

## AA 218 – Practice Exam 2

20 points

### Problem 1

1) (2 points) Show that the nonlinear first order PDE

$$U_x + U_y - (U_y)^2 = 0$$

is invariant under a one-parameter dilation group and work out the infinitesimals of the group.

2) (1 points) The first derivative in  $x$  transforms infinitesimally according to

$$\tilde{U}_{\tilde{x}} = U_x + s\eta_{\{x\}}$$

Work out  $\eta_{\{x\}}$  for the group.

3) (1 points) The first derivative in  $y$  transforms according to

$$\tilde{U}_{\tilde{y}} = U_y + s\eta_{\{y\}}$$

Work out  $\eta_{\{y\}}$  for the group.

4) (2 points) Write down the invariance condition for the equation and show that it is satisfied for the group.

5) (4 points) Determine the invariants of the group and use these to reduce the equation to a first order ODE.

### Problem 2

1) (2 points) Show that the nonlinear second order PDE

$$U_x + U_y - (U_y)^2 - xU_{yy} = 0$$

is also invariant under the group in problem 1.

2) (2 points) Work out the second extension  $\eta_{\{yy\}}$  for the group.

3) (2 points) Write down the invariance condition for this equation and show that it is satisfied.

4) (4 points) Use the group to reduce the PDE to a second-order ODE.