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On the Metric of Space-time

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Maxwell's equations are obeyed in a one-parameter group of isotropic gravity-free flat space-times whose metric depends upon the value of the group parameter. This value can be determined by experiment; if it is zero, the metric is Minkowski's. If it is non-zero, sources break Poincare invariance and local frequencies of electromagnetic waves change as they propagate. If the group parameter is positive, velocity-independent red shifts develop and the group parameter plays a role similar to that of Hubble's constant in determining the relation of these red-shifts to propagation distance. In the resulting space-times, the velocity-dependence of "Doppler shifts" is a function of propagation distance. If the group parameter and Hubble's constant have the same order of magnitude, observed frequency shifts in radiation received from stellar sources can imply source velocities quite different from those implied in Minkowski space. In such space-times electromagnetic waves received from bodies in distant Kepler orbits undergo frequency shifts indistinguishable from the effects attributed to the presence of dark matter and dark energy in Minkowski space.

Keywords: Space-time Metric, Hubble, Dark Matter, Astrophysics, Electrodynamics, Red Shift

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1. Introduction

In 1909 Bateman and Cunningham proved that Maxwell's equations are invariant under inversions and the transformations of a fifteen-parameter conformal Lie group. **Bateman** (1909); **Cunningham** (1909). Cunningham connected their work to the studies of Einstein's special relativity that were then just beginning, and wrote one of the first books dealing with special relativity. **Cunningham** (1914). Properties of conformal euclidian three-space are investigated in **Robertson** (1925), the doctoral thesis of Bateman's student, H. P. Robertson. However, several decades elapsed before the special conformal subgroup of the Bateman-Cunningham group received further attention. Then, just as WW II ended, **Hill** (1945) showed that this subgroup contains a one-parameter subgroup whose infinitesimal transformations define a relation identical to Hubble's law if one identifies the value of the group parameter with Hubble's constant.

Hill's discovery was overlooked for over 60 years. It is not mentioned in a summary of properties of the Bateman-Cunningham group written by **Fulton, Rohrlich & Witten**(1962). In the year 2000, unaware of Hill's discovery, Hoyle, Burbidge, and Narlikar used the Bateman-Cunningham Lie group in their development of a generalized relativistic cosmology, and in 2010, Tomilchik argued that Hubble's relations are a consequence of properties of transformations of the special conformal subgroup of the Bateman-Cunningham group. **Hoyle, Burbidge & Narlikar** (2000); **Tomilchik** (2010). Hill's paper is however listed in the review of papers dealing with conformal invariance written by **Kastrup** (2008). This review led the present author to the realization that the value of Hill's group parameter can be determined by experiment.**Wulfman** (2010). An extensive list of recent references to research on the physical role of conformal transformations is contained in a recent article on the quantum mechanics of accelerated relativistic particles. **Calixto, *et al*** (2012) . The present paper describes the effects of the transformations of Hill's subgroup upon electromagnetic waves and Minkowski's metric. It is shown that the transformations of this one-parameter group of isotropic transformations convert the well known plane-wave solutions of Maxwell's equations into a one-parameter family of plane-wave solutions with wave-numbers that slowly change as the waves propagate. The same transformations establish that Minkowski's metric is a member of a one-parameter family of metrics. This property of Hill's relation suggests that measurements of spectra believed to be made in Minkowski space may actually be measurements made in a space-time with a metriic defined by a non-zero value of the group parameter. To investigate this possibility the results of imagined spectral measurements are expressed in two sets of coordinates related by the transformations of the group. Frequency shifts that would be observed in Minkowski space are thereby related to frequency shifts that would be observed in these other space-times. In developing these relations we have made use of properties of Lie transformation groups that can be found in the recent texts **Cantwell** (2002), **Hydon** (2000), **Stephani** (1989), **Wulfman** (2011).

The final section of the present paper contains a very brief discussion of the relation between metrics applicable in the absence of, and in the presence of gravitational fields. In this we exploit analyses in **Milne** (1935) and **Page** (1936a,b). These deal with two problematic aspects of Einstein's special relativity that arise from Einstein's definition of equivalent observers: its dependence upon measurements made with widely separated rods and clocks, and the requirement that these have no relative accelerations.. Neither requirement can be met at astronomically relevant separations in a uniformly expanding universe - in which widely separated objects are expected to have substantial relative accelerations. These are accelerations that would be observed if Hubble's relations describe the results of observations made in any of the isotropic space-times allowed by the Bateman-Cunningham invariance of Maxwell's equations, space-times in which there are no gravitational fields and no repulsive forces.

2. The Isotropic Transformations of the Special Conformal Group

To uncover implications of Hill's discovery we make use of the one-parameter group of isotropic special conformal transformations to interconvert vectors **X**, and **X'**, with

$$\mathbf{X} = (x^1, x^2, x^3, x^4),\ x^4 = ct,\quad \mathbf{r} = (x^1, x^2, x^3), \qquad (2.1a\text{-}c)$$

and

$$\mathbf{X'} = (x'^1, x'^2, x'^3, x'^4),\ x'^4 = ct',\quad \mathbf{r'} = (x'^1, x'^2, x'^3), \qquad (2.1d\text{-}f)$$

If one defines the dilatation generator

$$D_3 = \Sigma_1^3\, x^k \partial/\partial x^k = r\partial/\partial r\,, \qquad (2.2a,b)$$

then the Lie generator of the group can be expressed as

$$C_4 = (r^2 + x^{4\,2})\partial/\partial x^4 + 2x^4 D_3\,. \qquad (2.2c)$$

Here, and in the following, italicized variables may take on initial values, final values, and all values in between, as the group parameter γ varies from zero to its final value. Changing γ to -γ, and interchanging r, r' and x^4, x'^4 produces the inverse transformations. The transformations can be carried out by the operator $\exp(\gamma C_4)$. It produces the projective transformations

$$(x^4 - r) \to x'^4 - r' = \exp(\gamma C_4)(x^4 - r) = (x^4 - r)/(1 - \gamma(x^4 - r)),$$

and (2.2d-i)

$$(x^4 + r) \to x'^4 + r' = \exp(\gamma C_4)(x^4 + r) = (x^4 + r)/(1 - \gamma(x^4 + r)).$$

Acting on r, x^4, $\exp(\gamma C_4)$ produces the transformation

$$r \to r' = r/D, \quad x^4 \to x'^4 = (x^4 - \gamma(x^{4\,2} - r^2))/D,$$

(2.2j-m)

$$D = (1-\gamma x^4)^2 - (\gamma r)^2 = (1-\gamma(x^4+r))(1-\gamma(x^4-r)),$$

With its inverse

$$r' -> r = r'/D', \quad x'^4 -> x^4 = (x'^4 + \gamma(x'^{4\,2} - r'^2)/D',$$

(2.2n-s)

$$D' = (1+\gamma x'^4)^2 - (\gamma r')^2 = (1+\gamma(x'^4 + r'))(1+\gamma(x'^4 - r')).$$

The transformations of the Bateman-Cunningham group do not alter angles between spatial vectors, and those generated by C_4 do not alter the origin of coordinates. They thus convert

$$\mathbf{X} = (r \sin(\theta)\cos(\phi),\ r \sin(\theta)\sin(\phi),\ r \cos(\theta),\ x^4), \tag{2.3a}$$

to

$$\mathbf{X'} = (r' \sin(\theta)\cos(\phi),\ r' \sin(\theta)\sin(\phi),\ r' \cos(\theta),\ x'^4). \tag{2.3b}$$

If **X** has its origin at the vertex of a light cone where a source is located, then **X'** will originate at the same source. If the time coordinates have non-negative values, we will say the coordinates are those of points on "source cones" - on which, $x^4 = r$ and $x'^4 = r$.' A continously emitting source produces a continuum of light cones with a common time axis. The vertex of any one of these may be considered that of a source cone. The origins of a source cone will be assumed to have zero velocity if coordinates on it are not being compared with coordinates on another.

The first extension of C_4 is

$$C_4^{[1]} = C_4 + 2r(1-v_r^2)\partial/\partial v_r\ ,\quad v_r = dr/dx^4. \tag{2.4a,b}$$

It generates transformations of radial velocities in which:

$$v_r -> v_r' = (Av_r + B)/(A + Bv_r).$$

Their inverse is (2.4c-e)

$$v_r = (Av'_r - B)/(A - Bv'_r).$$

In these relations

$$A = \partial r'/\partial r = \partial x'^4/\partial x^4 = (1+\gamma x'^4)^2 + (\gamma r')^2 = ((1-\gamma x^4)^2 + (\gamma r)^2)/D^2,$$

(2.4f-n)

$$B = \partial r'/\partial x^4 = \partial x'^4/\partial r = 2(\gamma r')(1+\gamma x'^4) = 2\gamma r\,(1-\gamma x^4)/D^2.$$

D is the function defined in eqn (2.2). It may be replaced by D'. Equations (2.4c-n) express the action of exp($\gamma C_4^{[1]}$) and exp($-\gamma C_4^{[1]}$) on v_r. Because of the $(1-v_r^2)$ factor in $C_4^{[1]}$, the effects of these transformations generated by $C_4^{[1]}$ weaken as v_r approaches the speed of light, 1, which they cannot alter. Because both C_4 and $C_4^{[1]}$ are quadratic functions of γr, γx^4, they generate transformations that are weak functions of both the coordinates and the group parameter near sources – terrestrial as well as celestial.

As C_4 annihilates,

$$I_1 \equiv (x^{4\,2} - r^2)/r, \tag{2.5a}$$

the transformations generated by it leave this function invariant. Similarly, the transformations (2.4c-e) generated by $C_4^{[1]}$ leave invariant the function

$$I_2 = ((x^4 - r)/(x^4 + r))((1+v_r)/(1-v_r))^{1/2}, \tag{2.5b}$$

The Lorentz invariant analogs of I_1 and I_2 are

$$(x^{4\,2} - r^2), \quad ((x^4 - r)/(x^4 + r))^{1/2}(1 + v')^{1/2}/(1 - v')^{1/2}. \tag{2.5c.d}$$

Note that they are not $\gamma \to 0$ limits of the invariants (2.5a,b), which state a general relation that is valid for all values of γ.

Because C_4 does not commute with any of the generators $\partial/\partial x^k$, the transformations generated by $C_4^{[1]}$ are not translation invariant. A translation that moves the origins of the vectors **X** from **O** to

$$\mathbf{A} = (a^1, a^2, a^3, a^4)$$

converts **X** to (2.6a,b)

$$\mathbf{X} + \mathbf{A}.$$

It moves the origin of the vectors **X'** from **O** to

$$\mathbf{X_A}' = \mathbf{X}' + \mathbf{A}, \tag{2.6c}$$

which is not the same as **(X + A)'**.

Spatial translations together with time translations produce source cones with parallel axes. The finite translations change C_4 to

$$C_{4tr} = ((\boldsymbol{r} + \mathbf{a})\cdot(\boldsymbol{r} + \boldsymbol{a}) + (x^4 + a^4)^2)\partial/\partial x^4 \tag{2.7}$$

$$+ 2x^4 D_3 + 2a^4 D_3 + 2(x^4 + a^4)\Sigma_1^3 a^k \partial/\partial x^k.$$

The generator of extended radial Lorentz transformations is

$$K_r^{[1]} = r\partial/\partial x^4 + x^4\partial/\partial r + (1- v_r^2)\partial/\partial v_r . \quad (2.8a)$$

If α is the group parameter, the operator of finite transformations, $\exp(\alpha K_r^{[1]})$, carries out the transformations:

$$x^4 + r -> e^{\alpha}(x^4 + r), \quad x^4 - r -> e^{-\alpha}(x^4 - r),$$

(2.8b-d)

$$v_r -> \tan(\alpha) v_r .$$

3. Electromagnetic Plane Waves with Evolving Wavenumbers

In this Section we show that C_4 generates transformations which define electromagnetic plane waves whose frequencies are a function of propagation distance. These are waves that are potentially observable at great distances from sources.

An electromagnetic plane wave with wave number k propagating in the direction of $\mathbf{x}^3$, may be defined by the equation

$$\Psi = (\mathbf{E} + \mathbf{H})\cos(k(x^3 - x^4) + \Phi_0)$$

(3.1a-e)

$$\mathbf{E}\cdot\mathbf{x}^3 = 0, \quad \mathbf{H}\cdot\mathbf{x}^3 = 0, \quad \mathbf{E}\cdot\mathbf{H} = 0, \ |\mathbf{E}| = |\mathbf{H}| = \text{const} .$$

Both **curl**(**E**) and **curl**(**H**) vanish, and the function $\cos(k(x^3 - x^4) + \Phi_0)$ satisfies the wave equation which reduces to

$$(\partial/\partial x^3 - \partial/\partial x^4)(\partial/\partial x^3 + \partial/\partial x^4)\cos(k(x^3 - x^4) + \Phi_0) = 0. \quad (3.1h)$$

Replacing x^3 by r converts these plane waves to spherical waves.

Now, on setting $r = x^3$, the inverse of eqns (2.2d,e) is

$$X^3 - x^4 = (x'^3 - x'^4)/(1 - \gamma(x'^3 - x'^4)). \quad (3.2)$$

Consequently, in the X' system the wave defined by eqns (3.1) becomes defined by the eqn

$$\Psi'(x'^3, x'^4) = (\mathbf{E'} + \mathbf{H'})\cos(k((x'^3 - x'^4)/(1+\gamma(x'^3 - x'^4))) + \Phi_0), \qquad (3.3a)$$

with

$$\mathbf{E'} = \mathbf{E}, \ \mathbf{H'} = \mathbf{H}. \qquad (3.3b,c)$$

One thus need only require that Ψ' continues to satisfy the wave equation. Eqn (3.2) ensures that it does everywhere that the denominator does not vanish. Because **curl**(**E'**) and **curl**(**H'**) vanish, no external potentials or currents are required to produce the EM waves defined by (3.3). The wave that carries the fields can be considered to have wavenumber,

$$k' = k/(1 - \gamma(x'^3 - x'^4)). \qquad (3.3d)$$

At a stationary point with coordinate x'^3_0 the waves oscillate with a frequency

$$\omega = k'c = kc/(1 - \gamma(x'^3_0 - x'^4)). \qquad (3.3e)$$

This frequency very slowly decreases with time if γ is the order of magnitude of Hubble's constant.

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4. The One-parameter Family of Metrics

Einstein based his theory of special relativity on his analysis of the relation of Lorentz transformations to observations. **Einstein** (1906). Minkowski soon recognized that the Lorentz transformations could be used to define the metric of a space-time, the space-time of special relativity that now bears his name. **Minkowski** (1910). It is invariant under the Lorentz transformations, the rotations, and the space-time translations that comprise the Poincare group.

Equations (2.3f-n) imply that the transformations generated by $C_4^{[1]}$ establish the relations

$$dr = A\,dr' + B\,dx'^4, \quad dx^4 = B\,dr' + A\,dx'^4. \qquad (4.1a,b)$$

These, together with eqns (2.2i-m) imply that, when expressed in an x'^4, **r'** spherical polar coordinate system, the Minkowski metric

$$ds^2 = (dx^4)^2 - (dr^2 + r^2(\sin^2(\theta)d\theta^2 + d\phi^2)), \qquad (4.2a)$$

becomes

$$ds'(\gamma)^2 = G(\gamma)((dx'^4)^2 - (dr'^2 + r'^2(\sin^2(\theta)d\theta^2 + d\phi^2)). \qquad (4.2b)$$

The components of the metric tensor $G(\gamma)$ are

$$g_{ij}(\gamma) = \pm\, \delta_{ij} \{(1- \gamma(r' - x'^{4}))(1+ \gamma(r'+x'^{4}))\}^{2}.$$

(4.2c,d)

$$= \pm\, \delta_{ij} \{(1 - \gamma(x^{4} - r))(1- \gamma(r + x^{4}))\}^{-2}.$$

Minkowski's metric tensor is thus G(0).

The G(γ) metric is invariant under the transformations of the rotation group. If γ is not zero, G(γ) and $ds'(\gamma)^2$ are not invariant under the translations and Lorentz transformations of the Poincare group. Thus a G(γ) metric expresses the properties of a space-time in which each source reduces Poincare invariance to a rotational invariance about itself. Poincare invariance becomes an approximate invariance of space-time. On each light cone the continuous group of covariant transformations is the three-parameter group SO(3). Its generators and other seven generators of the Poincare group define a realization of the Poincare group. If Q denotes any one to these seven generators, the generators of this realization are

$$Q\gamma \;\underline{=}\; \exp(\gamma C_4)\, Q \exp(-\gamma C_4). \qquad (4.3)$$

Each of these operators generate finite transformations that can convert coordinates r', x'^{4}, of points on a given source cone to corresponding points on a one-parameter family of other source cones. The transformations defined by the first extensions of the generators C_4 and Q determine velocities, v_r', of objects or observers at these points.

5. Interpretation of Observed Spectral Shifts

The C_4 transformations relating primed and unprimed variables define mappings which relate two sets of coordinates, (r, t, v_r) and (r', t', v'_{rr}), of the same point. These passive transformations will be considered to become active transformations when the points are attached to moving physical objects. Motions of physical points in Minkowski space will then be described by changes in the value of the (r, t, v_{rr}) coordinates of the points, and motions in G(γ) space-times will be described by changes in the value of the (r', t', v'_{rr}) coordinates of the same physical points.

A point Π fixed on an electro-magnetic wave can be assigned the pair of coordinates $(r_\pi$, $x^4{}_\pi)$, and $(r'_\pi, x'^{4}{}_\pi)$. Let Π_1 and Π_2 be two such points separated by one wavelength and centered at Π on a monochromatic EM wave propagating in otherwise field-free space-time. Let an observing apparatus at a point P momentarily on the wave's source cone have coordinates r_{ob}, $x^4{}_{ob}$, v_{rob}, and r'_{ob}, $x'^4{}_{ob}$, $v_r{}'_{ob}$. Let ν and ν' be frequencies, and λ and λ' wavelengths measured at P in the two systems. Then measuring frequencies in the two sets of coordinates determines the times $\Delta t = \Delta x^4/c$ (= $\Delta r/c$), and $\Delta t' = \Delta x'^4/c$ (= $\Delta r'/c$), that it takes one wavelength to pass the point P. The observations thereby relate time intervals at P in the two systems to frequency and wavelength measurements in the two systems via the equations

$$\nu/\nu' = \lambda'/\lambda = \Delta x'^4/\Delta x^4. \quad (5.1a\text{-}c)$$

We shall assume that the group parameter γ is small enough to ensure that approximating the last ratio by dx'^4/dx^4 leads to no detectable error. Then,

$$\nu/\nu' = \lambda'/\lambda = dx'^4/dx^4 = \partial x'^4/\partial x^4 + (\partial x'^4/\partial r)\, dr/dx^4. \quad (5.2a\text{-}c)$$

In this, dr/dx^4 is the radial velocity, v_r, of the observing apparatus at r, t, in Minkowski space. Eqns (5.2), (2.3) together imply that

$$\lambda'/\lambda = a' + b'v_r. \quad (5.3a)$$

with

$$a' = (1+\gamma x'^4)^2 + (\gamma r')^2, \quad b' = 2(\gamma r')(1+\gamma x'^4). \quad (5.3.b,c)$$

To simplify notation, define

$$\tau = \gamma x^4 = \gamma\, ct, \quad \tau' = \gamma x'^4, \quad \rho = \gamma r, \quad \rho' = \gamma r'. \quad (5.3d\text{-}h)$$

On source cones, $\tau = \rho$ and $\tau' = \rho'$.

When expressed in terms of the velocity and position of the apparatus in G(γ) space-time, eqn (5.3a) becomes

$$\lambda'/\lambda = a' + b'(a'v'_r - b')/(a' - b'v'_r). \quad (5.3i)$$

These equations state that λ'/λ is a function of the time, x'^4, as well as r'. However on a source cone $x'^4 = r'$, so for observations of any particular source

$$b' \to bb = 2\rho'(1+\rho'), \quad a' \to 1 + bb. \quad (5.4a\text{-}c)$$

If v' is zero, eqns (5.3) imply that

$$Z'_0 = (\lambda'/\lambda)|_{v'=0} - 1 \equiv \lambda'_0/\lambda_{ref} - 1$$

$$= (a'^2 - b'^2)/a' - 1 = bb/(1+ bb) . \qquad (5.4d\text{-}g)$$

Thus

$$Z'_0 = 2\rho'(1+ \rho')/(1+2\rho'(1+ \rho')),$$

$$= 2\rho'(1 - \rho')+ O((\gamma r')^4). \qquad (5.4h,i)$$

We suppose that γ is so small that reference wave-lengths, λ_{ref}, produced and observed by local sources, have the same observable value in Minkowski space as in $G(\gamma)$ space-times. Eqns (5.4) then relate observables in a $G(\gamma)$ space-time.

If $\gamma > 0$, eqns (5.4) predict red-shifts that depend solely upon the distance from the source to the observer. They may be compared with the Hubble relations

$$c(\lambda'_{obs}/\lambda_{obs} - 1) = H_0 R = V,$$

with (5.5a-d)

$$H_0 = (2.19 \pm 0.56) \times 10^{-18} \text{ sec}^{-1}, \; H_0/c = (7.3 \pm 1.9) \times 10^{-25} \text{ km}^{-1},$$

the relations which are considered to apply in Minkowski space when $V << c$, and peculiar velocities are zero. If one supposes that $2\gamma = h_\gamma H_0$ where h_γ is a dimensionless constant of order 10^1, then $\gamma = h_\gamma 10^{-25}$ km^{-1}. **Figure 1** depicts the dependence of Z_0 upon ρ'.

When v' is non-zero, eqns (5.3) through (5.4) define additional, velocity-dependent, shifts

$$Z'_{v'} \equiv \lambda'_{v'} /\lambda'_0 - 1. \qquad (5.6a)$$

As eqn (5.4f) implies

$$bb = Z'_0/(1- Z'_0), \qquad (5.6b)$$

eqn (5.3) implies

$$Z'_{v'} = v'_r Z'_0/(1- v'_r Z'_0). \qquad (5.7)$$

On applying eqn (5.6b) to eqn (5.3), one finds that the full expression for Z' becomes

$$Z' = \lambda' /\lambda_{ref} - 1 = Z'_0 (1 + v'_r)/(1- v'_r Z'_0).$$

Thus (5.8a-c)

$$\lambda'/\lambda_{ref} - 1 = 2\rho'(\rho'+1)(1+ v'_r)/\{1+ 2\rho'(\rho'+1)(1- v'_r)\}.$$

This expression relating Z' to v'_r and $\rho' = \gamma r'$ is plotted in **Figure 2**, in which $Zp \equiv Z'$ and $V_p \equiv v'_r$. Equation (5.8c) is to be compared with the Hubble equations that apply when peculiar velocities are radial velocities. Then one has

$$\lambda/\lambda_{ref} - 1 = (H_0/c)r + v_r \ . \quad (5.9)$$

Figure 3 displays the relation between ρ', v'_r, and Z' when Z' < 0.02. Particularly noteworthy is the weak dependence of Z' upon v'_r that is evident for values of $P \equiv \rho'$ that are < 0.1.

The general relation between the velocities in equations (5.8c) and (5.9) is a consequence of the non-linear distance dependence of velocity-dependent frequency shifts in G(γ) space-times. If observations have established the frequency shifts that correspond to many values of R and $v_{r\,pec}$ - or to many values of r' and v'_r, - it could, in principle, be possible to determine which of the two relations most accurately describes the observations.

Very surprisingly it appears that, for values of v << 1, the necessary observations have been made. They are the subject of the next Section.

6. Observations of Distant Keplerian Motions

In 1914, V. Slipher discovered the existence of internal rotations in nebulae. **Slipher** (1914). When **F. Zwicky**(1933) observed that motions in the Coma cluster of nebulae do not obey the virial theorem, he suggested that the failure to obey it might be due to unobserved "dunkel Materie", or to a failure of the known laws of physics. The failure is now known to be very general, and it is commonly attributed to the presence of unobserved mass, and less commonly, to a modification of Newtonian dynamics. The following paragraphs investigate the possibility that the apparent failure to obey the virial theorem could arise from interpreting observations in a G(γ) space-time as observations in Minkowski space.

We follow **Peebles** (1993) and exploit the fact that for circular orbits the mean values in the virial theorem become single values. For a body of mass m and orbital speed v orbiting a center with effective mass M >> m, the speed is related to r, the radius of the orbit by the well known equation

$$v = (MG/r)^{1/2}. \qquad (6.1)$$

When observations of luminosities and 21 cm microwave intensities establish that the luminous mass of a nebula appears to lie within a sphere of radius $r_0 < r$, an especially clear analysis of the apparent failure of this eqn is obtained if the rotational velocities are observed "edge-on". In most cases the observed tangential velocities are indistinguishable from those of bodies moving on circular orbits with center at the center of mass of the nebulae. Tangential velocities of bodies in Kepler orbits then become radial velocities along the line of sight of the observer, which is orthogonal to the line from the orbiting body to the nebular center. When the velocities of such motions of massive bodies, and hydrogen, in spiral galaxies have been deduced from observed Doppler shifts it has become abundantly evident that massive bodies moving on arcs with radii greater than r_0 , have velocities that appear to change but little as r increases. This is a result that would be obtained if there is sufficient dunkel Materie within all the surrounding spheres of radii > r_0 to keep MG/r_0 independent of the value of r_0.

The discussion of eqns (5.9) suggests that when observations such as these are expressed in r', x'^4, v'_r coordinate systems, the implications may be quite different. We therefore ask what would be observed in a G(γ) space-time if a distant orbiting body of mass m is moving on a circular Kepler orbit centered on a body of mass M at rest with respect to the observer.

In answering this question we consider the orbiting bodies to be origins of source cones, on which observed orbital tangential velocities become radial velocities of the observer relative to the source. If the observations are being made in an r', x'^4, v'_r, system, then the spectral shifts Z' arising from v'_r will satisfy the relation

$$Z' = \{ 2P'(1 + P')(1 + v'_r)\}/\{(1 + 2P'(1 + P')(1 - v'_r)\}, \qquad (6.2)$$

with $P' = \gamma r'$, and r' = D, the distance from the body of mass m to the observer. The velocity v'_r is the radial velocity of the observer with respect to the body. Let r, r', represent radial coordinates of the central mass in the source-cone system, and let $\underline{v}$, $\underline{v}'$ represent the value of its tangential velocity, $\underline{\mathbf{v}}$, in the source-cone system. Then, $\underline{v}'_r = - v'_r$, and we replace eqn (6.1) with the eqn

$$v'_r = \pm\, (MG/\underline{r}')^{1/2},$$

in which (6.3a-c)

$$\underline{r}' = \exp(\gamma\, C_4^{[1]})\underline{r}\ , \quad \underline{v}'_r = \exp(\gamma\, C_4^{[1]})\underline{v}_r\ .$$

It is revealing to assign values to MG that are those of two of the galaxies investigated in the paper "Dark Matter in Spiral Galaxies" by **van Albada** & **Sancisi** (1986). The authors analyze orbital motions observed in five galaxies whose distance from earth ranges from 3.2 to 9.2 mpc. The observed trajectories of bodies orbiting these galaxies have radii ranging from a few, to 36 kpc, and deduced orbital velocities, $v = V/c$, less than 10^{-3}. The orbital radii are such a small fraction of the distance D, that the distance from the observers to their centers is at most 1% greater than D. As a consequence, the first factor, $2\rho'$, in eqn (6.2), gives Z' a linear dependence upon D, the distance from earth to the galaxy, a dependence like that of the Hubble relation (5.5a) if one supposes that 2γ and H_0/c have the same order of magnitude.

However, the relation of Z' to peculiar velocities is entirely different than is supposed in the Hubble relations

$$cZ = H_0R + V_{Pec}\ ,\ H_0R = V. \qquad (6.4a,b)$$

In the cases considered here, the peculiar velocities have the same magnitude as the velocities of the center of mass of the nebulae relative to the source. Consequently, for v' values of the order of 10^{-4} to 10^{-3}, the factors $(1 \pm v'_r)$ in eqn (6.2) differ from unity by less than 0.1%. The resulting effect on values of Z' is displayed in **Table 1**. It contains values of Z' that would result from measuring spectra of radiation emitted from bodies with velocities $\underline{\mathbf{v}}'$ orbiting centers with masses equal to those of NGC 2403 and NGC 2841, the galaxies with the smallest, and the largest, mass of the galaxies considered by van Albada and Sancisi. The radii $\underline{r}'$ of the orbits, and the orbital velocities, $\underline{v}'$, of the orbiting bodies, are labeled R_{Newt} , V_{Kep} in the Table. In it, the hundred-fold change in the radii R_{Newt} changes the local orbital velocities V_{Kep} by a factor of 10. If the values of Z' are interpreted as arising from peculiar velocities in Minkowski space, they produce the values denoted V_{Mink} . For NGC 2403, over the whole range of R_{Newt}, the value of Z' and V_{Mink} vary by .02%. For NGC 2841 they change by .23%. These results are a direct consequence of the very weak dependence of Z' upon velocity that is depicted in **Figure 3**. When Z' becomes as large as 0.2, increasing the velocity from 0 to 10% of the velocity of light makes a 10 % change in Z'.

The results in the Table, taken together with the analysis in the previous Section, make it evident that if earth-based observations are actually being made in a $G(\gamma)$ space-time with $2\gamma \simeq H_0/c$, it is not necessary to modify Newtonian dynamics, or introduce dark matter, to explain the observations; they are the consequence of a weak dependence of Z' on V_{Kep}. This conclusion applies to observed frequency shifts in the radiation received from bodies orbiting a large category of distant galaxies.

The $V_{Kep} \rightarrow 0$ entries in **Table 1** illustrate another revealing property of $G(\gamma)$ metrics: observations in a $G(\gamma)$ space-time do not require the presence of dark energy, or any other type of mass, to explain the spectra of radiation received from objects moving at non-relativistic velocities in regions arbitrarily distant from centers of gravitational attraction. This implies that current estimates of the total mass of the universe may require considerable revision.

7. Concluding Remarks

The mere fact that Maxwell's equations allow a $G(\gamma)$ metric implies that Minkowski's metric cannot be assumed *a priori* to be the metric of physical gravitation-free space-time. Each source of electromagnetic radiation breaks the Poincare symmetry of space-time with A Minkowski metric. Thus, observed shifts in the wavelengths of stellar radiations ought not be assumed to imply the relation of frequency shifts to recessionary velocities and peculiar velocities that are codified in Hubble's relations.

The analyses in the previous pages also establish that the need to introduce dark matter or a modified Newtonian physics could be eliminated if frequency shifts are related to velocities in a space-time in which the $G(\gamma)$ metric is one in which the value of γ has a value approximately half that of H_0/c.

To date, observations made over distances as great as several AU have confirmed the view that Minkowski's metric is the metric of gravity-free space-time, but it has not seemed at all possible to determine whether the metric is equally valid over distances relevant to Hubble's relation. However, the value of the group parameter γ, can be determined by measuring successive return times and frequency shifts within radar wave-pulses sent to receding spacecraft as they move outward beyond the solar system.**Wulfman** (2010). The uncertainty in the resulting value of the group parameter can be expected to be less than the uncertainty in the value of Hubble's constant.

If the value turns out to be indistinguishable from zero, the known range of validity of Minkowski's metric will be greatly extended, and much existing astrophysics and cosmology will receive further validation. If the experiments confirm a non-zero value for γ, they will establish that sources of electromagnetic radiation, in fact, break Poincare and Lorentz invariance in a way that is both subtle and fundamental. Such results would have another consequence. In **Tomilchik** (2010) and **Wulfman** (2010, 2011), it is argued that the values of the "Pioneer Anomalies" roughly correspond to the value of the Hubble constant because the anomalies are primarily a consequence of the Bateman-Cunningham invariance of Maxwell's equations. However, **Turyshev *et al*** (2012) argue that the anomalies are due to an incorrect assessment of the reflectivity of the Pioneer vehicles. The proposed experiments would determine the extent to which each explanation is correct. <u>More importantly, the experiments would establish the metric governing the transmission of electromagnetic waves over distances of at least 70AU</u>.

If it turns out that the value of 2c and Hubble's constant are indistinguishable, the standard interpretation of the spectra of light propagating from sources over distances on the scale of megaparsecs will need to be revised. The analysis set forth in Section 6 establishes that such results would make it possible to reduce the number of hypotheses currently required to relate astrophysical observations. It is also evident that such experimental results would provide a foundation for cosmologies that extend that of Hoyle, Burbridge, and Narlikar..

There are revealing connections between the work reported here and the analyses presented in Milne's *Relativity Gravitation and World Structure*, **Milne** (1935), which stimulated much discussion at the time; c.f., **Temple** (1939) and **Whitrow** (1996). Milne lays the foundations for his "kinematic relativity" by imagining that two observers use light signals to compare measurements of time intervals made using their own clocks. The distance to an object is measured by the time it takes for a light signal to go to and return from the object. Though his resulting definition of "equivalent observers" does not depend upon Einstein's use of hypothetical rigid measuring rods, Milne shows that <u>if</u> his equivalent observers have <u>constant</u> relative velocity, their observations will agree because their time and distance measurements are related by Lorentz transformations. As his arguments assume translation invariance, the resulting theory is one in which equivalent observers are those whose observations are covariant under the transformations of the Poincare group.

At several points Milne replaces his remote observer with a mirror that reflects signals back to a local observer, but he does not use it to eliminate the need for a distant clock. Thus, he does not eliminate the dependence of special relativity upon the hypothesized use of measuring instruments that could not then, and cannot now, be carried to distances as astronomically close as that to Alpha Centauri. The foundations of the first theory of relativity to eliminate a dependence upon such imaginary experiements are developed in **Page** (1936a,b). In these two papers, Page also deals with another important aspect of Milne's work. In his book, Milne points out that Hubble's equation $V = H_0R$: implies that observers at points where R is different have different radial velocities, V. On light cones, R is proportional to time, and Hubble's equation requires that such observer's have relative accelerations. The invariance transformations of Maxwell's equations that convert Minkowski space to $G(\gamma)$ space-times have a similar property, for a different reason: they convert constant radial velocities $v_r < 1$ to radial velocities $v'_r < 1$ that are position-dependent and time-dependent.

The experiments we have proposed would determine a fundamental property of space-time metrics without relying upon hypothesized properties of distant clocks and rigid measuring rods. They would thereby avoid the underlying problems associated with the fact that accelerations in a $G(\gamma)$ space-time may, in principle, differently affect instruments, as well as measurements made with instruments, distant from one another. If γ is nonzero, points in space-times with "local" $G(\gamma)$ metrics have relative accelerations dV_r/dt that are not constant. Though these may be minute when they are the same as those implied by Hubble's equations, they are not due to gravitational fields, and the relation between $G(\gamma)$ metrics and the metrics of general relativity is not the same as the connection between Minkowski's metric and the metrics of general relativity. In this connection, it should be noted that the inverse of transformation (4.2c), defined by eqn (4.2d), is undefined on lines where its factors vanish. The transformation is nonsingular at all other points, and Einstein's equivalence principle is there satisfied because the inverse of eqns (4.1) converts the $g_{jj}(\gamma)$ into the functions $g_{jj}(0)$ - all of whose first derivatives $\partial g_{jj}(x)/\partial x^m$ vanish. (c.f. **Weinberg**(1972)).

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TABLE 1

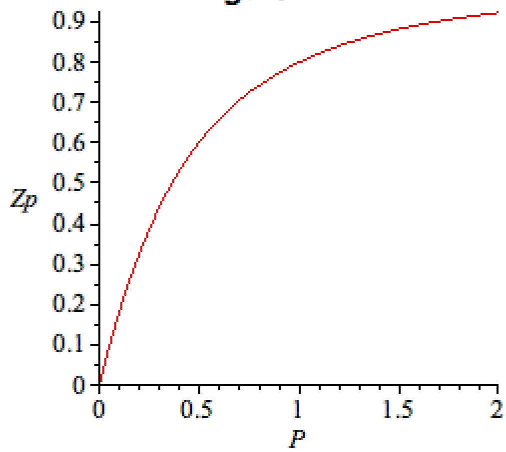
	D (Mpc)	MG (10^{11}km)	R_{Newt} (10^{17}km)	V_{Kep} (10^{-4}c)	V_{Mink} (10^{-4}c)	Z' (10^{-4})
NGC 2403	3.2	1.176	1.543	8.730	0.741h	0.7412h
			3.086	6.173	0.741h	0.7411h
			6.172	4.365	0.741h	0.7409h
			154.3	0.873	0.741h	0.7407h
			infinite	0.000	0.741h	0.7406h
NGC 2841	9.0	9.555	1.543	24.89	2.088h	2.0881h
			3.086	17.60	2.087h	2.0865h
			6.172	12.44	2.086h	2.0855h
			154.3	2.489	2.083h	2.0834h
			infinite	0.000	2.083h	2.0828h

Velocities of bodies moving in circular orbits outside of spherically distributed masses of galactic magnitude

The first D and MG entries apply to NGC2403. The lower entries apply to NGC2841. V_{Kep} is the computed classical velocity of a star in a circular orbit of radius R_{Newt} , and center at the center of mass of the indicated galaxy. Z' is the spectral shift that would be observed at distance D from the star, measured in G(γ) space-time with $2\gamma = h\ H_0/c$. V_{Mink} is the velocity that would be deduced from the relation V = cZ' valid in Minkowski space when V << c. The assumed value of H_0/c is 7.5×10^{-25} km^{--1}.

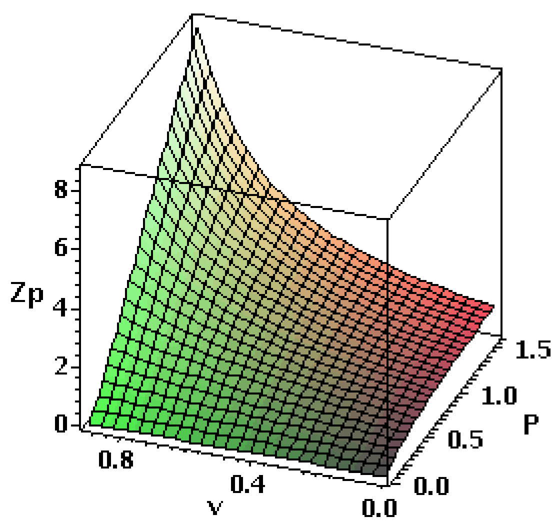
Figures:

Figure 1



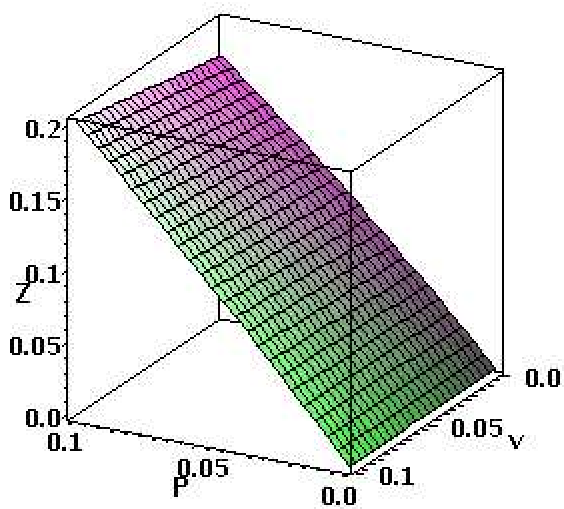
Distance dependence of velocity-independent red shifts. *P* is r' times the group parameter.

Figure 2



Dependence of total red shift on *P* and the velocity v'

Figure 3



Dependence of red shifts less than 0.2 upon *P* and v'

[END]