**MS&E 319/CS 369X: Topics in Network Algorithms. Winter 2005-06**
Course URL: `http://www.stanford.edu/~ashishg/network-algorithms`.
Instructor: Ashish Goel, Stanford University.

**HW 1. Given February 26, 2006. Due 3/8/2006 before class.**

*Collaboration policy: You can discuss the problem with another student, but you must obtain and write the final solution by yourself. Also schedule an appointment with me to discuss your homework.*

1. In class, we discussed how buffers of size $B = O(\log Nd)$ suffice for obtaining a packet injection and forwarding schedule that took time $O(c + dB)$. What would the buffer requirement be if we wanted to use the same ideas to obtain a schedule that took time $(1 + \epsilon)c + O(dB)$?

2. Consider a rectilinear (i.e. Manhattan-like) road network where each intersection is a stop-sign. Assume that automobiles cross an intersection in FIFO order. Suppose an adversary can inject automobiles in the system over arbitrary routes and at arbitrary times, subject only to the $(w, r)$ constraints discussed in class where $r$ is the maximum injection rate through any stop sign and $w$ is the burst parameter. Assume that travel between two stop signs take time 0 and crossing a stop sign takes time 1. Design a simple set of streets and stop-sign intersections that result in instability for some $r < 1$ and $w$. Give the adversarial pattern of injections that causes this instability. Also think about the following questions, the answers to which are not obvious to me at this time and which are not part of this homework. If you make any progress, let me know.

   (a) Can you cause instability using the "one car from each direction" rule which is actually used in practice?

   (b) Is it possible to obtain a universally stable protocol for street-sign intersections which respects the constraint that cars cannot physically go through each other? Either give such a protocol or prove that none exists.

3. Draw as many parallels as you can between the work of Kelly, Maulloo, and Tan and the primal-dual rate allocation algorithms using the exponential cost metric (e.g.. the paper by Garg and Konemann). One possible direction is to look at the weighted proportional fairness condition in the former work and interpret it as the duality condition in the latter. You have the option of describing your answer to me verbally when you meet me, or of writing the answer down and submitting it.

4. Let $P_j^*$ denote the optimum value of the $j$-th prefix $P_j(x)$, subject to the constraints $x \in S, x \geq 0$, where $S$ is an arbitrary set.

   (a) Prove that the sequence $P_j^*/j$ is non-decreasing in $j$.

(b) Suppose now that the set $S$ is given by $\sum_i c_i x_i \leq 1$ where $c_i > 0$. Prove that the optimum solution for maximizing the prefix $P_j(x)$ is two-valued.

(c) Suggest an iterative primal-dual programming based approach to maximize the prefix $P_j^*$, using intuition from the paper by Garg and Konemann discussed in class. Do not prove or analyze the scheme.