Entropy and the Shannon Capacity of Queueing Systems

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I Summary

A number of results in the classical theory of point processes assert that the Poisson process is a "fixed point" for the operations of (1) random splitting, (2) independent superposition and (3) random translation of points. That is, one may begin with a Poisson process (or a finite number of independent Poisson processes) and as a result of performing each of the above operations obtain a Poisson process (or a finite number of independent Poisson processes).

Classical results in queueing theory also assert that the Poisson process is a fixed point for various queueing systems in the following sense: If the arrival process to such a queueing system is a Poisson process, then so is the equilibrium departure process. Examples of such queueing systems include the first-come-first-served (FCFS) exponential server queue (the M/M/1 queue), a queue which dispenses i.i.d. services with a general distribution and has either of the following service disciplines (1) last-come-first-served with pre-emptive resume (M/GI/1-LCFS) or (2) processor sharing (M/GI/1-PS), Jackson Networks, and others which incorporate traffic of different classes.

Further, it has been demonstrated that as a fixed point the Poisson process is an attractor: That is, the Poisson process results as the distributional limit of repeatedly subjecting an arbitrary stationary and ergodic point process to (most of) the above operations. See [2, 3, 4].

Since the Poisson process has maximum entropy among all processes of a given rate, one asks the following natural question: Are the operations mentioned above entropy increasing in nature?

In this paper we show that a variety of queueing systems are indeed entropy increasing in nature. The methods used to establish this are a combination of elementary techniques of information theory and queueing theory. We discuss the connection of these ideas with the recent work of Anantharam and Verdu [1] on the information capacity of queues.

References

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