STANFORD UNIVERSITY B. Cantwell 3/31/20
Department of Aeronautics and Astronautics
AA218 - Introduction to Symmetry Analysis

Topics covered

1) Introduction to symmetry
2) Symmetry of functions, dimensional analysis
3) Review of ODEs, first-order PDEs, state-space analysis in 2-D and 3-D
4) Introduction to one-parameter Lie groups, examples of groups
5) Infinitesimal transformations, group operators, Lie series expansion of a function
6) **`IntroToSymmetry.m`** software package
7) Multi-parameter groups, Lie algebras
8) Application to first-order ODE's, integrating factors, differential functions
9) Extended groups, invariance of higher-order ODEs, reduction of order
10) Similarity variables for PDEs, reduction of dimension
11) Invariant groups of the classical equations of mathematical physics
12) Nonlocal groups, use of symmetries to generate solitary wave solutions
13) Noether's theorem and the connection between symmetries and conservation laws

Tentative list of examples.

1) Several examples from dimensional analysis
2) The two-body problem, symmetries in Kepler's laws
3) The geometry of light transmission through apertures
4) Laminar boundary layers
5) Sound propagation through a shear layer
6) Thermal gradient shocks in nonlinear heat conduction
7) Elliptic curves and flow patterns
8) Similarity rules for turbulent shear flows
9) Problems in nonlinear wave propagation, solitary waves
10) Flexural waves in a thin elastic beam

Grading - Homeworks will be assigned each week. Following guidelines set out by the Faculty Senate, grading will be on an S/NC basis. Substantial completion of the homeworks is needed to achieve a satisfactory grade. You will need to work problems to learn the methods of the course which do take practice to master. But during the second half of the course I will reduce the homework load to place more emphasis on a creation of your own making. I will suggest a number of possible projects but I am very open to your suggestions; it could be a problem derived from your research, or some other area of interest you might have from finance, biology, physics, chemistry, control theory, the grid, etc, Symmetry methods can be applied to virtually any field. This does not mean you have to solve some very complex problem. Your project might be exposition of an example from the literature where symmetry methods are applied. It could be something of current interest, such as say, the problem of viral spread or the problem of maintaining a supply chain in the face of disruption, etc.

e-Resources – If you need free access to journal articles through the Stanford system this link, https://library.stanford.edu/using/connecting-e-resources will help you set that up on your home computer.

Course material – Materials for all my courses are available at my website at https://web.stanford.edu/~cantwell/. The folder AA218_Course_Material includes a folder containing a pdf of the course text, *Introduction to Symmetry Analysis*, a folder with pdfs of the lectures, and a folder where homework assignments will be posted. In addition, selected papers related to the course material are also included in a Resources folder.

Software – There is a Mathematica package on my website that can be used to find the symmetries of ODEs and PDEs. It can be downloaded as a zip file or as individual files from: https://web.stanford.edu/~cantwell/SymmetryAnalysisSoftware. Instructions for use of the software can be found in Appendix 4 of the text. You should all be able to access the cluster computers at Stanford remotely using https://cluster-checkout.stanford.edu/. Both Mac and Windows systems should have Mathematica available (thanks to Eric Zelikman for that information).

Suggested prerequisites – Math 53 and Math 131 or equivalent background in differential equations. If you have questions or concerns, please contact me at cantwell.brianj@gmail.com.