+ and h_- indicate flame heights for θ values $+\pi/2$ and $-\pi/2$ respectively. Thus

$$\alpha_c = \frac{Pe}{2\pi} \exp(\pm\frac{Peh_\pm}{2})K_0(\frac{Peh_\pm}{2}). \tag{36}$$

For large Pe values, using the asymptotic form of K_0, it follows that

$$h_+ \approx \frac{Pe}{4\pi\alpha_c^2}, \tag{37}$$

indicating the expected linear dependence of flame height on Peclet number. Figure 3 is a plot of flame height h_+ as a function of Pe showing the linear behavior for large Pe. This, and the dependence on α_c are in reasonable agreement with Im et al.'s (1990) results. Finally, Figure 4 shows variation of h_- with Peclet number for different values of α_c. At large Peclet numbers, the size of the back-diffusion region decreases as anticipated. In this limit, the two important mechanisms are streamwise convection and cross stream diffusion as in the original Burke-Schumann flame. However, for low
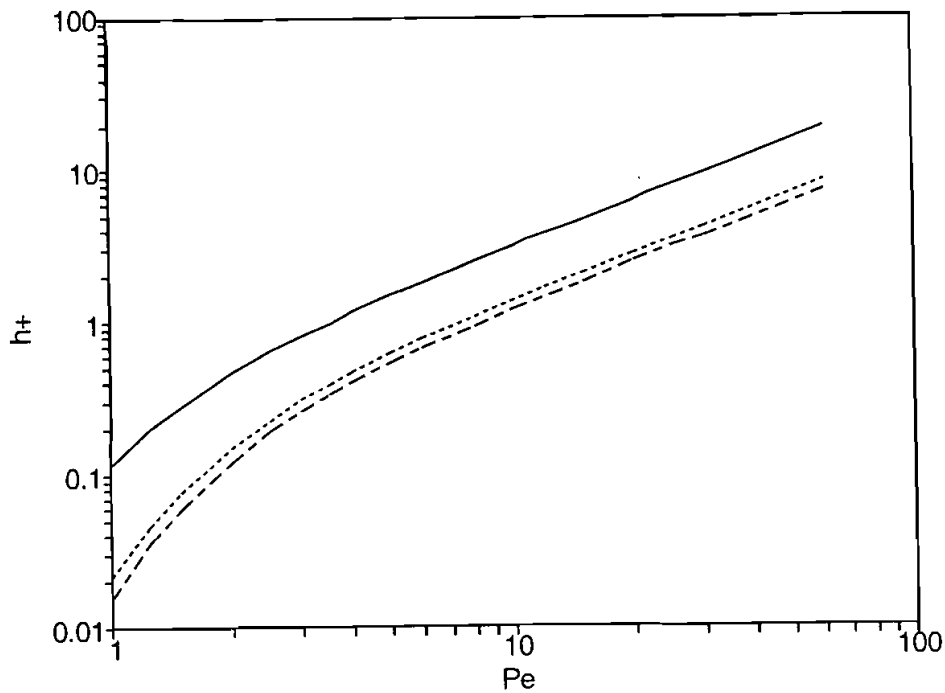
FIGURE 3   Flame height, h_+ variation with Peclet number, Pe. _____ : $\alpha_c = 0.50$; - - - - - : $\alpha_c = 0.75$; — · — : $\alpha_c = 0.80$.

values of Pe, h_- increases with Pe, contrary to what one expects intuitively. Note also that h_+ increases at approximately the same rate at low Peclet numbers. The reason for this behavior is that for low Peclet numbers, increasing Pe gives rise to an increased source strength tending to push the location of the α_c contour away from the source. Indeed for small values of Pe, it may be easily shown that the locus of the flame is given by

$$r \approx \frac{2}{Pe} \exp(\frac{-2\pi\alpha_c}{Pe}),$$

(38)

representing a circle for which h_+ and h_- are identical. In this limit, the only mechanism is diffusion, streamwise and cross-stream components being equally important. As Peclet number increases beyond this regime, the associated increase in the strength of streamwise convection tends to reduce h_-. The resulting competition gives rise to a maximum in h_-. For fixed Pe, the back diffusion flame height $h-$ decreases with increasing freestream oxidizer concentration. Based on these results it is suggested that back diffusion in real burners could be of importance. In fact measurements by Eckbreth and Hall (1979) in low speed propane diffusion flames indicate the presence of strong diffusional effects at short distances from the burner exit. They report measurements of 15.5% nitrogen and a temperature of 800°K at 1mm distance downstream of the propane tube exit plane. Although they suggest that actual nitrogen concentrations could be lower than that measured experimentally, they conclude that significant concentrations are present indicating strong diffusional effects. However, as suggested by Chung and Law (1984), their results may not be entirely conclusive due to excessive entrainment.
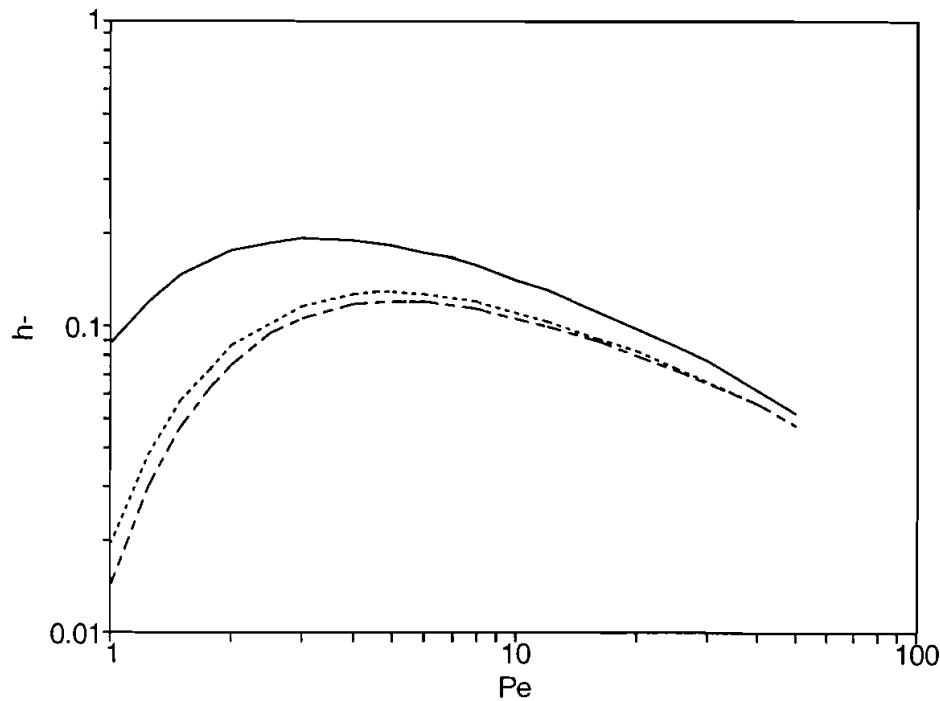
FIGURE 4   Flame height, h_-, indicating size of back diffusion region as function of Peclet number, Pe.
————— : $\alpha_c = 0.50$; - - - - - : $\alpha_c = 0.75$; — · — : $\alpha_c = 0.80$.

CONCLUSIONS

In this paper it is shown how Lie group theoretical methods can be successfully applied to the study of limiting forms of diffusion flames. The relatively simple solution in terms of two parameters (Pe, α_c) provides valuable insight into diffusion flame behavior at various Peclet numbers. It predicts the correct behavior of flame height h_+ with respect to variations in α_c and Peclet number. The unusual behavior of the size of the back diffusion region h_- for very low Peclet numbers is explained by examining the competing effects of diffusion and convection. In the future, analytical methods will be used to study back diffusion in real burners. The solution can also be used to study flame tip opening phenomena using perturbation methods following the technique of Im et al. (1990).

ACKNOWLEDGEMENTS

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