# AA 218 – Not a Mid-Term Exam

May 12, 2020

20 points

**Problem 1 - 8 points**

Consider the transformation

$$\tilde{x} = x + s/2$$
$$\tilde{y} = y\left(1 + s/(2x)\right)^2$$

1) (2 points) Show by composition that the transformation is a Lie group.

2) (2 points) Determine the infinitesimals of the group.

3) (2 points) Determine the invariant of the group

4) (2 points) Determine an invariant family.

**Problem 2 - 8 points**

Consider the first order ODE

$$x^2 y_x - 2xy - y^2 = 0$$

1) (4 points) Show that the equation is invariant under the group in Problem 1.

2) (4 points) Use this group to construct an integrating factor for the equation.

**Problem 3 - 4 points**

Consider the second order ODE

$$x^2 y_{xx} - 4xy_x + 6y - y^2 = 0$$

1) (3 points) Show that the equation is invariant under the group in Problem 1.

2) (1 point) Based on inspection would you say the equation is solvable? If so, why?