Brief Reports

Oscillation Frequencies of Freely Suspended Water Drops

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An investigation of water-drop oscillations was undertaken through the analysis of high-speed films of water drops suspended in a large vertical wind tunnel. Measurements of the natural frequency of vibration of suspended water drops versus diameter were made for drops ranging in size from 1 to 7 mm. A theoretical formula derived by Lord Rayleigh in 1879 predicts that, for small spheroidal oscillations of a water drop, the frequency is proportional to the diameter raised to the —3/2 power. Although our drops were suspended at their terminal velocity and were significantly deformed, the data agree with both the power dependence and the coefficient of proportionality.

The natural oscillations of water drops are of interest, since they may provide a mechanism for drop breakup in clouds. In particular, the effect of an electric field in promoting large distortions and drop oscillations with subsequent breakup has been the subject of various investigations [*Ausman and Brook*, 1967; *Billings and Holland*, 1969]. However, there have been few quantitative experimental studies of small-amplitude natural drop oscillations, although *Rayleigh* [1945] derived the appropriate formulas governing these vibrations almost a hundred years ago. It is the purpose of this investigation to provide accurate new data on the oscillation frequencies of water drops over a wide range of sizes and to correlate these data with the more limited results of other studies [*Blanchard*, 1948; *Brook and Latham*, 1968] and with the theory of Lord Rayleigh.

Experimental arrangement. The data analyzed consisted of film strips taken with a high-speed 16-mm camera with close-up attachments for high magnification and a framing rate of up to 4000 frames/sec [*Spengler and Gokhale*, 1971]. The drops were photographed in a large vertical wind tunnel [*Spengler*, 1971; *Spengler and Gokhale*, 1970] capable of suspending hundreds of drops simultaneously, and thus a great number of oscillating drops were observed. Only the clearest instances of pure oscillation were chosen, since on a two-dimensional film strip the possibility of rotations and other distortions due to turbulence can cause some confusion. Frequency was determined simply by counting the number of frames that were taken as the drop underwent deformation and returned to its original shape and then by dividing the framing rate by this number. The absolute maximum error involved should be of the order of 5 out of 50 frames for each oscillation, or about 10% deviation is possible in each frequency measurement. Figure 1 shows selected frames taken from a film strip of a large (5 mm) oscillating drop.

Since many drops were suspended at once, it was not possible to determine the diameter of a particular drop before suspension. This situation caused some difficulty, since the larger drops deform when suspended at their terminal velocity, and it is difficult to know the exact equivalent spherical diameter. Fortunately, during their oscillations most drops passed through a spherical shape, and their diameter was measured then. The random error in the diameter measurements is estimated at less than 10%. However, the possibility of a large (up to 15%) systematic error in the size measurements remains, because of an imprecise knowledge of the camera magnification.

Results. The results of this and other investigations are summarized in the log-log graph

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of Figure 2. The solid line represents the Rayleigh formula

$$f = [n(n-1)(n+2)]^{1/2}(2T/\pi^2 \rho d^3)^{1/2} \quad (1)$$

where f is the frequency, T is the surface tension, ρ is the density, d is the diameter, and n gives the mode of vibration. Using $T = 74$ dynes/cm, $\rho = 1$ g/cm^3, and $n = 2$ for the principal mode of vibration, we have

$$f = 11.0d^{-1.5} \quad (2)$$

where f is in hertz and d is in centimeters. The dashed line in the graph is a least-squares fit to the data from 29 observations of oscillating drops and is given by

$$f = (11.7 \pm 0.8)d^{-1.47 \pm 0.06} \quad (3)$$

The uncertainties shown in (3) are calculated from the least-squares fitting [see, e.g., *Bevington*, 1969] and represent only the random errors involved. As mentioned above, since the magnification on the films is not known exactly, a systematic error of up to 15% may exist in the diameter measurements of all the drops. This error will have no effect on the determination of the exponent in (3), and the agreement with the Rayleigh formula is seen to be quite good, even though the range in frequency was from 20 to 200 Hz. However, a 15% change in the diameter would change the coefficient in (3) by about 20%. Therefore the rather small deviation in this parameter from the Rayleigh formula may be fortuitous. The quantitative results are summarized in Table 1.

Discussion. For most drops the cause of the oscillations was turbulence, either from the air flow in the wind tunnel itself or from passage close to the wake of another drop. In five instances the vibrations were stimulated by the coalescence of two drops. Two other possible causes of oscillations, electric fields and the shedding of eddies [*Gunn*, 1949], were not studied.

All drops analyzed quantitatively exhibited vibrations that caused them to be, in the words of *Rayleigh* [1945, p. 371], 'alternately compressed and elongated in the direction of the axis of symmetry.' For the smaller drops (d = 1–3 mm), this type of oscillation was the only one seen. However, the larger drops (d = 4–7 mm), which are significantly deformed from spherical shape even when they were not oscillating, exhibited various other shapes, but no attempt was made to analyze these complicated patterns of distortion. Higher-order modes of

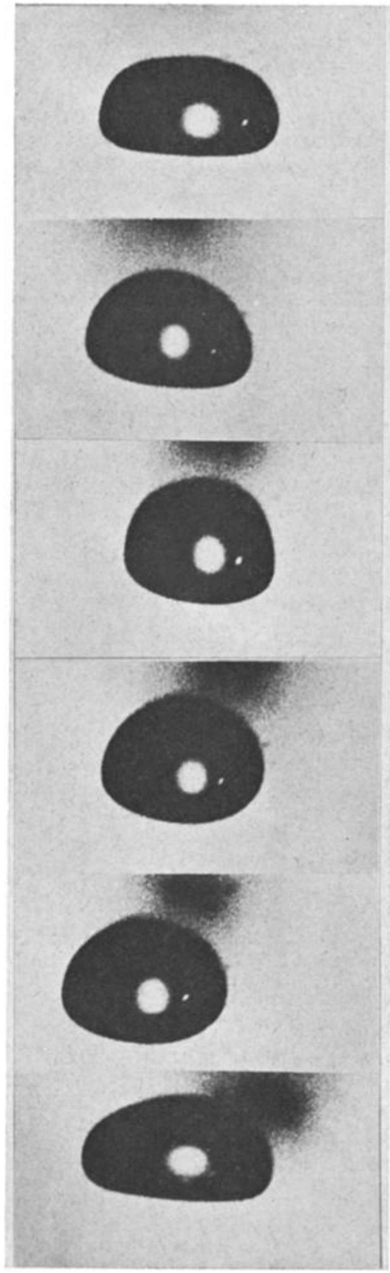

Fig. 1. Several stages in the vibration of a 5-mm water drop. In this case the total elapsed time from the beginning to the end of one oscillation cycle was 38 msec.

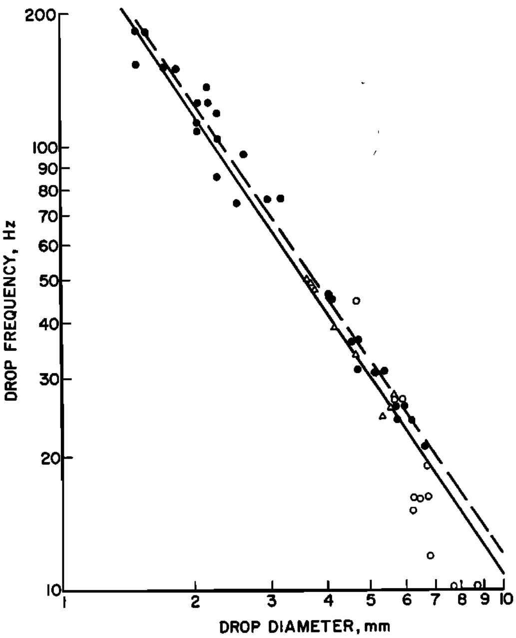

Fig. 2. Results from three different experimental investigations of the frequency of vibration of a water drop as a function of drop diameter. Open circles, results of *Blanchard* [1948]. Solid circles, results of this paper. Triangles, results of *Brook and Latham* [1968]. Broken line, least-squares fit to the data from this paper. Solid line, the Rayleigh theory.

vibration were not observed, although microwave studies [*Brook and Latham*, 1968] indicate that they may exist.

In all instances the amplitude of vibration was relatively small; that is, the maximum ratio of the major axis a to the minor axis b was less than 1.8 even for the larger drops. The large distortion, $a/b = 2.0$, has been discussed elsewhere [*Billings and Holland*, 1969].

Figure 2 exhibits results from three different investigations of the frequency of vibration for a freely suspended water drop. The data from *Blanchard's* [1948] study of large drops (d = 5.0–10.0 mm) indicates a substantial deviation from Rayleigh's formula, especially for the very largest drops. This deviation could be due to experimental uncertainty, since the photographs were obtained with a stroboscope (set at approximately 80 Hz), and the subsequent determination of the frequency of oscillation would appear to be quite difficult. A more likely reason for the discrepancy is that Blanchard measured more complicated forms of deformation, since he describes his drops as having the form of prolate ellipsoids with the major axis horizontal. This description does not correspond to the spheroidal deformations that are described by Rayleigh and that constitute the bulk of the observations in this investigation.

TABLE 1. Summary of Results

	Rayleigh's Theory	Least-Squares Fit to Exper. Data	Deviation
Coefficient	11.0	11.7 ± 0.8	0.7 (6%)
Exponent	1.50	1.47 ± 0.06	0.03 (2%)

The triangles of Figure 2 are the data points from the optical measurements of *Brook and Latham* [1968]. Their data consisted of high-speed (240 frames/sec) photographs of small spheroidal deformations, and they obtained good agreement with the theory in the range of their investigation (d = 3.5–7.0 mm). However, our results, as illustrated by the closeness of the exponent in the least-squares calculation, suggest that the Rayleigh formula applies over a much wider range, down to and including drops of the order of 1 mm in diameter. This agreement over a frequency range of an order of magnitude is interesting, since the 3/2 power law is the first and simplest approximation for an oscillating drop. In particular, when the effects of gravity and air flow are neglected, the 3/2 power law represents the very small deformation of an inviscid incompressible spherical drop. The work described in this paper suggests that the law has a more general range of validity, including water drops falling at their terminal velocity, as in a rain cloud.

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