AA 218 – Virtual Exam 2
June 2020

20 points

Problem 1

1) (2 points) Show that the first order PDE

$$U_t + 6UU_x = 0$$

is invariant under a one-parameter dilation group and work out the infinitesimals of the group.

2) (2 points) Write down the invariance condition for this equation.

3) (2 points) The first derivative in t transforms infinitesimally according to

$$\tilde{U}_{\tilde{t}} = U_t + s\eta_{\{t\}}$$

Work out $\eta_{\{t\}}$ for the given group.

4) (2 points) The first derivative in x transforms infinitesimally according to

$$\tilde{U}_{\tilde{x}} = U_x + s\eta_{\{x\}}$$

Work out $\eta_{\{x\}}$ for the given group.

5) (2 points) Show that the invariance condition is satisfied by the group.

6) (3 points) Determine the invariants of the group and use these to generate similarity variables and reduce the equation to a first order ODE.

Problem 2

1) (4 points) Show that the nonlinear third order KdV equation

$$U_t + 6UU_x + U_{xxx} = 0$$

is also invariant under the group in problem 1.

2) (3 points) Use the group to generate similarity variables and reduce the PDE to a third-order ODE.