

## COURSE NOTES

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**Course Assistant:** Fushen Liu  
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Office Hours: To be announced

### References (Available in the Reserve Section):

- [1] R. I. Borja, *Plasticity Modeling and Computation*, Lecture Notes, 2007 (handed out in class).
- [2] Y. C. Fung, *A First Course in Continuum Mechanics*, Second Edition, Prentice-Hall, 1977.
- [3] R. Hill, *The Mathematical Theory of Plasticity*, Clarendon Press, Oxford Classic Series, 1998.
- [4] J.C. Simo and T.J.R. Hughes, *Computational Inelasticity*, Springer-Verlag, 1998.
- [5] C. S. Desai and H. J. Siriwardane, *Constitutive Laws for Engineering Materials*, Prentice-Hall, 1984.

### Notes:

1. The course outline on page 2 is tentative and subject to change.
2. The course deals with the basic aspects of inelastic material behavior from a modeling standpoint. We will present the mathematical theory of plasticity, covering classical and advanced plasticity models as well as the numerical implementation of these models. We will also discuss shear band-type instability problems and loss of uniqueness of the field equations.
3. Homework problem sets will be assigned on a regular basis and will typically be due one week later. Computing assignments will require some knowledge of Matlab, C, or Fortran.
4. There will be a one-hour midterm examination (May 14) and a take-home final examination. Weighting for the final grade is: Final Examination: 40%, Mid-Term Examination: 30%, Homework and Computing Assignments: 30%.

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## COURSE OUTLINE

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### 1. Introduction

The big picture; structure of a nonlinear FE program.

### 2. Plasticity in one dimension

One-dimensional nonlinear problems; basic numerical solution procedures; notion of isotropic and kinematic hardening in one dimension.

### 3. Deviatoric Plasticity

Deviatoric plasticity; flow rule; isotropic, kinematic, and combined hardening; plastic dissipation.

### 4. Integration Algorithm for Deviatoric Plasticity

Radial return algorithm; consistent tangent operator; introduction to general return mapping algorithm.

**Midterm Examination – May 14**

### 5. Two- and Three-Invariant Plasticity Models

Mohr-Coulomb, Drucker-Prager, cap models, critical state models; three-invariant models; return-mapping in principal directions; spectral decomposition.

### 6. Some Advanced Plasticity Models

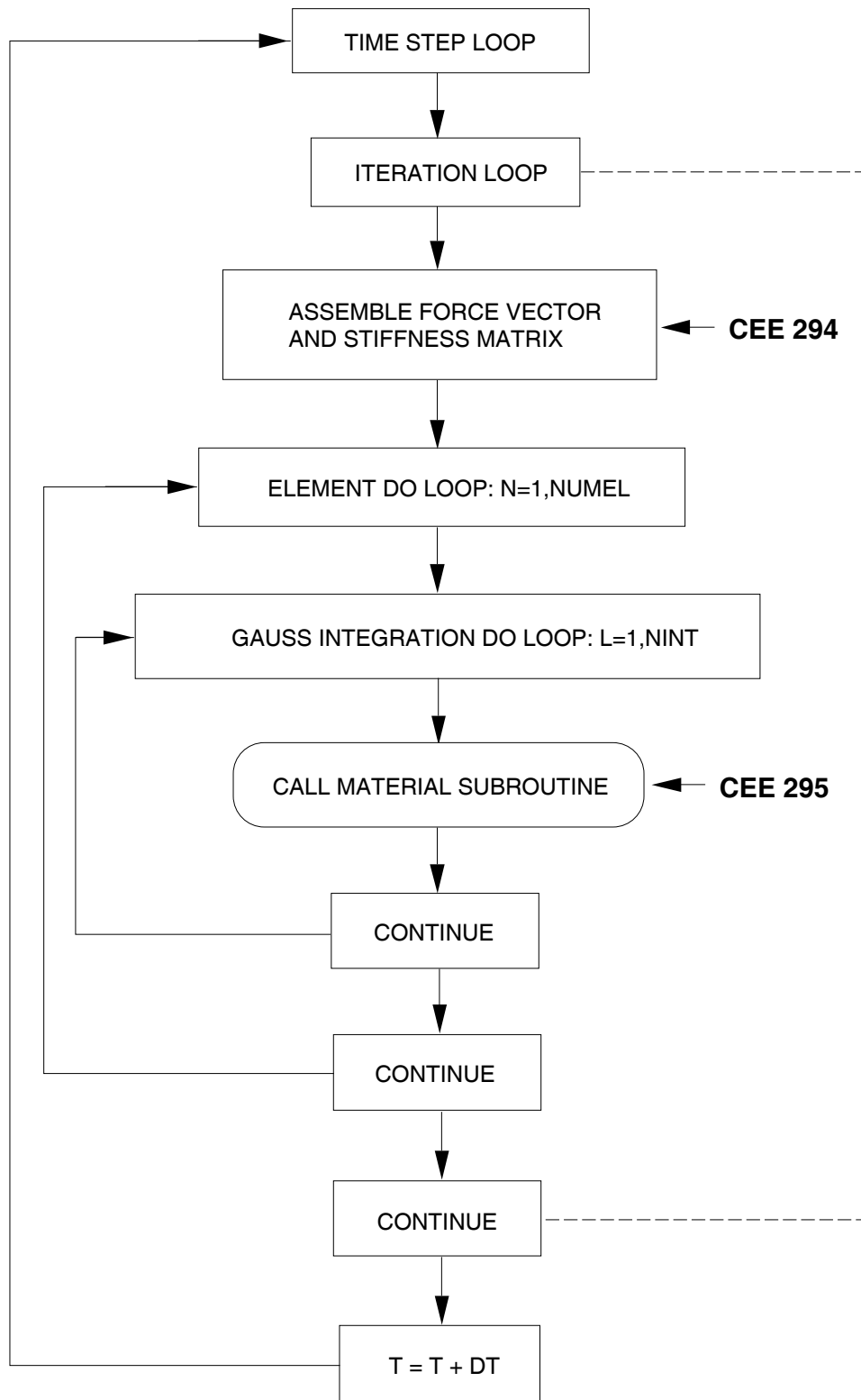
Nested yield surfaces; bounding surface plasticity; multi-surface plasticity

### 7. Material Instability and Bifurcation

Limitations of classical plasticity models; length scale; viscoplastic regularization; strain localization into deformation bands.

**Take-Home Final Examination**

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Structure of a time-domain nonlinear FE program.